RAMS Analysis of Hybrid Redundancy System of Subsea Blowout Preventer Based on Stochastic Petri Nets

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Abstract

This paper presents the stochastic Petri net (SPN) model of the hybrid redundancy system. The capacity of fault detection is taken into account in the model. Besides, the method to perform reliability, availability, maintainability and safety (RAMS) analysis has been proposed based on the isomorphic Markov chain of the SPN model. The presented methodology is demonstrated by a case study of the output subsystem of the subsea blowout preventer. RAMS analysis of the presented case is conducted and the effects of the diagnostic rate and repair rate on the performance are studied. The results show that high diagnostic rate can improve the reliability, availability and safety of the system.

Keywords: *Subsea blowout preventer, Stochastic Petri nets, RAMS, Fault detection*

1. Introduction

Redundancy technique is widely used in various safety-critical systems with high reliability and availability [1]. It makes one or more duplicates of critical elements or functions of a system in order to improve the reliability. Hardware redundancy, software redundancy, time redundancy and information redundancy are four major forms of redundancy techniques. There are passive technique, active technique and hybrid approach in hardware redundancy [2]. Passive hardware redundancy achieves fault tolerance without any action and it does not detect a fault, but mask a fault. Triple modular redundancy (TMR) is the most common form of passive hardware redundancy, which employs majority voting. If any one of the three components fails, the other two components can correct and mask the fault. When there are two failures of the components, the redundant system fails. On the contrary, active hardware redundancy detects the existence of a fault, and then performs some action to remove faulty hardware from the system. So, it includes fault detection, fault location and fault recovery. Hybrid hardware redundancy is combination of passive and active techniques. It integrates voting scheme, fault detection and reconfiguration together. There are lots of methods to implement hybrid redundancy. However, its general form can be expressed as “N modular redundancy + standby redundancy” [3]. Although the reliability of the system can be improved by increasing the number of the standby modules, the cost and complexity of the system will also increase. Therefore, it’s important to find a balance between the reliability and the cost of the system [4].

The most common hybrid redundancy system is made up of a TMR system (including three working modules) and N standby modules. The standby modules are used to replace the faulty modules of TMR system, which is the core of the hybrid system. Figure 1 is a common structure of hybrid redundant system using TMR technique. It is mainly composed of three TMR modules and two standby modules. Each module is completely independent in this system. When the system is perfect, the voter performs 2 out of 3 voting. The detector compares each module with the voted result to find out if anyone is different from the others. Any faulty module in TMR system will be replaced by the standby module through the selector switch. After that, if another module fails, the remaining standby module will be employed. Therefore, reliability of the system can be improved by increasing the number of the standby modules.

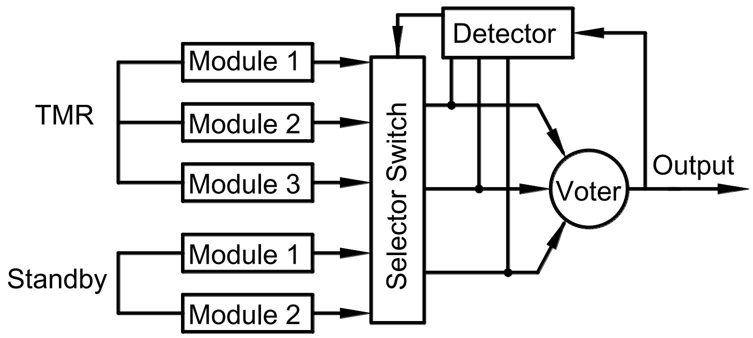

Figure 1. Architecture of the Hybrid Redundancy System

Because redundant systems have complex characteristics, such as parallelism, synchronization and resource sharing, Petri nets are suitable for modeling and analysis of redundant systems [5]. This paper presents a SPN model for the dependency analysis of hybrid redundancy system with imperfect detection. The paper is structured as follows. Section 2 describes the model formulation and evaluation in detail. Section 3 presents a case study of output subsystem of subsea blowout preventer control system. Section 4 summarizes the paper.

2. Model Formulation and Evaluation

2.1. Stochastic Petri Nets

The concept of Petri nets (PNs) was first introduced by Carl A. Petri [6]. Now, PNs become graphical and mathematical modeling tools for analysis of the static and dynamic systems. A Petri net (PN) is a 5-tuple [7], $PN = (P, T, F, W, M_0)$.

$P = (P_1, P_2, P_3, \cdots, P_m)$ is a finite set of places(drawn as circles).

$T = (T_1, T_2, T_3, \cdots, T_q)$ is a finite set of transitions(drawn as rectangles).

$F \subseteq (P \times T) \cup (T \times P)$ is a set of arcs. An arc connects a transition to a place or a place to a transition with a directed arrow.

$W : F \to N - \{\varnothing\}$ is a weight function.

$M_0 : P \to N$ is the initial marking, with $P \cap T = \varnothing, P \cup T \neq \varnothing$.

When an enabled transition is fired, appropriate tokens are removed from the input places to the output places. This flow can change the token distribution of PNs, leading to subsequent firing of other transitions.

A stochastic Petri net (SPN) is defined as a timed PN whose transition firing periods are exponentially distributed random variables [8]. The variable means that an enabled transition can fire after an exponentially distributed time delay. So, an SPN model is a 6-tuple, $SPN=(P,T,F,W,M_0,\lambda)$. The first five variables in the expression have the same meanings as those of a classical PN, $\lambda=(\lambda_1,\lambda_2,\lambda_3,\cdots,\lambda_{|T|})$ is a set of average firing rates of transitions [9]. Besides, each transition t_i corresponds to a constant firing rate λ_i. Because of the memoryless property of exponential distribution of firing delays, live and bounded SPNs are isomorphic to finite continuous-time Markov chains (MC) [10]. SPNs integrate the advantages of PNs with Markov processes. According to the reachable graph, the corresponding isomorphic MC can be extracted and the states of the Markov chain are the markings in the reachable graph [11, 12]. By solving the linear equations derived from the Markov chain, performance measures of the SPN models can be calculated. SPN is widely used in performance analysis of various areas such as railway station protection system [13], wireless opportunistic schedulers [14], ABR in ATM LANs [15] and reconfigurable manufacturing system [16], *et al*.

2.2. Modeling

Based on the operating principles of the hybrid redundancy system, its SPN model with partial detection is shown in Figure 2. Meanings of all places and transitions in the figure are as follows.

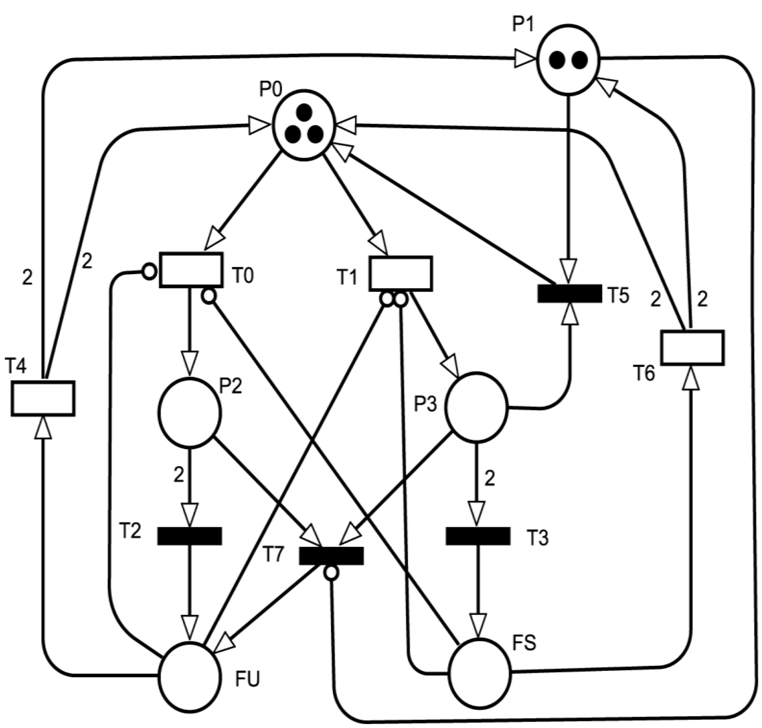

Figure 2. SPN Model of the System

The tokens in place P0 mean the number of the modules in working state. The number of standby modules is represented by the tokens in place P1. The detected failure fault and undetected fault are represented by place P3 and P2 respectively. FU means the state of the dangerous failure of the system, while FS is the state of safe failure. Transition T0 and T1 represents the occurrence of undetected fault and detected fault respectively. Transition T4 and T6 mean the repair actions. The immediate transition is represented by a narrow and filled rectangle. The unmarked weight of the arc is 1. The reachable graph of the SPN model is shown in Figure 3. The states caused by the immediate transitions have been deleted. According to the reachable graph, the Markov chain is shown in Figure 4.

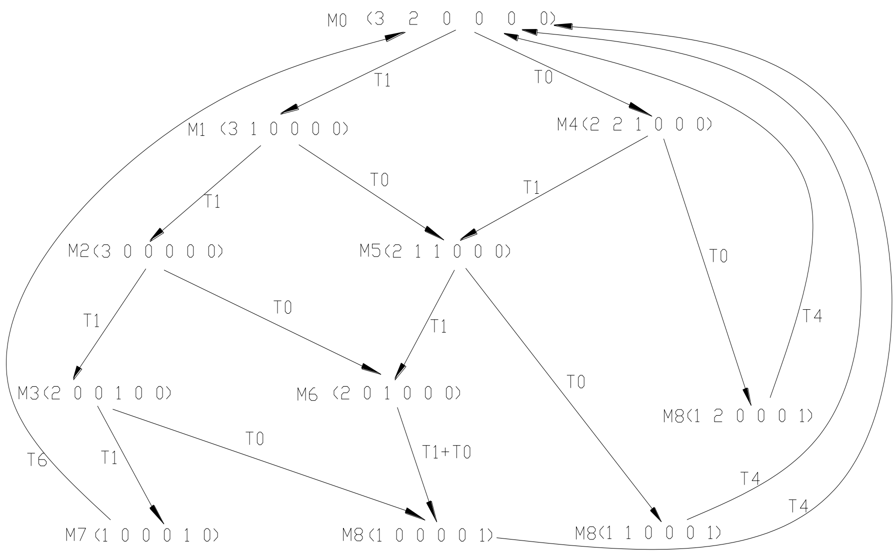

Figure 3. Reachable Graph of SPN Model

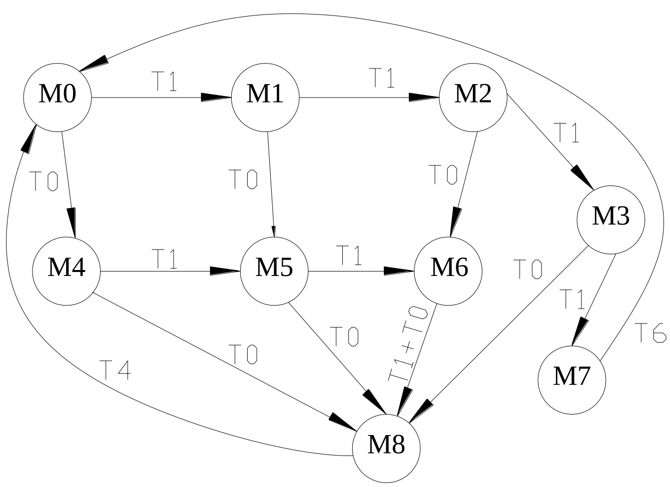

Figure 4. State Transition Graph of Markov Chain

Based on Figure 4, the state transition matrix Q can be obtained. Therefore, the dependency index like reliability, availability, maintainability and safety can be calculated according to the definitions in the next section.

$$Q=\begin{bmatrix} 1-3\lambda & 3c\lambda & 0 & 0 & 3(1-c)\lambda & 0 & 0 & 0 & 0 \\ 0 & 1-3\lambda & 3c\lambda & 0 & 0 & 3(1-c)\lambda & 0 & 0 & 0 \\ 0 & 0 & 1-3\lambda & 3c\lambda & 0 & 0 & 3(1-c)\lambda & 0 & 0 \\ 0 & 0 & 0 & 1-2\lambda & 0 & 0 & 0 & 2c\lambda & 2(1-c)\lambda \\ 0 & 0 & 0 & 0 & 1-2\lambda & 2c\lambda & 0 & 0 & 2(1-c)\lambda \\ 0 & 0 & 0 & 0 & 0 & 1-2\lambda & 2c\lambda & 0 & 2(1-c)\lambda \\ 0 & 0 & 0 & 0 & 0 & 0 & 1-2\lambda & 0 & 2c\lambda \\ \mu & 0 & 0 & 0 & 0 & 1-\mu & 0 & 0 & 0 \\ \mu & 0 & 0 & 0 & 0 & 0 & 1-\mu & 0 & 0 \end{bmatrix} \quad (1)$$

2.3. Dependency Analysis

Reliability is defined as the probability that a system or component can perform its required functions under stated conditions for a specified period of time. Mathematically, it is expressed as follows,

$$R(t)=\Pr\{T>t\}=\int_t^{\infty} f(x)dx \quad (2)$$

where $f(x)$ is the failure probability density function and t is the length of the period of time.

Availability is defined as the probability that the system is operating at a specified time t. The equation for availability is as a ratio of the expected value of the uptime of a system to the aggregate of the expected values of up and down time,

$$A=\frac{E[Uptime]}{E[Uptime]+E[Downtime]} \quad (3)$$

Maintainability is the probability that performing successful maintenance within a given time. That is, it describes the ease and speed with which a system can be restored to operational state after a failure. Its equation is shown as follows [17]:

$$M(t)=1-e^{-\mu t} \quad (4)$$

Safety means freedom from unacceptable risk of harm [18]. It is the probability of a system’s sate without dangerous failures. Therefore, in the system with fault detection system, when the fault is detected, it enters fail-safe state.

3. Case Study

Subsea Blowout Preventer (BOP) stack plays a critical role in providing safety during the drilling activities. Devastating losses could be caused by the failures of subsea BOP system. Therefore, extreme reliability is required for its control system. Redundant technique is widely used in the development of subsea BOP stack and its control system [19].

Output system of subsea BOP control system is applied to control all kinds of solenoid valves in hydraulic control system. The functions of subsea BOP stack is determined by the solenoid valves. In order to improve its reliability, output system of subsea BOP is developed based on the hybrid redundancy technique shown in Figure 1. As shown in Figure 1, the output of the voter is directly connected to the valves. SPN model of the output system is depicted in Figure 2. Here, the failure rate is chosen to be $\lambda = 3.27\mathrm{E}-06$ and the repair rate $\mu = 0.1$. The diagnostic coverage factor c is assumed to be 90%.

RAMS of the system are shown in Figure 5-Figure 8. As shown in Figure 5, reliability of the system decreases rapidly in the first 20000 hours and the values will be 0 in the end. Figure 6 shows that availability of the system decreases very slowly over time. According to Equation (4), maintainability of the system is not relevant to the diagnostic coverage factor. Therefore, the effects of diagnostic coverage factor on reliability, availability and safety are studied. It can be seen from the figures, increasing the diagnostic coverage factor can improve the performance of the system, which has the greatest influence on safety of the system. To improve safety of the system, the diagnostic coverage factor should be increased. As shown in Figure 7, maintainability increases as time goes by. The relationship between repair rate and maintainability demonstrates that if the repair rate is high, the maintainability is high too. Hence, in order to improve maintainability of the system, great efforts should be made to increase the repair rate. At last, the effects of repair rate on availability are also researched shown in Figure 9. Repair rate is very important to improve the availability of the system.

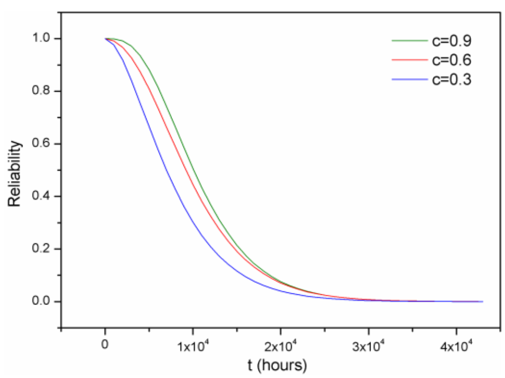

Figure 5. Reliability of the System

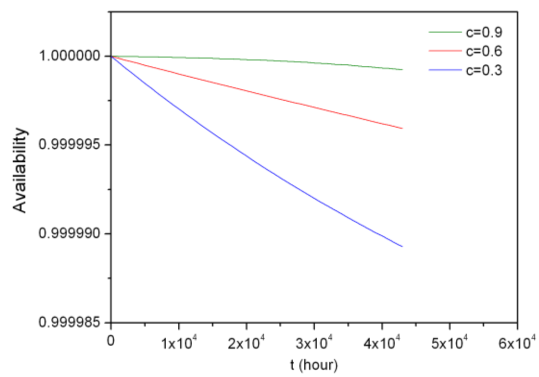

Figure 6. Availability of the System

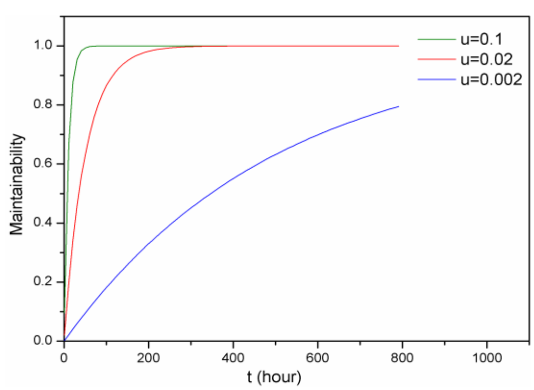

Figure 7. Maintainability of the System

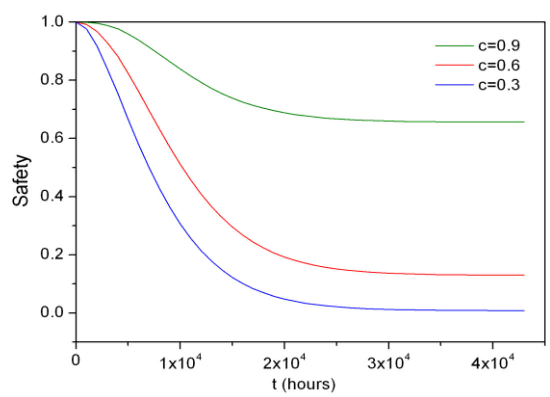

Figure 8. Safety of the System

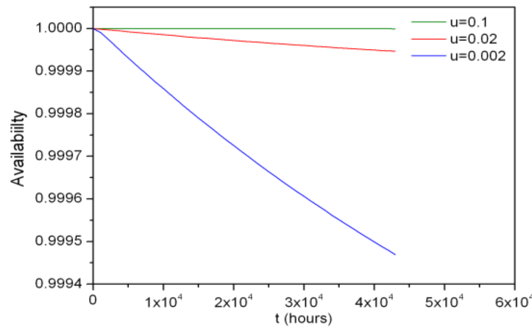

Figure 9. Effects of Repair Rate on the Availability of the System

4. Conclusions

In this paper, hybrid redundancy system with TMR is introduced and its SPN model is proposed. The Markov chain is obtained based on the reachable graph of the model. The methodology to evaluate the dependability of the system has also been presented. At last, a case study of the output subsystem of the subsea BOP control system is introduced. RAMS of the system are obtained based on the developed model. Because the diagnostic rate is not related to maintainability, its effects on reliability, availability and safety of the system are studied. The results show that high diagnostic rate can improve dependability of the system. The effects of repair rate on the availability demonstrate that increasing the repair rate can improve availability of the system.

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