Gabor Filter Using on Graph Regularized Non-negative Matrix Factorization for Face Recognition

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Abstract

Gabor Filter is a famous linear filter which is usually used for edge extraction. In this paper, we introduced Gabor Filter into Graph regularized Non-negative Matrix Factorization for face recognition. Thanks to the Gabor Filter, important information of the original picture can be highlighted so the recognition rate can be improved. We have performed a series of experiments on NMF algorithm with different Gabor Filter to exhibit the improvement on recognition rate. Next, we make some discussion on the accuracy of the neighbor graph construction method under different Gabor Filters. We also focus on the recognition rate of the Gabor Filter based GNMF algorithm with and without knowledge. Experiments demonstrate the effectiveness of our method.

Keywords: face recognition, manifold, Non-negative Matrix Factorization (NMF), Gabor Filter

1. Introduction

Face recognition is very challenging because of noise, illumination [1]-[2]. Nonnegative matrix factorization (NMF) is a widely-used method for low-rank approximation of a nonnegative matrix (matrix with only nonnegative entries), where nonnegative constraints are imposed on factor matrices in the decomposition. There are large bodies of past work on NMF [2].

NMF [3] was firstly proposed by Lee and Seung in Nature, and then they gave a detail discussion in NIPS [4]. Later a large number of variants have been proposed to improve NMF; most of them are focus on introducing an additional parameter that balances the reconstruction and other constraints, such as sparseness constraint [5], discriminant information [6] and manifold [7]. He et al.[8] proposed the famous Locality Preserving Projections algorithm (LPP) which introduced a nonlinear manifold embedded into original data space, later Cai et al.[7] proposed Graph regularized Non-negative Matrix Factorization (GNMF) which introduced manifold into NMF, GNMF construct an weight matrix \( W \) to encode the geometrical information and seek a matrix factorization which respects the graph structure.

The Gabor Filter function is a linear filter for edge detection. The frequency and direction of the Gabor filter are similar with the human visual system [9]-[10]. The study shows that Gabor filter is very suitable for texture expression and separation.

In this paper, we introduce the Gabor Filter into Graph regularized Non-negative Matrix Factorization for face recognition, and we will give a series of experiments on the effect of Gabor Filter, The remainder of this paper is organized as follows. In Section 2, we give a brief review of GNMF, in Section 3; Gabor Filter is introduced into GNMF; comparison and experiment are presented in Section 4; finally, we will give some concluding in Section 5.
2. An Introduction to GNMF

Cai et al.[7] proposed GNMF which introduced manifold into NMF. In this section, we give a brief introduction to GNMF.

Consider a graph with \( n \) vertices where each vertex corresponds to a data point. We construct one weight matrix \( W \) which encodes the geometrical information of the data \( W \). The detail of the construction method of \( W \) will be discussed in detail in Section 3.

By defining \( L = D - W \) where \( D \) is a diagonal matrix whose entries are column (or row, since \( W \) is symmetric) sums of \( W \), \( D_{ii} = \sum_j W_{ij} \). \( L \) is called graph Laplacian. By vectoring the original data point \( X_i \) onto the low-dimensional space \( V_i = \mathbb{R}^r \), the discrete approximation function of the smoothness can be computed as Eq. (1):

\[
\mathbf{g} = VLV^T
\]

So GPNMF aims to find the non-negative matrices \( U = [u_{ij}] \in \mathbb{R}^{n \times r} \), \( V = [v_{ij}] \in \mathbb{R}^{r \times m} \) which minimize the following objective function as Eq. (2):

\[
\min_{U \geq 0, V \geq 0} F = \|X - UV\|_F^2 + \lambda VLV^T
\]

where \( \| \cdot \|_F \) denotes the matrix Frobenius norm.

By introducing the Lagrange multiplier and the KKT conditions, the update rules can be deduced as Eq. (3):

\[
\begin{align*}
U_{ij} &= U_{ij} \cdot \frac{(XV^T)_{ij}}{(UVV^T)_{ij}} \\
V_{ij} &= V_{ij} \cdot \frac{(U^TX)_j + \lambda (VW)_j}{(U^TUV)_j + \lambda (VD)_j}
\end{align*}
\]

3. Gabor Filter based Graph Regularized Non-negative Matrix Factorization

In this section, we impose Gabor Filter into Graph regularized Non-negative Matrix Factorization, which will improve the recognition rate for face recognition.

3.1. Gabor Filter

Gabor Filter are conjointly optimal in providing the maximum possible resolution for information in both spatial and frequency domains. So Gabor Filter can serve as excellent band-pass filters for un-dimensional signals [11].

The kernels of Gabor filter have a real and an imaginary component representing orthogonal directions [12]. The two components may be formed into a complex number or used individually. The complex of the Gabor kernels is defined as Eq. (4):

\[
g(x, y; \lambda, \theta, \psi, \sigma, \gamma) = \exp(-\frac{x^2 + y^2}{2\sigma^2}) \exp[i(2\pi \frac{x'}{\lambda} + \psi)]
\]

The real part of the Gabor kernels is defined as Eq. (5):

\[
g(x, y; \lambda, \theta, \psi, \sigma, \gamma) = \exp(-\frac{x^2 + y^2}{2\sigma^2}) \cos(2\pi \frac{x'}{\lambda} + \psi)
\]

And the imaginary part of the kernels is defined as Eq. (6):

\[
g(x, y; \lambda, \theta, \psi, \sigma, \gamma) = \exp(-\frac{x^2 + y^2}{2\sigma^2}) \sin(2\pi \frac{x'}{\lambda} + \psi)
\]

Where
\[
\begin{align*}
x' &= x \cos \theta + y \sin \theta \\
y' &= -x \sin \theta + y \cos \theta
\end{align*}
\] (7)

In this equation, \( \lambda \) represents the scale of the sinusoidal factor which is considered as the scales of the wave length, \( \theta \) represents the orientation of the normal to the parallel stripes of a Gabor function, \( \psi \) is the phase offset, \( \sigma \) is the sigma/standard deviation of the Gaussian envelope and \( \gamma \) is the spatial aspect ratio, and specifies the elasticity of the support of the Gabor function.

Because the Gabor Filter has a real and an imaginary component, so we can choose the real part of the Gabor kernels (Filter) as the even (cosine) Gabor filter, denoted \( I'_R = g_R(I) \), where \( I \) denoted the gray level distribution of an image, \( I'_R \) denoted the result corresponding to the Gabor filter representation of the real part of Gabor kernel. We can also choose the imaginary part of the Gabor kernels as the odd (sine) Gabor Filter, denoted as \( I'_I = g_I(I) \), where \( I'_I \) denoted the result corresponding to the Gabor filter representation of the imaginary part of Gabor kernel.

Figure 1 shows the real part of the Gabor kernels at five scales (\( \lambda = \{1, 2, 3, 4, 5\} \)) and eight orientations (\( \theta = 0^\circ, 45^\circ, 90^\circ, 135^\circ, 180^\circ, 225^\circ, 270^\circ, 315^\circ \)) with the following parameters: \( \psi = 0 \), \( \sigma = \lambda / 2 \) and \( \gamma = 1 \). We can see that the gray block of each image become large with the scale (\( \lambda \)), different \( \theta \) makes different orientations. Figure 2 shows the imaginary part of the Gabor kernels (normalized within \( [0, 1] \) if necessary) at three scales (\( \lambda = \{3, 4, 5\} \)) and eight orientations (\( \theta = 0^\circ, 45^\circ, 90^\circ, 135^\circ, 180^\circ, 225^\circ, 270^\circ, 315^\circ \)) with its parameters the same as Figure 1.
3.2. The Neighbor Graph Construction Method

He et al.[8] and Cai et al.[7] gave us an easy way to construct the neighbor graph. According to the knowledge of classified information belong to the data set, there are two construction methods:

(1) Construction method with knowledge:
If we own the classified information belonging to the data set, we can construct the neighbor graph easily by simply setting \( W_{ij} = 1 \) if \( x_i \in N(x_j) \), otherwise \( W_{ij} = 0 \). \( x_i \in N(x_j) \) if \( x_i \) and \( x_j \) belong to the same class.

(2) Construction method without knowledge:
If we don’t own the classified information belonging to the data set, we can construct the neighbor graph by setting \( W_{ij} = \exp(-\frac{||x_i - x_j||^2}{lt}) \) if \( x_i \in N(x_j) \), otherwise \( W_{ij} = 0 \). \( W_{ij} = \exp(-\frac{||x_i - x_j||^2}{lt}) \) is the heat kernel function we defined. Because we don’t own the classified information, we need to ‘guess’ the neighbor information \( N(x_j) \) of \( x_i \). Usually, Euclidean distance is performed as \( D = \|x_i - x_j\|^2 \), and the construction method is as follow:

Step 1: we need to vector all the images, and then compute all the Euclidean distance between \( x_i \) and all the other the other images.

Step 2: we need to choose \( p \) images which are the closest to \( x_i \), (According to the ascending order of the Euclidean distance and choose the first \( p \) images).

Consider this two method, we can get an accurate neighbor graph if we own the classified information belonging to the data set, but usually, we don’t have the classified information, so we have to take use of the method (2) to construction the neighbor graph. But the Euclidean distance is sensitive to the noise and illumination, so we take use of Gabor Filter, expecting good performance.

4. Experiments

In this section, we will perform a series of experiments to show the improvements if we take use of Gabor Filter under different parameters. Our experiments were performed on the ORL database [13]. The nearest neighbor (NN) classifier was used for all face recognition experiments. Figure 3 shows one face image comes from ORL with its dimension as \( 112 \times 92 \), and the parameters of its corresponding image to the real part of Gabor kernel are set to different orientation \((\theta = 0^\circ, 45^\circ, 90^\circ, 135^\circ, 180^\circ, 225^\circ, 270^\circ, 315^\circ)\) and scale \((\lambda = [1, 2, 3, 4, 5])\), we set all the other parameters the same as Figure 1 and Figure 2. We can see that if the scale is small \((\lambda = 1)\), little change was occurred. But if the scale is large, we will lost much detail information, leaving those important information such as outline information, which will helpful to classify; we will also notice that if the orientation is not a right angle \((\theta = 45^\circ, 135^\circ, 225^\circ, 315^\circ)\), we will lost much more detail information, the reason is not concern.

Because there are two part of Gabor Filter: the real part and the imaginary part, so we will take use of three type of Gabor Filter: the even (cosine) Gabor filter; the odd (sine) Gabor Filter; And we can also choose both the real part (even Gabor filter) and the imaginary part (odd Gabor Filter) as the amplitude of the Gabor Filter, which is computed as \( I_A' = \sqrt{g_I'(I) + g_R'(I)} \), we will call \( I_A' \) as amplitude image. For each \( I'(I_R', I_I \text{ or } I_A') \) under the different scale and different orientation, we will compute the means of all
$I'$ under different orientation as $I_{\text{mean}}'$, and vector $I_{\text{mean}}'$ as the input of the NMF or GNMF.

Figure 3. The Corresponding Images to the Real Part of Gabor Kernel on One Face Image Comes from ORL with its Dimension as 112×92

Figure 4 shows the corresponding Gabor images of one image comes from ORL with its dimension as 112×92 under different scales ($\lambda = [1,2,3,4,5]$). The first row shows the mean even image, the second row shows the imaginary image and the third row shows the amplitude image. We can see that the even image will preserve much more detail information while the amplitude image will lost the most information (especially under large $\lambda$). We will perform a series of experiments to choose the best parameters of NMF and GNMF.

Figure 4. The Corresponding Mean Images to the Three Types of Gabor Kernel on One Face Image under Different Scales

4.1. Experiments of NMF with and without Gabor Filter

We chose a widely used ORL face image database [13] consisting of 40 people and each for 10 images, we choose all the 400 face images in our experiments, and all images in the database are aligned by the centers of eyes and mouth and the size of each image is 112×92. Examples of ORL database are shown in Figure 5.
In this section, comparison between NMFs with and without Gabor filter are performed, we set parameter $r$ for the NMF algorithm the same as the class number, so in these experiments we set $r = 40$ for ORL, we set the scales and the orientations of Gabor kernels the same as previous. We will randomly select train set and test set from each class, the train number $k$ for one class range from 5 to 9, all the experiments were performed 10 times, and the mean recognition rate are list in Table 1.

Table 1. The Mean Recognition Rates (%) of NMF for different Gabor Filter and Different Wavelength

<table>
<thead>
<tr>
<th>runfunction</th>
<th>Even Gabor NMF</th>
<th>Imaginary Gabor NMF</th>
</tr>
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<tbody>
<tr>
<td>$k$</td>
<td>$\lambda$</td>
<td>1</td>
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<tr>
<td>5</td>
<td>$91.6$</td>
<td>$93.4$</td>
</tr>
<tr>
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<td>$93.6$</td>
<td>$95.1$</td>
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<tr>
<td>7</td>
<td>$94.1$</td>
<td>$95.3$</td>
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<td>8</td>
<td>$95.3$</td>
<td>$96.3$</td>
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<tr>
<td>9</td>
<td>$96$</td>
<td>$96.3$</td>
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</tbody>
</table>

Table 1. The Mean Recognition Rates (%) of NMF for Different Gabor Filter and Different Wavelength (Continued)

<table>
<thead>
<tr>
<th>runfunction</th>
<th>Amplitude Gabor NMF</th>
<th>NMF</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k$</td>
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<tr>
<td>9</td>
<td>$95.3$</td>
<td>$96.3$</td>
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</tbody>
</table>

Table 1 shows the mean recognition rate for different Gabor Filter and different Wavelength under different train number. From Table 1, we can see that the recognition rate of NMF under even Gabor Filter perform best under the wavelength $\lambda = 2$; the recognition rate of NMF under Imaginary Gabor Filter perform best under the wavelength $\lambda = 3$; the recognition rate of NMF under Amplitude Gabor Filter perform best under the wavelength $\lambda = 3$. Usually, all three Gabor Filters perform better than NMF without Filter, but we also see that if the wavelength is unsuitable, (such as wavelength $\lambda = 4,5$ in Even Gabor Filter and wavelength $\lambda = 1,2,4,5$ in Imaginary Gabor Filter), the recognition rate would be even worse than NMF without Filter, we can see that for all the
wavelength ($\lambda = 1, 2, 3, 4, 5$), Amplitude Gabor Filter perform better than NMF, and for small wavelength ($\lambda = 1, 2, 3$), Even Gabor Filter performs also better.

4.2. Experiments of GNMF with Classified Information

In this experiment, we will perform experiments on GNMF, all three type of Gabor Filter are chosen, along with GNMF without Gabor Filter, to illustrate the improvement of Gabor Filter under different type and different parameters. All the experiments were performed 10 times, and the mean recognition rates are list in Table 2. Notice Gabor Filter will affect the construction of neighbor graph if without knowledge, so in this experiment, we choose the neighbor graph construction method with knowledge, and we will give some discuss on the effect of Gabor Filter on the construction of neighbor graph under different Gabor Filter.

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<tr>
<th>runfuncruntion</th>
<th>Even Gabor GNMF</th>
<th>Imaginary Gabor GNMF</th>
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<tbody>
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<td>$k$</td>
<td>$\lambda$</td>
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</tr>
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<tr>
<td>9</td>
<td>98.3</td>
<td>96.5</td>
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</table>

Table 2 shows the mean recognition rates of GNMFs with classified information for different Gabor Filter and different Wavelength under different train number. From table 2, we can see that the recognition rates of GNMF under even Gabor Filter perform best under the wavelength $\lambda = 1, 2, 3$, the recognition rate of NMF under Imaginary Gabor Filter with the wavelength $\lambda = 3$; the recognition rates of NMF under Amplitude Gabor Filter perform best under the wavelength $\lambda = 3$. Usually, all three Gabor Filters perform better than GNMF without Filter, but we also see that if the wavelength is unsuitable, (such as wavelength $\lambda = 4, 5$ in Even Gabor Filter and wavelength $\lambda = 1, 2, 4, 5$ in Imaginary Gabor Filter), the recognition rate would be even worse than GNMF without Filter, we can see that for all the wavelength ($\lambda = 1, 2, 3, 4, 5$). Amplitude Gabor Filter perform better than NMF, and for small wavelength ($\lambda = 1, 2, 3$), Even Gabor Filter performs also better.

4.3. Experiments of GNMF with different Neighbor Graph Construction Method
In this experiment, we will perform experiment on the accuracy of neighbor graph construction method with and without Gabor Filter; table 3 shows the accuracy of neighbor graph construction method, there are four benchmarks to measure the accuracy: true positive rate (TP), true negative rate (TN), false positive rate (FP), false negative rate (FN). We compute the true positive rate (TP) as: the right connections we get/the right connections; true negative rate (TN) as: the right connectionless we get/the right connectionless; the false positive rate (FP) as: the wrong connection we get/the right connections; the false negative rate (FN) as: the wrong connectionless we get/the connectionless. According the compute method, we expect large TP and TN, small FP and FN. We set all the parameters the same as pervious experiments, and we set the train number to 5. All the experiments were performed 10 times, and the mean accuracy are list in table 3.

### Table 3. The Mean Accuracy (%) of Neighbor Graph Construction Method for Different Gabor Filter and Different Wavelength

<table>
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<tr>
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<th>Even Gabor GNMF</th>
<th>Imaginary Gabor GNMF</th>
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<tbody>
<tr>
<td>( \lambda )</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>TP</td>
<td>64</td>
<td>61.8</td>
</tr>
<tr>
<td>TN</td>
<td>99.2</td>
<td>98.8</td>
</tr>
<tr>
<td>FP</td>
<td>0.8</td>
<td>1.1</td>
</tr>
<tr>
<td>FN</td>
<td>36</td>
<td>48.2</td>
</tr>
</tbody>
</table>

Table 3 shows the rates for neighbor construction method under different Gabor Filters and different parameters. According the compute method, we expect large TP and TN, small FP and FN. From table 3, we can see that the rates under even Gabor Filter perform best under the wavelength \( \lambda = 1 \), with the large TP and TN, small FP and FN; the rates under Imaginary Gabor Filter perform best under the wavelength \( \lambda = 3 \), but other parameters for wavelength perform even worse; all the wavelength perform better for the rates under Amplitude Gabor Filter, and wavelength \( \lambda = 3 \) perform best.

### 4.4. Experiments of GNMF without Classified Information

We have performed GNMF with classified information in Section 4.2; also make some discussion on the effect of neighbor graph construction method under different Gabor Filters in Section 4.3. So in this section, we will perform experiments of GNMF without classified information.
Table 4. The Mean Recognition Rates (%) of Gnmfs without Classified Information

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<th>Imaginary Gabor GNMF</th>
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Table 4. The Mean Recognition Rates (%) of Gnmfs without Classified Information (Continued)

<table>
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<td>93.5</td>
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Table 4 shows the mean recognition rate of GNMFs without classified information. From Table 4, we can see that the recognition rate of GNMF under even Gabor Filter perform best under the wavelength \( \lambda = 1 \); the recognition rate of NMF under Imaginary Gabor Filter perform best under the wavelength \( \lambda = 3 \); the recognition rate of NMF under Amplitude Gabor Filter perform best under the wavelength \( \lambda = 3 \). Usually, all three Gabor Filters perform better than GNMF without Filter, but we also see that if the wavelength is unsuitable, the recognition rate would be even worse than GNMF without Filter, we also notice that compared with NMF and GNMF with knowledge, GNMF without knowledge is sensitive with parameters, so we need select parameters ingeniously to get better performance.

5. Conclusion

In this paper, we introduced Gabor Filter into Graph regularized Non-negative Matrix Factorization for face recognition. There are three types of Gabor filters, such as: Even Gabor Filter, Imaginary Gabor Filter and Amplitude Gabor Filter. We have performed a series of experiments on NMF algorithm with different Gabor Filter to exhibit the improvement on recognition rate. Next, we make some discussion on the accuracy of the neighbor graph construction method under different Gabor Filters. We also focus on the recognition rate of the Gabor Filter based GNMF algorithm with and without knowledge. Experiments demonstrate the effectiveness of our method.

Acknowledgments

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References


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Yuhao Liu, He is working at Shanghai Maritime University, China. He received his Ph.D degree in 2014. His major research interests are Face Recognition.