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Frequency Tracking of Power Grid based on Phase Difference and UKF

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Abstract

For frequency tracking problem of non-stationary signals in power grid, we propose a unscented Kalman filter (UKF) and the weighted phase difference smoothing algorithm. It improved poor initial conditions sensitive situations of robustness UKF. In order to improve the accuracy of the algorithm, real-time and noise immunity, a suboptimal multiple fading factor and weighted smoothing method is added to improve UKF for frequency tracking of power signals. At last, The simulation results are given. It shows that this method has better accurate, real-time tracking of various frequency power signal and its performance is superior to similar documents tracking algorithm.

Keywords: *UKF algorithm; frequency tracking; weighted smoothing phase difference method; robustness*

1. Introduction

In the power system, the dynamic behavior of the frequency directly affects the security and stability of power quality and system [1]. In fact, the power signal is affected by various factors such as electromagnetic transient, non-linear devices, harmonic load, high-frequency or low-frequency electromagnetic interference in the real-time monitoring of the process, which will change the frequency of a certain time, a non-stationary signal. Real-time tracking frequency, to achieve the system running status assessment and control, has important applications [2-3].

Currently frequency tracking algorithm has been proposed mainly using zero-crossing detection, wavelet transform, fast Fourier transform, the minimum mean square error method, adaptive notch filter [2-6] and so on. Aspects of these algorithms in tracking speed, accuracy, computation, to achieve the degree of difficulty, suppress harmonics and noise immunity and other advantages and disadvantages, but generally it is difficult taking into account the needs of both real-time and accuracy. For this reason, some scholars have proposed algorithm which is simple in structure and has a certain frequency tracking accuracy of the Kalman filter algorithm to track the frequency of the signal and have achieved some results. But the system's initial state error is large or high degree of nonlinearity model, fixed set of parameters cannot respond [7]. The frequency of mutations severely affect the accuracy of filtering and even lead to filter divergence. Some scholars use Sigma Point Kalman filter algorithm to improve the accuracy of frequency tracking. But Sigma Point Kalman filtering accuracy susceptible to system noise, lack of adaptability, prone to non-sampling local effects, but to a

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certain extent, increased the complexity of the algorithm [8]. To enhance the stability of the Kalman filter algorithm, [9] introduced the robust factor, effectively enhance the stability of the system and improve the dynamic characteristics of the algorithm. Phase-locked loop and adaptive notch filter has better real-time [6, 10]. [11] proposed a quadrature phase-locked loop structure, improve the algorithm accuracy, on this basis, based on adaptive notch filter algorithm [12-13] to further simplify the algorithm implementation steps. These algorithms to meet different application environments corresponding requirements, but under abnormal conditions interfere with the grid suppression, much depends on the algorithm model itself, if the algorithm works in high-noise or high harmonic conditions, the output will be varying degrees of impact.

To solve these problems, this paper proposes a kind of unscented Kalman filter (UKF) and the weighted phase difference smoothing method. This algorithm to a certain extent, reduce the impact of the original signal frequency tracking process to ensure the accuracy of the tracking algorithm and improve the traditional UKF algorithm accuracy. It overcomes the shortcomings of traditional methods such as accuracy is not high, anti-interference ability is poor, unstable and even divergent and so on.

2. Algorithm Description

2.1. Weighted Phase Difference Smoothing

N observations complex sinusoidal signal superimposed complex Gaussian white noise can be expressed as [14]

$$s_k = Ae^{j(w_0 k+q)} + n_k, \quad k = 0,1,\mathrm{L}\ ,N-1 \tag{1}$$

where A is the amplitude; a signal frequency; phase are unknown constant; is the variance of the zero mean complex Gaussian white noise.

The formula (1) is rewritten as

$$s_k = Ae^{j(w_0 k+q)} + (1+\hat{n}_k), \quad k = 0,1,\mathrm{L}\ ,N-1 \tag{2}$$

where $\hat{n}_k = \frac{n_k}{A} e^{-j(w_0 k+q)}$, $\hat{n}_k$ is complex white Gaussian noise with the variance equals to s_n^2/A^2. Let $\hat{n}_k = u_k + ju_k$, we have

$$1+\hat{n}_k = \sqrt{(1+u_k)^2 + u_k^2}\, e^{j\tan^{-1}\left(\frac{u_k}{1+u_k}\right)} \tag{3}$$

When the SNR is large, that is, there is $s_n^2/A^2 = 1$, we get

$$1+\hat{n}_k \approx e^{j\tan^{-1}(u_k)} \approx e^{ju_k} \tag{4}$$

(2) can be rewritten as

$$s_k \approx Ae^{j(w_0 k+q+u_k)}, \quad k = 0,1,\mathrm{L}\ ,N-1 \tag{5}$$

Where u_k is real white Gaussian noise with the variance of $s_n^2/2A^2$. The argument s_k can be calculated by the formula (6) as follows

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$$f(x_k) = w_0 k + q + u_k, \quad k = 0,1,\Lambda, N-1 \tag{6}$$

Based on the formula (6), it introduces a weighted smoothing phase difference method that can reach a CR boundary where the phase difference is defined as follows

$$\Delta f(k) = f(x_{k+1}) - f(x_k)$$

That is

$$\Delta f(k) = w_0 + u_{k+1} - u_k, \quad k = 0,1,\Lambda, N-2 \tag{7}$$

From (7), it shows that the problem is converted to estimates the mean from colored Gaussian noise process w_0. The stochastic process equivalent to a sliding average model where coefficients are 1 and -1. Therefore, the maximum likelihood estimate of w_0 is equivalent to the minimum variance unbiased estimation of linear model, i. e., minimize J,

$$J = (\Delta f - w_0 l)^T \mathbf{C}^{-1} (\Delta f - w_0 l) \tag{8}$$

where $\Delta f = [\Delta f(0)\ \Delta f(1)\ \Lambda\ \Delta f(N-2)]$, $l = [1\,1\,\Lambda\,1]^T$, $\mathbf{C} \in \mathbf{R}^{(N-1)\times(N-1)}$ is covariance matrix of $\Delta f(k)$.

Frequency is estimated at

$$w_0 = \frac{l^T \mathbf{C}^{-1} \Delta f}{l^T \mathbf{C}^{-1} l} \tag{9}$$

where covariance matrix $\mathbf{C}$ is a three-diagonal matrix as follows

$$\mathbf{C} = \frac{s_n^2}{2A^2} \begin{bmatrix} 2 & -1 & 0 & 0 & \Lambda & 0 \\ -1 & 2 & -1 & 0 & \Lambda & 0 \\ \Lambda & \Lambda & \Lambda & \Lambda & \Lambda & \Lambda \\ 0 & 0 & 0 & -1 & \Lambda & 2 \end{bmatrix}$$

Then we have

$$\mathbf{C}^{-1}_{ij} = \frac{2A^2}{s_n^2} \left[\min(i, j) - \frac{i \cdot j}{N} \right], \quad 1 \le i,j \le N-1 \tag{10}$$

and

$$l^T \mathbf{C}^{-1} l = \frac{N(N^2-1)A^2}{6 s_n^2} \tag{11}$$

$$l^T \mathbf{C}^{-1} \Delta = \frac{N(N^2-1)A^2}{6 s_n^2} \sum_{k=0}^{N-2} y_k \Delta f(k) \tag{12}$$

where

$$y_k = \frac{\frac{3}{2}N}{N^2-1}$$

From equation (9), we yield

$$\hat{w}_0 = \sum_{k=0}^{N-2} y_k \Box f(k) \tag{13}$$

where $\sum_{k=0}^{N-2} y_k = 1$. $\hat{w}_0$ is unbiased estimates. Estimated variance is

$$\mathrm{var}(\hat{w}_0) = \frac{6}{\frac{A^2}{s_n^2} N(N^2-1)} \tag{14}$$

The estimated variance is exactly equal to CR. y_k is a window function on a point-symmetric $k = N/2-1$.

2.2. Multiple Fading Factor Adaptive UKF

Examine the system state equation and measurement equation satisfied

$$X(k) = \Phi(k-1)X(k-1) + \Gamma(k-1)W(k-1) \tag{15}$$

$$Z(k) = H(k)X(k) + V(k) \tag{16}$$

Where is $X(k) \in \mathbf{R}^n$ is state sequence, $\Phi(k) \in \mathbf{R}^{n' n}$ is transition matrix, $\Gamma(k) \in \mathbf{R}^{n' p}$ is system noise matrix, $Z(k) \in \mathbf{R}^m$ is measurement sequence, $H(k) \in \mathbf{R}^{m' n}$ is measurement matrix, $W(k) \in \mathbf{R}^p$ is system noise sequence. At the same time, $W(k)$ and $V(k)$ are not corrected to the Gaussian white noise with the statistical properties as follows

$$\begin{cases} E[W(k)] = 0, Cov[W(k), W(j)] = Q(k)\delta_{kj} \\ E[V(k)] = 0, Cov[V(k), V(j)] = R(k)\delta_{kj} \\ Cov[W(k), V(j)] = 0 \end{cases} \tag{17}$$

where δ_{kj} is $\mathrm{Kronecker} - \delta$ function, the covariance matrix $Q(k) \in \mathbf{R}^{m' n}$ is non-negative definite matrix and $R(k) \in \mathbf{R}^{m' m}$ is positive definite matrix.

The basic idea is to gradually eliminate errors caused by the filtering system model parameters or inaccurate due to noise equivalent to filter estimation error. By introducing a fading factor to increase predict the mean square error matrix, thereby increasing the filter gain matrix, and ultimately achieve increasing the use of newly measured data weighting purposes [15]. Among them, a typical adaptive fading filter equations are as follows

$$\hat{X}(k/k-1) = \Phi(k-1)\hat{X}(k-1) \tag{18}$$

$$P(k/k-1) = r(k)\Phi(k-1)P(k-1)\Phi(k-1)^T + \Gamma(k-1)Q(k-1)\Gamma(k-1)^T \tag{19}$$

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$$e(k) = Z(k) - H(k)\hat{X}(k/K-1) \tag{20}$$

$$K(k) = P(k/k-1)H(k)^T \left[H(k)P(k/k-1)H(k)^T + R(k)\right]^{-1} \tag{21}$$

$$\hat{X}(k) = \hat{X}(k/k-1) + K(k)e(k) \tag{22}$$

$$P(k) = \left[I - K(k)H(k)\right] P(k/k-1) \tag{23}$$

where

$$\begin{cases} r(k) = \max\left(1, \dfrac{1}{m}\bar{r}(k)\right) \\ \bar{r}(k) = tr\left\{\hat{C}(k) - N(k)\,M(k)^{-1}\right\} \end{cases} \tag{24}$$

$$N(k) = H(k)G(k-1)Q(k-1)G(k-1)^T H(k)^T + R(k) \tag{25}$$

$$M(k) = H(k)F(k-1)P(k-1)F(k-1)^T H(k)^T \tag{26}$$

where $r(k)$ is scalar fading factor, m is the dimension of sequence, $e(k)$ is new information sequence, and the theoretical value of covariance satisfies

$$\hat{C}(k) = E\left[e(k)e(k)^T\right] = H(k)P(k/k-1)H(k)^T + R(k) \tag{27}$$

$\hat{C}(k)$ is covariance estimates of new information, the new covariance estimates obtained by the fenestration of time is as follows [16]

$$\hat{C}(k) = \frac{1}{N}\sum_{j=0}^{N-1} e(k-1)e(k-1)^T \tag{28}$$

Where N is the length of the sliding window.

Compared with the conventional Kalman filter, the filter is fading in the formula (19) to increase the fading factor entry $r(k)$. Because $r(k) \geq 1$, $P(k-1)$ is improved $r(k)$ times, thereby expanding the filter gain matrix, That is an increase of newly measured data weight in the filtering estimation to achieve suppression filter divergence and improve filtering accuracy. It should be noted that in order to ensure $r(k) \geq 1$, let $r(k) = 1$, it degenerates to Kalman filter.

The basic principles of various fading filtering is the same, through the introduction of a fading factor to increase further predict the mean square error, thus improving filter data when the newly measured weight, so the difference between the respective fading filtering mainly reflected in the fading factor selection.

General fading filtering through the introduction of a scalar fading factor to regulate $P(k/k-1)$. Algorithm is simple, but lead to the same degree of regulation for each channel, reducing the filter estimation accuracy [17]. Since the estimated mean square error array diagonal elements characterize the corresponding state variable estimation accuracy and estimation accuracy of each state variable is different, this article is the mean square error matrix of this feature is introduced to replace the multiple fading factor based on the

estimated single scalar fading factor, in order to ensure that all channels have a different ability to regulate and improve the overall performance of the filter.

Let filtering estimated mean square error matrix $P(k)$ is as follow

$$P(k)=\begin{bmatrix} p(1,1) & & \\ & \ddots & \\ & & p(n,n) \end{bmatrix} \tag{29}$$

where $p(i,i)$ is estimation accuracy of the i-th state variables.

$\lambda(k)$ can be described as

$$\lambda(k)=\begin{bmatrix} \lambda_{1,1} & & \\ & \ddots & \\ & & \lambda_{n,n} \end{bmatrix} \tag{30}$$

where $\lambda_{i,i}$ is the fading factor corresponding to the i-th state variable

$$\lambda_{i,i} = c(i)\cdot r(k) \tag{31}$$

where $\lambda_{i,i} \geq 1$. $r(k)$ is the fading factor calculated by (19), it can be simplified to

$$\begin{cases} \bar{r}(k)= tr[N(k)]/tr[M(k)] \\ r(k)= \max(1,\bar{r}(k)) \end{cases} \tag{32}$$

$c(i)$ is the assigned weight factor by $\lambda_{i,i}$ satisfies

(1) $r(k)=1$,

$$c(i)=1 \tag{33}$$

(2) $r(k)>1$,

$$\begin{cases} \bar{c}(i)= p(i,i)\Big/\left(\frac{1}{n}\sum_{j=1}^{n} p(i,i)\right) \\ c(i)= \max(1,\bar{c}(i)) \end{cases} \tag{34}$$

So, Multiple fading factor $\lambda(k)$ can be written as

$$\lambda(k)= r(k)\begin{bmatrix} c(1) & & \\ & \ddots & \\ & & c(n) \end{bmatrix} \tag{35}$$

If $\lambda(k)=\mathbf{I}$, adaptive filtering based on multiple fading factor then degenerated into a conventional Kalman filtering. Step prediction covariance matrix is modified as

$$P(k/k-1) = \lambda(k)\Phi(k-1)P(k-1)\Phi(k-1)^T + \Gamma(k-1)Q(k-1)\Gamma(k-1)^T \tag{36}$$

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Figure 1 shows the adaptive UKF based on the weighted phase smoothing difference algorithm principle

Weighted phase difference signal	$\hat{\omega}_0 = \sum_{k=0}^{N-2} \psi_k \Delta\phi(k)$
Prediction	$\hat{X}(k/k-1) = \Phi(k-1)\hat{X}(k-1)$
Covariance Of new information	$\varepsilon(k) = Z(k) - H(k)\hat{X}(k/K-1)$ $\hat{C}(k) = E\left[\varepsilon(k)\varepsilon(k)^T\right] = H(k)P(k/k-1)H(k)^T + R(k)$ $N(k) = H(k)\Gamma(k-1)Q(k-1)\Gamma(k-1)^T \bullet H(k)^T + R(k)$
Fading factor update	$M(k) = H(k)\Phi(k-1)P(k-1)\Phi(k-1)^T H(k)^T$ $\rho(k) = tr\left[\hat{C}(k) - N(k)\right] / tr\left[M(k)\right]$ $c(i) = p(i,i) \Big/ \left\{\frac{1}{n}\sum_{j=1}^{n} p(i,i)\right\}$
Fading factor	$\lambda(k) = \rho(k) \bullet \begin{bmatrix} c(1) & & \\ & \ddots & \\ & & c(n) \end{bmatrix}$
Correct	$P(k/k-1) = \lambda(k)\Phi(k-1)P(k-1)\Phi(k-1)^T + \Gamma(k-1)Q(k-1)\Gamma(k-1)^T$
Filter Update	$K(k) = P(k/k-1)H(k)^T \bullet \left[H(k)P(k/k-1)H(k)^T + R(k)\right]^{-1}$ $\hat{X}(k) = \hat{X}(k/k-1) + K(k)\varepsilon(k)$ $P(k) = \left[1 - K(k)H(k)\right]P(k/k-1)$

Figure 1. Schematic Overall Algorithm

3. Simulation Results and Analysis

Without loss of generality, the actual power signal with the harmonic (three times the main consideration and 5th harmonic) and white noise, with the following general form

$$s(t) = 220\sqrt{2}\sin\left(2\pi\int_{t_0}^{t}(f_0 + \Delta f)dt + \varphi_1\right) + h_{n3}\sin\left(6\pi\int_{t_0}^{t}(f_0 + \Delta f)dt + \varphi_3\right)$$
$$+ h_{n5}\sin\left(10\pi\int_{t_0}^{t}(f_0 + \Delta f)dt + \varphi_5\right) + h_n w(t)$$

Where the frequency of power grid is $f = f_0 + \Delta f$, f_0 is the frequency of the fundamental wave and Δf is the frequency fluctuation. φ_1, φ_3, φ_5 are the fundamental, the third harmonic, fifth harmonic of the initial phase. $w(t)$ is zero mean and one variance of the Gaussian white noise. h_{n3}, h_{n5}, h_n are signal to noise ratio of the third harmonic, 5 harmonic relative amplitude and Gaussian white noise. In this paper, the following three models will be considered.

Model 1: fix frequency, $f = f_0 = 50\text{Hz}$.

Model 2: Slowly varying frequency, $f = f_0 + \Delta f = 50 + 0.5\sin(4\pi t)\text{Hz}$.

Model 3: fast varying frequency

$$f = \begin{cases} 50 & t \le 0.4\text{s} \\ 54 & 0.4\text{s} < t \le 0.6\text{s} \\ 48 & 0.6\text{s} < t \le 1\text{s} \end{cases}$$

Frequency signal model for 1,2,3, in relative amplitude harmonics in the simulation were set to 0.2 and 0.1, the Gaussian white noise signal to noise ratio is set to 30dB, weighted smoothing window is set to 20, the sampling time Ts = 0.001s.

For Model 1, the transient signal frequency tracking results is shown in Figure 2. When there is noise, although there is a slight fluctuation frequency tracking, the absolute maximum error is less than 0.0005Hz which is better than UKF. Compared with existing research results in the signal model with the same parameters, the literature [3, 18-19] tracking error is 0. 001 Hz, 0. 01 Hz, 0. 05 Hz. Thus, the algorithm of this work has better noise immunity and higher calculation accuracy in tracking constant frequency power signal with noise.

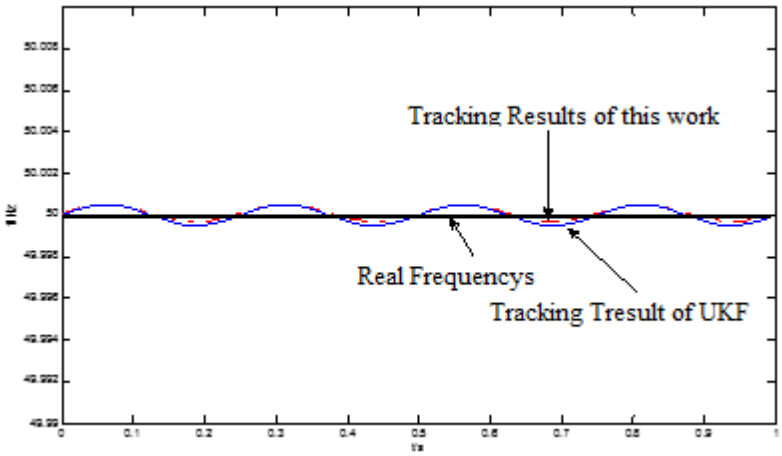

Figure 2. Tracking Results for Model 1

For signal frequency dynamics model 2 and model 3, track results shown in Figure 3 and Figure 4. Simulation results show that tracking varying frequency, the absolute error of frequency tracking is less than 0. 003 Hz, but at the same time it has the time-delay of about 0.025s. Under the same circumstances, in the literature [3,4] time-delay was 0. 06s and 0.05s,

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frequency tracking errors were 0.015Hz and 0.05Hz. Thus, the tracking accuracy of the proposed method is higher than the algorithm described in the literature.

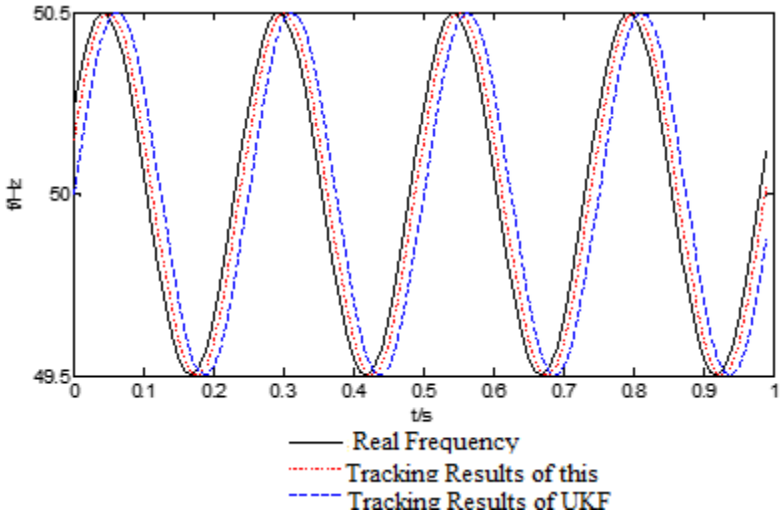

Figure 3. Tracking Results for Model 2

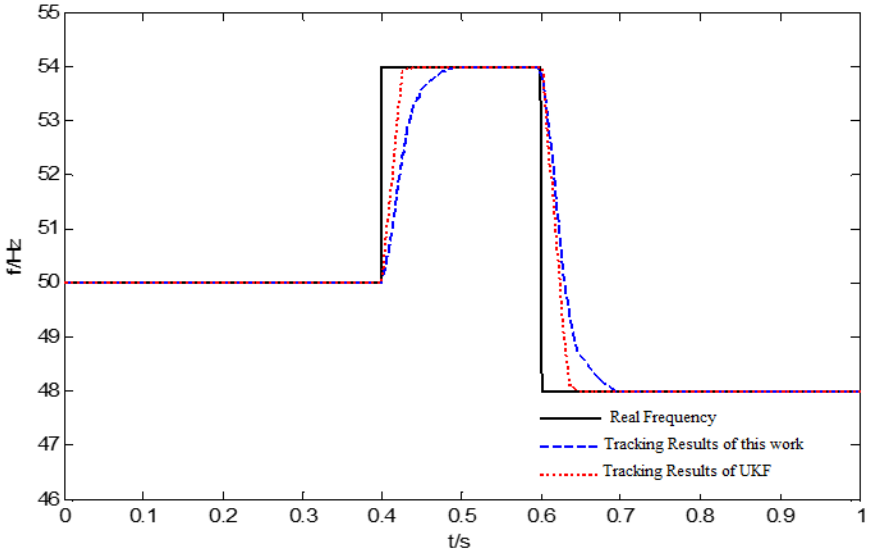

Figure 4. Tracking Results for Model 3

4. Conclusion

In this paper, the adaptive Kalman filter with multiple fading factor and a new covariance estimator is designed based on weighted phase difference smoothing. One step prediction mean square error matrix is adjusted online by the use of multi-factor fading, so as to achieve self-adjust the gain matrix and improve filtering accuracy. It filter out noise in the power signal and interference effectively, and greatly enhance real-time, accuracy and stability of the frequency tracking for power grid. Experimental results show that the algorithm can quickly track grid frequency.

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