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Research on Approaching Error Boundary of Variable Universe Adaptive Fuzzy Controller

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Abstract

To confirm the fuzzy rules satisfying given approaching error of variable universe adaptive fuzzy controller, this paper designs a universal approaching error boundary of adaptive fuzzy controller. With explicating the structure of variable universe adaptive fuzzy controller, the approaching error boundary of Single-Input and Single-Output or Multi-Input and Multi-Output fuzzy control system should be ascertained respectively for physical condition. The model simulation results indicate the better performance of the new method in comparison with the traditional controller with more accuracy and practicability.

Keywords: *Adaptive Fuzzy Control; Approaching Error Boundary; Variable Universe*

1. Introduction

For the accuracy of control, the more meticulous universe is and the more fuzzy rules are, the better control consequence is. Due to the restriction of objective term, it should possess finite fuzzy rules in practical operational process. On the engineering application point of view, the less rules are, the more convenient employment is. Above all, it's needful to gain a method on determining fuzzy rule library boundary [1-4].

Fuzzy control has been proved one kind of universal approach instrument in [5], which is indicated the extremity of controller approaching error should be decreased arbitrarily. In practical application, we wish to design a universe partition way for satisfying approaching precision in accordance with demand, and it's the work this article will accomplish.

The rest of this paper is structured as follows: next section we firstly analyse the structure of variable universe adaptive fuzzy controller. Section 3 designs the approaching error boundary of Single-Input and Single-output (SISO) adaptive fuzzy controller. The approaching error boundary of Multi-Input and Multi-output (MIMO) adaptive fuzzy controller will be discussed in Section 4.

2. Structure of Variable Universe Adaptive Fuzzy Controller

The structure of a variable universe adaptive fuzzy controller with input factor $\alpha_i(x_i^k)$ $(i=1,2,...,n)$ and output factor $\beta(y^k)$ is expressed in Figure 1 [5-8].

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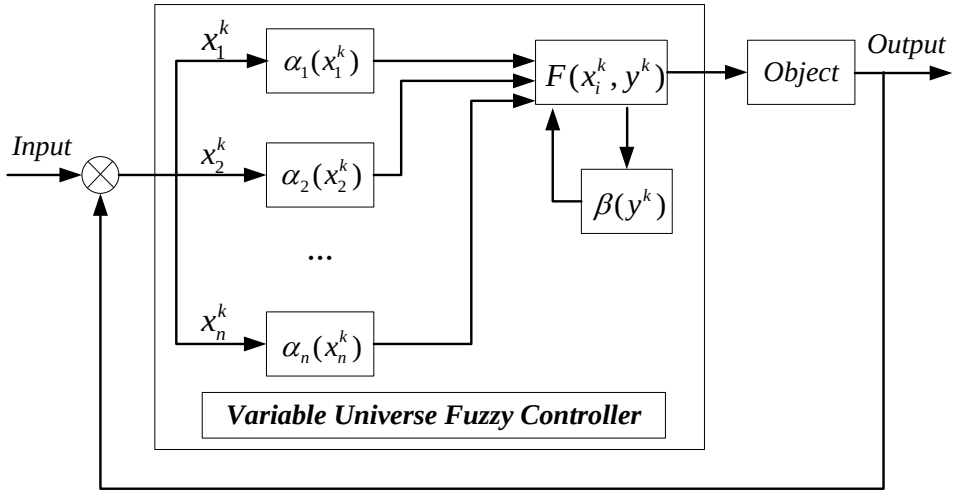

Figure 1. Variable Universe Fuzzy Controller

With the operation of $\alpha_i(x_i^k)$ and $\beta(y^k)$, the input variable universe U_i^k and the output variable universe V^k shrink or expand, and the shape of membership functions $\mu_{A_{ij}^0}$ becomes narrow or wide, as shown in Fig. 2 [9-18].

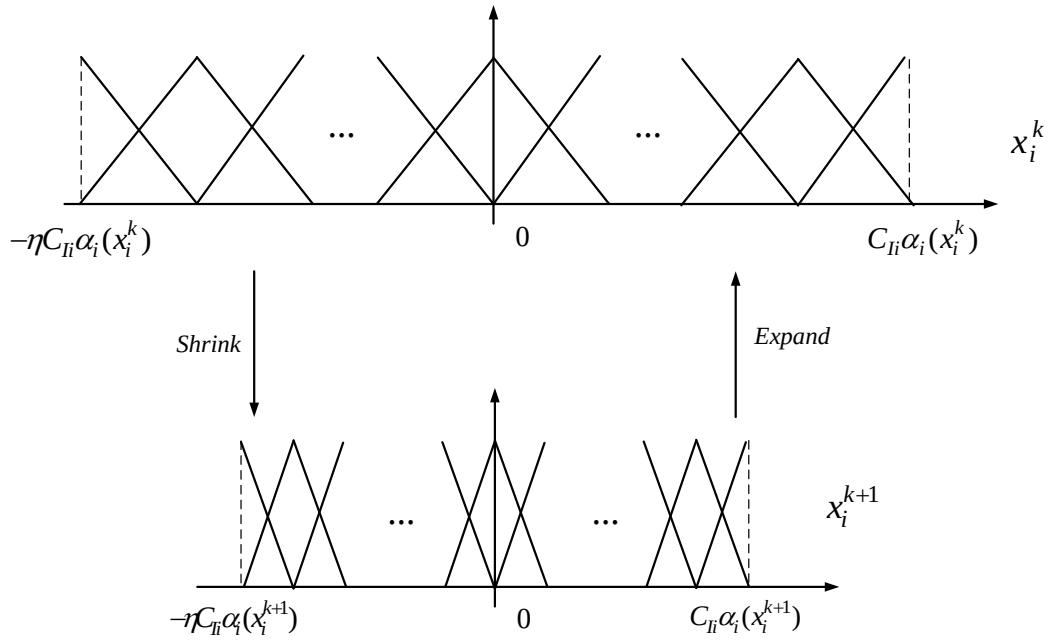

Figure 2. Shrinkage and Expansion of Universe

Suppose that $\bar{x}_{ij}^k$ and $\bar{y}_j^k$ are the centers of fuzzy sets A_{ij}^k and B_j^k respectively, the algorithm of variable universe fuzzy control is:

Step 0: $k=0$, input is $\boldsymbol{x}^0=(x_1^0, x_2^0, ..., x_n^0)$, according to fuzzy rules expression, the output of controller is:

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$$y^1 = f(\boldsymbol{x}^0) = \frac{\sum_{j=1}^{m} \bar{y}_j^0 \prod_{i=1}^{n} \mu_{A_{ij}^0}(x_i^0)}{\sum_{j=1}^{m} \prod_{i=1}^{n} \mu_{A_{ij}^0}(x_i^0)} \tag{1}$$

Step 1: $k=1$, input is $\boldsymbol{x}^1 = (x_1^1, x_2^1, ..., x_n^1)$, and $\bar{x}_{ij}^1 = \alpha(x_i^1)\bar{x}_{ij}^0$, $\bar{y}_j^1 = \beta(y^1)\bar{y}_j^0$, the output of controller is:

$$y^1 = f(\boldsymbol{x}^0) = \frac{\sum_{j=1}^{m} \bar{y}_j^0 \prod_{i=1}^{n} \mu_{A_{ij}^0}(x_i^0)}{\sum_{j=1}^{m} \prod_{i=1}^{n} \mu_{A_{ij}^0}(x_i^0)} \tag{2}$$

Step *k*: when *k*, input is $\boldsymbol{x}^k = (x_1^k, x_2^k, ..., x_n^k)$, and $\bar{x}_{ij}^k = \alpha(x_i^k)\bar{x}_{ij}^0$, $\bar{y}_j^k = \beta(y^k)\bar{y}_j^0$, the output of controller is:

$$y^{k+1} = f(\boldsymbol{x}^k) = \frac{\sum_{j=1}^{m} \bar{y}_j^k \prod_{i=1}^{n} \mu_{A_{ij}^k}(x_i^k)}{\sum_{j=1}^{m} \prod_{i=1}^{n} \mu_{A_{ij}^k}(x_i^k)} \tag{3}$$

and:

$$x_i^k \in A_{ij}^k \Leftrightarrow \frac{x_i^k}{\alpha_i(x_i^k)} \in A_{ij}^0 \tag{4}$$

$$\mu_{A_{ij}^k}(x_i^k) = \mu_{A_{ij}^0}\left(\frac{x_i^k}{\alpha_i(x_i^k)}\right) \tag{5}$$

meanwhile, $\bar{y}_j^k = \beta(y^k)\bar{y}_j^0$.

The final output of controller should be expressed as:

$$y^{k+1} = f(\boldsymbol{x}^k) = \beta(y^k) \frac{\sum_{j=1}^{m} \bar{y}_j^0 \prod_{i=1}^{n} \mu_{A_{ij}^0}\left(\frac{x_i^k}{\alpha_i(x_i^k)}\right)}{\sum_{j=1}^{m} \prod_{i=1}^{n} \mu_{A_{ij}^0}\left(\frac{x_i^k}{\alpha_i(x_i^k)}\right)} \tag{6}$$

3. Approaching Error Boundary of SISO Adaptive Fuzzy Controller

Suppose that fuzzy rules library of one fuzzy control system R_i: if x is A_i $(i = 1, 2, ..., N)$, then y is B, the output is:

$$y^{k+1} = f(\boldsymbol{x}^{\boldsymbol{k}}) = \beta(y^k) \frac{\sum_{j=1}^{N} y_j^0 \mu_{A_j}^0\left(\frac{x^k}{\alpha(x^k)}\right)}{\sum_{j=1}^{N} \mu_{A_j}^0\left(\frac{x^k}{\alpha(x^k)}\right)} = \frac{\sum_{j=1}^{N} y_j^k \mu_{A_j}^k\left(x^k\right)}{\sum_{j=1}^{N} \mu_{A_j}^k\left(x^k\right)} \tag{7}$$

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Theorem 1. Suppose $U^0 = \cup_{j=1}^{N-1}[x_j^0, x_{j+1}^0]$ is a compact set in $\boldsymbol{R}$, if $A_1^0, A_2^0, ..., A_N^0$ are the sets of complete states, ordered and consistent sets, $g(x)$ is a differentiable continuous function in U^0, the output $f(\boldsymbol{x}^k)$ as (7) satisfies:

$$\left\| g(\boldsymbol{x}^k) - f(\boldsymbol{x}^k) \right\|_\infty \le \frac{1}{8} \left\| \frac{\partial^2 g}{\partial x^2} \right\|_\infty [\alpha(x^0)]^2 h^2 \tag{8}$$

where $h = \max_{1 \le j \le N-1} | x_j^0 - x_{j+1}^0 |$.

Prove:

As $A_i^0 (i = 1, 2, ..., N)$ satisfies complete states, ordered and consistent for universe U^0, and $A_i^k (i = 1, 2, ..., N)$ are standard partitions of complete states, ordered and consistent for universe U^k:

$$y^{k+1} = f(\boldsymbol{x}^k) = \frac{\sum_{j=i}^{i+1} y_j^k \mu_{A_j}^k \left(x^k \right)}{\sum_{j=i}^{i+1} \mu_{A_j}^k \left(x^k \right)} \tag{9}$$

with regard to $\forall x^0 \in U^0 = \cup_{j=1}^{N-1}[x_j^0, x_{j+1}^0]$, by reason of

$$x^k = \alpha(x^k) x^0 \in \alpha(x^k) U^0 = \alpha(x^k) \cup_{j=1}^{N-1} [x_j^0, x_{j+1}^0] \tag{10}$$

and $x^k \in \cup_{j=1}^{N-1}[\alpha(x^k) x_j^0, \alpha(x^k) x_{j+1}^0]$.

In accordance with the monotone constraint of original rules, the contradiction should not be occurred among rules in universe shrink or expand course, so:

$$x^k \in \cup_{j=1}^{N-1}[\alpha(x_j^k) x_j^0, \alpha(x_{j+1}^k) x_{j+1}^0] = \cup_{j=1}^{N-1}[x_j^k, x_{j+1}^k] \tag{11}$$

and for $\forall x^k \in \cup_{j=1}^{N-1}[x_j^k, x_{j+1}^k]$, all $\mu_{A_j^k}(x_j^k) + \mu_{A_{j+1}^k}(x_{j+1}^k) = 1$, (9) should be shorthanded as:

$$y^{k+1} = f(\boldsymbol{x}^k) = \sum_{j=i}^{i+1} y_j^k \mu_{A_j}^k \left(x^k \right) \tag{12}$$

As Mean-Value Theorem:

$$f(\boldsymbol{x}^k) = \mu_{A_j}^k (x^k) g(x_j^k) + \mu_{A_{j+1}}^k (x^k) g(x_{j+1}^k) \tag{13}$$

Definition 1.

$$R_{temp}(x) = g(x) - f(x) \tag{14}$$

if $x^k = x_j^k$, then $\mu_{A_j}^k (x_j^k) = 1$, and $\mu_{A_{j+1}}^k (x_j^k) = 0$.

if $x^k = x_{j+1}^k$, then $\mu_{A_j}^k (x_{j+1}^k) = 0$, and $\mu_{A_{j+1}}^k (x_{j+1}^k) = 1$.

Above all $f(x_j^k) = g(x_j^k)$, $f(x_{j+1}^k) = g(x_{j+1}^k)$ and $R_{temp}(x_j^k) = R_{temp}(x_{j+1}^k) = 0$.

Definition 2.

$$R_1(x^k) = K(x^k)(x^k - x_j^k)(x^k - x_{j+1}^k) \tag{15}$$

where $K(x^k)$ is a 2nd differentiable continuous function in U^k.

Definition 3.

$$\phi(t) = g(t) - f(t) - K(x^k)(x^k - x_j^k)(x^k - x_{j+1}^k) \tag{16}$$

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where $t = x_j^k, t = x_{j+1}^k$ and $t = x^k$ are roots of $\phi(t) = 0$. As Mean-Value Theorem, $\phi''(\xi) = 0$ (where $\xi \in (x_j^k, x_{j+1}^k)$ is connected with x^k), so $K(x^k) = \frac{1}{2} g''(\xi)$:

$$R_1(x^k) = \frac{1}{2} g''(\xi)(x^k - x_j^k)(x^k - x_{j+1}^k) \tag{17}$$

The two sides of (17) execute infinite operation, and command $h = \max_{1 \le j \le N-1} |x_j^0 - x_{j+1}^0|$:

$$\left\| g(\boldsymbol{x}^{\boldsymbol{k}}) - f(\boldsymbol{x}^{\boldsymbol{k}}) \right\|_\infty \le \frac{1}{8} \left\| \frac{\partial^2 g}{\partial x^2} \right\|_\infty [\alpha(x^k)]^2 h^2 \tag{18}$$

Finally:

$$\left\| g(\boldsymbol{x}^{\boldsymbol{k}}) - f(\boldsymbol{x}^{\boldsymbol{k}}) \right\|_\infty \le \frac{1}{8} \left\| \frac{\partial^2 g}{\partial x^2} \right\|_\infty [\alpha(x^0)]^2 h^2 \tag{19}$$

4. Approaching Error Boundary of MIMO Adaptive Fuzzy Controller

Without loss of generality, double-input single output is discussed. Suppose that fuzzy rules library of one fuzzy control system R_i: if x_1 is A_{i_1} and x_2 is A_{i_2}, then y is B_i $(i = 1, 2, ..., N)$, the output of system is:

$$y^{k+1} = f(\boldsymbol{x}^{\boldsymbol{k}}) = \beta(y^k) \frac{\sum_{j=1}^{N} y_j^0 \prod_{i=1}^{2} \mu_{A_{ij}}^0 \left(\frac{x_i^k}{\alpha_i(x_i^k)} \right)}{\sum_{j=1}^{N} \prod_{i=1}^{2} \mu_{A_{ij}}^0 \left(\frac{x_i^k}{\alpha_i(x_i^k)} \right)} \tag{20}$$

Theorem 2. Suppose U^0 is a compact set in $\boldsymbol{R}^2$, if $A_{i1}^0, A_{i2}^0, ..., A_{iN}^0 (i = 1, 2)$ are the sets of complete states, ordered and consistent sets, $g(x)$ is a 2nd differentiable continuous function in U^0, the output $f(\boldsymbol{x}^k)$ as (20) satisfies:

$$\left\| g(\boldsymbol{x}^{\boldsymbol{k}}) - f(\boldsymbol{x}^{\boldsymbol{k}}) \right\|_\infty \le \frac{1}{8} \left[\left\| \frac{\partial^2 g}{\partial x_1^2} \right\|_\infty [\alpha_1(x_1^0)]^2 h_1^2 + \left\| \frac{\partial^2 g}{\partial x_2^2} \right\|_\infty [\alpha_2(x_2^0)]^2 h_2^2 \right] \tag{21}$$

where $h_i = \max_{1 \le j \le N-1} |x_{ij}^0 - x_{i(j+1)}^0|$.

Prove:

As $U^0 = U_1^0 \times U_2^0 = \cup_{j=1}^{N_1 - 1} [x_{1j}^0, x_{1(j+1)}^0] \times \cup_{j=1}^{N_2 - 1} [x_{2j}^0, x_{2(j+1)}^0]$,

$\forall x_i^0 \in U_i^0 = \cup_{j=1}^{N_i - 1} [x_{ij}^0, x_{i(j+1)}^0]$, so:

$$x_i^k = \alpha_i(x_i^k) x_i^0 \in \alpha_i(x_i^k) U_i^0 = \alpha_i(x_i^k) \cup_{j=1}^{N_i - 1} [x_{ij}^0, x_{i(j+1)}^0] \tag{22}$$

and $x_i^k \in \cup_{j=1}^{N_i - 1} [\alpha_i(x_i^k) x_{ij}^0, \alpha_i(x_i^k) x_{i(j+1)}^0]$.

On the basis of monotonicity,

$$x_i^k \in \cup_{j=1}^{N_i - 1} [\alpha_i(x_{ij}^k) x_{ij}^0, \alpha(x_{i(j+1)}^k) x_{i(j+1)}^0] = \cup_{j=1}^{N_i - 1} [x_{ij}^k, x_{i(j+1)}^k] \tag{23}$$

$A_i^k (i = 1, 2, ..., N)$ are standard partitions of complete states, ordered and consistent sets for universe U^k, and (22) should be written as:

$$y^{k+1} = f(\boldsymbol{x}^{\boldsymbol{k}}) = \beta(y^k)\frac{\sum_{j=1}^{i+!} y_j^0 \mu_{A_{1j}}^0\left(\frac{x_1^k}{\alpha_1(x_1^k)}\right)\mu_{A_{2j}}^0\left(\frac{x_2^k}{\alpha_2(x_2^k)}\right)}{\sum_{j=1}^{i+1} \mu_{A_{1j}}^0\left(\frac{x_1^k}{\alpha_1(x_1^k)}\right)\mu_{A_{2j}}^0\left(\frac{x_2^k}{\alpha_2(x_2^k)}\right)} \tag{24}$$

For $\forall x_i^k \in \cup_{j=1}^{N_i-1}[x_{ij}^k, x_{i(j+1)}^k]$, all $\sum_{j=1}^{i+1}\mu_{A_{1j}}^k\left(x_1^k\right)\mu_{A_{2j}}^k\left(x_2^k\right)=1$, so:

$$y^{k+1} = f(\boldsymbol{x}^{\boldsymbol{k}}) = \frac{\sum_{j=1}^{i+1} y_j^k \mu_{A_{1j}}^k\left(x_1^k\right)\mu_{A_{2j}}^k\left(x_2^k\right)}{\sum_{j=1}^{i+1} \mu_{A_{1j}}^k\left(x_1^k\right)\mu_{A_{2j}}^k\left(x_2^k\right)} \tag{25}$$

Shorthand:

$$f(\boldsymbol{x}^{\boldsymbol{k}}) = \sum_{j=1}^{i+1} y_j^k \mu_{A_{1j}}^k\left(x_1^k\right)\mu_{A_{2j}}^k\left(x_2^k\right) \tag{26}$$

As Mean-Value Theorem:

$$f(\boldsymbol{x}^{\boldsymbol{k}}) = \sum_{i=j}^{j+1} y_i^k \mu_{A_{1i}}^k(x_1^k)\mu_{A_{2i}}^k(x_2^k) g(x_{1i}^k, x_{2i}^k) \tag{27}$$

Definition 4.

$$R_2(x_1^k,\ x_2^k) = K_1(x_1^k)(x_1^k - x_{1j}^k)(x_1^k - x_{1(j+1)}^k) + K_2(x_2^k)(x_2^k - x_{2j}^k)(x_2^k - x_{2(j+1)}^k) \tag{28}$$

Definition 5.

$$\phi(s,t) = g(s,t) - f(s,t) - K_1(x_1^k)(s - x_{1j}^k)(s - x_{1(j+1)}^k) + K_2(x_2^k)(t - x_{2j}^k)(t - x_{2(j+1)}^k) \tag{29}$$

As Mean-Value Theorem:

$$K_1(x_1^k) = \frac{1}{2}\frac{\partial^2 g}{\partial x_1^2}\bigg|_{s=\xi, t=x_{2j}^k},\ K_2(x_2^k) = \frac{1}{2}\frac{\partial^2 g}{\partial x_2^2}\bigg|_{s=x_{1j}^k, t=\eta}$$

where $\xi \in (x_{1j}^k, x_{1(j+1)}^k)$ and $\eta \in (x_{2j}^k, x_{2(j+1)}^k)$ are connected with x_1^k, x_2^k.

$$\left\|g(\boldsymbol{x}^{\boldsymbol{k}}) - f(\boldsymbol{x}^{\boldsymbol{k}})\right\|_\infty \le \frac{1}{8}\left[\left\|\frac{\partial^2 g}{\partial x_1^2}\right\|_\infty [\alpha_1(x_1^k)]^2 h_1^2 + \left\|\frac{\partial^2 g}{\partial x_2^2}\right\|_\infty [\alpha_2(x_2^k)]^2 h_2^2\right] \tag{30}$$

$$\left\|g(\boldsymbol{x}^{\boldsymbol{k}}) - f(\boldsymbol{x}^{\boldsymbol{k}})\right\|_\infty \le \frac{1}{8}\left[\left\|\frac{\partial^2 g}{\partial x_1^2}\right\|_\infty [\alpha_1(x_1^0)]^2 h_1^2 + \left\|\frac{\partial^2 g}{\partial x_2^2}\right\|_\infty [\alpha_2(x_2^0)]^2 h_2^2\right] \tag{31}$$

Promote the conclusion for $n(n>2)$ dimension.

Theorem 3. Suppose U^0 is a compact set in $\boldsymbol{R}^n$, if $A_{i1}^0, A_{i2}^0, ..., A_{iN}^0 (i = 1, 2, ..., n)$ are the sets of complete states, ordered and consistent sets, $g(x)$ is a 2nd differentiable continuous function in U^0, the output $f(\boldsymbol{x}^k)$ as (7) satisfies:

$$\left\|g(\boldsymbol{x}^{\boldsymbol{k}}) - f(\boldsymbol{x}^{\boldsymbol{k}})\right\|_\infty \le \frac{1}{8}\sum_{i=1}^{n}\left\|\frac{\partial^2 g}{\partial x_i^2}\right\|_\infty [\alpha_i(x_i^0)]^2 h_i^2 \tag{32}$$

where $h_i = \max_{1\le j\le N-1} |\, x_{ij}^0 - x_{i(j+1)}^0 \,|$.

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5. Model Simulation

To improve the universe partition way for satisfying approaching precision in accordance with demand we proposed, we simulate a Single-Input and Double-Output asymmetric variable universe adaptive fuzzy landing control system as shown in Figure 3.

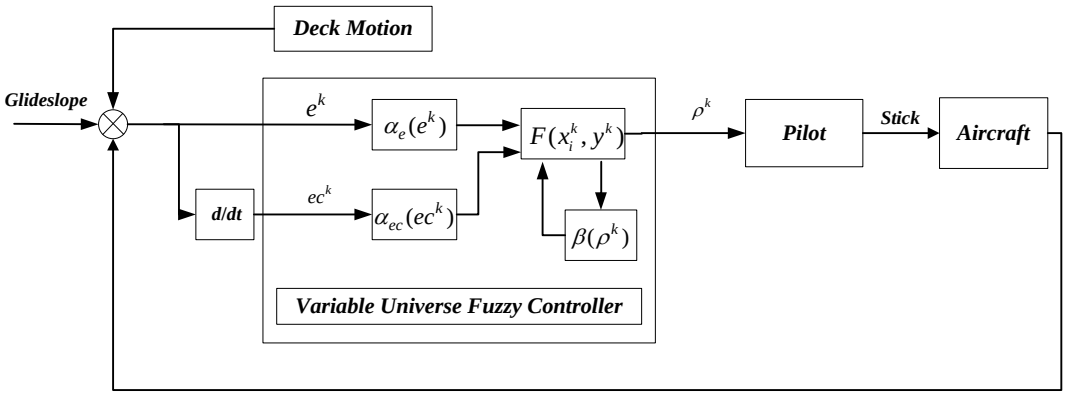

Figure 3. Variable Universe Fuzzy Trajectory Controller of Carrier-based Aircraft

The simulation conditions are designed as:

(1) The range from initial location to the point of ideal is 926m;

(2) The initial flight altitude of carrier-based aircraft is 66.67m with 10m deviation;

(3) The initial velocity of carrier-based aircraft is 69.96m/s;

(4) The initial angle of attack is 8.1°;

(5) The approaching error of system is $\varepsilon = 0.2\,\mathrm{m}$.

Fuzzy controller is defined as follow:

$$\rho^{k+1} = \beta(\rho^k)\frac{\sum_{j=1}^{36}\bar{\rho}_j^0 \mu_{A_{1j}^0}\left(\frac{e^k}{\alpha_e(e^k)}\right)\mu_{A_{2j}^0}\left(\frac{ec^k}{\alpha_{ec}(ec^k)}\right)}{\sum_{j=1}^{36}\mu_{A_{1j}^0}\left(\frac{e^k}{\alpha_e(e^k)}\right)\mu_{A_{2j}^0}\left(\frac{ec^k}{\alpha_{ec}(ec^k)}\right)} \tag{33}$$

Fig.4 is the response curves of flight path for different control patterns.

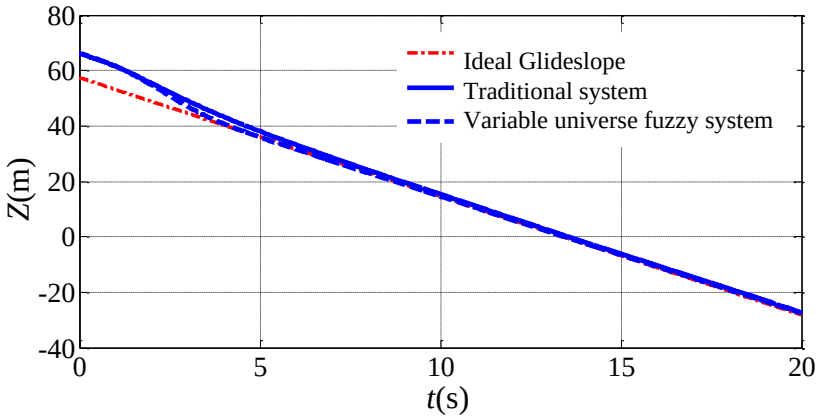

Figure 4. Response Curve of Flight Path

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From Figure 4, the approaching error of system is $\varepsilon = 0.2\,\mathrm{m}$ at the moment 3.8s under the variable universe adaptive fuzzy landing control system. It has the superiority complex on approximation error for the asymmetric variable universe adaptive fuzzy landing control system we designed.

6. Conclusion

This paper has presented an improved control manner for approaching error boundary of variable universe adaptive fuzzy controller. The simulation results show that compared with the traditional control manner, the new fuzzy method has better evaluation result for pilots, which means more accuracy and dynamic.

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