Research On Efficient Turbo Frequency Domain Equalization In STBC-MIMO System

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Abstract

An efficient Turbo Frequency Domain Equalization (FDE) based on symbol-wise minimum mean-square error (MMSE) filtering is proposed for a novel space-time block code (STBC) MIMO system. The transmitter sends a separate data block via STBC using two antennas per group to get diversity gain. The receiver can effectively utilize inter-antenna interference (IAI) and inter-symbol interference (ISI) followed by frequency domain equalization to process soft interference cancellation (SIC). After frequency domain filtering, the symbol Log-likelihood ratio (LLRS) calculated from the outputs of equalizer is as the inputs of the soft-in soft-out (SISO) decoder. Simulation results show that our proposed scheme provides a further substantial gain while not increasing complexity at the receiver.

Keywords: *MIMO; MMSE; single carrier; STBC; Turbo FDE*

1. Introduction

Multiple-input multiple-output (MIMO) is well known for the benefit of improving the wireless link performance through diversity gains and capacity. The performance of MIMO system can be further enhanced when perfect or partial channel state information is made available [1]. Space-time block code (STBC) is a kind of orthogonal encoded mode which integrate time domain and space domain which can increase frequency spectrum efficiency and resist fading effectively [2, 3].

Single carrier modulation with frequency domain equalization (SC-FDE) and orthogonal frequency division multiplexing modulation (OFDM) have been introduced to avoid the inter-symbol interference (ISI) due to multipath propagation [4-6]. There are similar complexity and performance between SCFDE and OFDM. However, OFDM suffers from high peak-to-average power ratio (PAPR) and is sensitive to carrier frequency offset. Meanwhile SC-FDE is seen as a powerful candidate for future broadband wireless systems. And Turbo frequency domain equalization is considered as one of the important equalization schemes since iteration theory is introduced into equalization in which equalizer and decoder have been jointly optimized by interchanging extrinsic information with each other [7, 8].

In this paper, we develop an iterative detection and decoding algorithm of SC-FDE based on [9] for a novel STBCMIMO wireless system. The receiver can effectively utilize inter-antenna interference (IAI) and inter-symbol interference (ISI) followed by frequency domain equalization to process soft interference cancellation(SIC), and symbol Log-likelihood ratio (LLR) is calculated as the inputs of the soft-in soft-out (SISO) decoder using the outputs of equalizer. So it can realize iterative channel equalization and channel decoding at each iteration. Theory analysis and simulation results both show that our proposed algorithm can improve the system performance remarkably compared with general MIMO system.

The rest of the paper is organized as follows. In Section II, a novel STBC-MIMO model and Space-time block coded scheme are illustrated. Then the proposed SISO turbo STBC-MIMO FDE is presented in section III. Section IV discusses results of the performance obtained through analysis and simulation. Finally, the conclusion is drawn in Section V. It's worth noting that vectors are indicated in bold and scalar parameters in normal font. Superscripts $*$, T , H and -1 denote complex conjugate, transpose, conjugate transpose and inverse, respectively. F represents the N × N normalized Fast Fourier transform (FFT) matrix. The expectation and covariance are expressed as E {} and cov {}, respectively. The operator diag applied to a length i vector returns an i × i square matrix with the vector elements along the diagonal. Ii and 0i denote an i × i identity matrix, an all-zeros row vector of length i, respectively.

2. System Overview

Concentrate on a single carrier turbo frequency-domain equalizer in a novel STBC-MIMO system model as depicted in Figure 1 and Figure 2. At the transmitter, the incoming input bit stream is encoded by a forward error correction (FEC) code. Then each packet of m encoded and interleaved bits is mapped onto an M-ary complex symbol with variance σ_s^2 and which belongs to a signal set $S\{s_i \in S \in | i = 1, 2, \ldots M\}$ and S represents a symbol set. After mapping, data can be distributed on P streams and processed respectively as following encode principle (2). Subsequently, each transmitted block is preceded by a cyclic prefix (CP) and distributed onto two antennas. In the paper, there are 2P transmit antennas (2 antennas per stream) and Q receive antennas.

We define that $x_p^{(t)}$ (p =1, 2..., P ; t = v, v + 1; v =0, 2, ...) is the t^{th} block signal on the p^{th} stream before space-time block coding. After transferring each signal block into frequency domain by N-point Fast Fourier Transform (FFT), corresponding signal is $X_p^{(t)} \times x_{pi}^{(t)} (i=1,2)$ is the t^{th} block signal on p_i^{th} transmitted antenna(i^{th} antenna on the p^{th} stream) after being encoded according to STBC principle. Where,

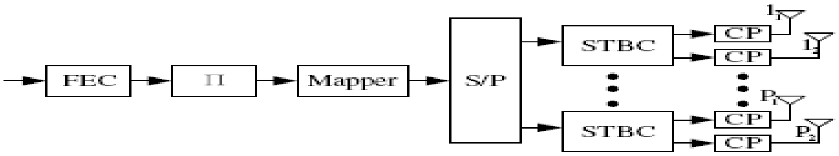

Figure 1. The Transmitter Structure

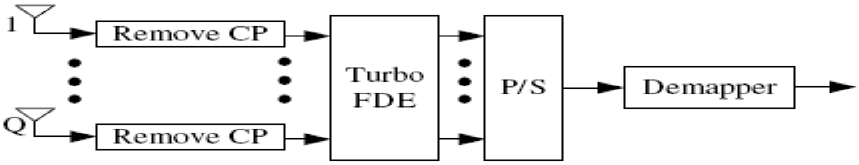

Figure 2. The Receiver Structure

$$\begin{cases} x_p^{(t)} = \left(x_p^{(t)}(1), x_p^{(t)}(2), \ldots x_p^{(t)}(N) \right)^T \\ X_{p_i}^{(t)} = \left(X_{p_i}^{(t)}(1), X_{p_i}^{(t)}(2), \ldots X_{p_i}^{(t)}(N) \right)^T \\ x_{p_i}^{(t)} = \left(x_{p_i}^{(t)}(1), x_{p_i}^{(t)}(2), \ldots x_{p_i}^{(t)}(N) \right)^T \end{cases} \quad \textbf{(1)}$$

Block-wise STBC principle is shown as follows[?],

$$\begin{cases} x_{p_1}^{(t)}(n) = x_p^{(t)}(n), x_{p_1}^{(t+1)}(n) = -x_p^{(t+1)^*}(n) \\ x_{p_2}^{(t)}(n) = x_p^{(t+1)}(n), x_{p_2}^{(t+1)}(n) = x_p^{(t)^*}(n) \end{cases} \quad \textbf{(2)}$$

At the receiver, after discarding the CP, the time domain signals on q^{th} antenna, for the t^{th} block can be expressed as $r_q^{(t)}$ (q=1,2,…,Q),and the corresponding received frequency domain signal is $R_q^{(t)}$ .Where,

$$\begin{cases} r_q^{(t)} = \left(r_q^{(t)}(1), r_q^{(t)}(2), \ldots r_q^{(t)}(N) \right)^T \\ R_q^{(t)} = \left(R_q^{(t)}(1), R_q^{(t)}(2), \ldots R_q^{(t)}(N) \right)^T \end{cases} \quad \textbf{(3)}$$

In order to express clearly system model ,we define

$$\begin{cases} x = \left[x_1^{(t)^T}, x_1^{(t+1)^T}, \ldots, x_P^{(t)^T}, x_P^{(t+1)^T} \right]^T \\ X(k) = \left(X_1^{(t)}(k), X_1^{(t+1)}(k), \ldots, X_P^{(t)}(k), X_P^{(t+1)}(k) \right)^T \\ R(k) = \left(R_1^{(t)}(k), R_1^{(t+1)^*}(k), \ldots, R_Q^{(t)}(k), R_Q^{(t+1)^*}(k) \right)^T \\ k = 1, 2 \ldots N \end{cases} \quad \textbf{(4)}$$

It is assumed that the communication channel is static over the observation interval (adjacent data both on t^{th} block and $(t+1)^{th}$ block) and perfectly known at both the transmitter and the receiver.

We define $\tilde{F}_{NUM} = \begin{pmatrix} F & & & \\ & F & & \\ & & \ddots & \\ & & & F \end{pmatrix}$,where the number of F is NUM, and F is the N×N unitary FFT matrix with $F_{(m,n)} = \frac{1}{N} e^{-i\frac{2\pi}{N}(m-1)(n-1)}$. Let $E_k = \left[e_k, e_k, \cdots, e_k \right]$, where e_k denotes the unit vector that can get k^{th} frequency domain signal at the k^{th} (k=0,…N-1) tone can be written as,

$$R(k) = H(k) E_k^H \tilde{F}_{2P} x + Z(k) = H(k) X(k) + Z(k) \quad \textbf{(5)}$$

Where,

$$H(k)=\begin{pmatrix} H_{1,1_1}(k) & H_{1,1_2}(k) & \cdots & H_{1,P_1}(k) & H_{1,P_2}(k) \\ H^*_{1,1_2}(k) & -H^*_{1,1_1}(k) & \cdots & H^*_{1,P_2}(k) & -H^*_{1,P_1}(k) \\ \vdots & \vdots & & \vdots & \vdots \\ H_{Q,1_1}(k) & H_{Q,1_2}(k) & \cdots & H_{Q,P_1}(k) & H_{Q,P_2}(k) \\ H^*_{Q,1_2}(k) & -H^*_{Q,1_1}(k) & \cdots & H^*_{Q,P_2}(k) & -H^*_{Q,P_1}(k) \end{pmatrix}$$

H_{q,p_i} is channel frequency impulse response at k^{th} frequency tone from the p_i^{th} transmit antenna to the q^{th} receive antenna. $Z(k)$ is the zero-mean complex AWGN sample with a variance of σ_z^2 at the k^{th} frequency tone.

3. Frequency Domain MMSE Turbo Equalization

In order to describe the algorithm clearly, $x=\left[x_1^{(t)^T}, x_1^{(t+1)^T}, \ldots, x_P^{(t)^T}, x_P^{(t+1)^T}\right]^T$ can be rewritten equally as $x=\left[x_1^T, x_2^T, \ldots, x_{2P}^T\right]^T$.

A. Soft ISI and IAI Cancellation

From (5), we can see that this FDE STBC-MIMO system can get frequency diversity together with multi-antenna diversity because each data symbol $x_i(n)$ is transmitted on every frequency tone according to the characteristic of Fast Fourier transform. At each iteration, prior information is fed back from SISO decoder to regenerate the expected interference for the desired symbol. The estimate of the desired symbol can be produced by a frequency domain MMSE filter after the ISI and IAI cancellation in the frequency domain. In the following, we assume the n^{th} symbol on the i^{th} ith antenna $x_i(n)$ $(i=1,2, \ldots 2P\ ;\ n=1, 2, \ldots N)$ is the desired symbol.

In the paper, the expected IAI and ISI for $x_i(n)$ can be presented as $\alpha_i(k)$ and $\beta_{i,n}(k)$, respectively,

$$\alpha_i(k)=H(k)E_k^H\tilde{F}_{2P}x_{\hat{i}} \qquad \textbf{(6)}$$

$$\beta_{i,n}(k)=H(k)E_k^H\tilde{F}_{2P}x_{\hat{i},\hat{n}} \qquad \textbf{(7)}$$

Where,

$$\begin{cases} x_{\hat{i}}=\left[\bar{x}_1^T,\cdots,\bar{x}_{i-1}^T,0^T,\bar{x}_{i+1}^T,\cdots,x_{2P}^T\right]^T \\ x_{\hat{i},\hat{n}}=\left[0^T,\cdots,0^T,\hat{x}_{\hat{i},\hat{n}}^T,0^T,0^T,\cdots 0^T\right]^T \\ \hat{x}_{\hat{i},\hat{n}}=\left[\bar{x}_i(1),\cdots\bar{x}_i(n-1),0,\bar{x}_i(n+1),\cdots\bar{x}_i(N)\right]^T \end{cases} \qquad \textbf{(8)}$$

$\bar{x}_j(n)$ denotes the priori expectation of $x_j(n)$ obtained from obtained from the extrinsic information passed by the decoder. According to (5), (6) and (7),after SIC, the signal in the frequency domain at k^{th} tone is written as,

$$\begin{aligned} Y_{i,n}(k) &= R(k) - \alpha_i(k) - \beta_{i,n}(k) \\ &= H(k) E_k^H \tilde{F}_{2P}\left(x - x_{\hat{i}} - x_{\hat{i},\hat{n}}\right) + Z(k) \\ &= H(k) E_k^H \tilde{F}_{2P}\left(x - \hat{x}_{i,n}\right) + Z(k) \end{aligned} \quad \textbf{(9)}$$

Considering all frequency tones, the soft interference cancellation model can be expressed as follows,

$$Y_{i,n} = R - \alpha_i - \beta_{i,n} = H\tilde{F}_{2P}\left(x - \hat{x}_{i,n}\right) + Z \quad \textbf{(10)}$$

After soft interference cancellation, signal to interference plus noise ratio (SINR) of signal $Y_{i,n}$ has been improved compared with original received data. Then frequency domain MMSE equalization is implemented, while soft interference is ignored in iteration zero since there is no priori information.

B. Efficient Frequency Domain MMSE Filtering

Symbol-wise MMSE criterion can be written as (11) in order to detect the desired symbol $x_i(n)$.

$$D_{i,n} = \arg\min_{D_{i,n}} E\left\{\left|D_{i,n}Y_{i,n} - x_i(n)\right|^2\right\} \quad \textbf{(11)}$$

According to the orthogonality principle, we have,

$$E\left\{\left(D_{i,n}Y_{i,n} - x_i(n)\right)Y_{i,n}^H\right\} = 0 \quad \textbf{(12)}$$

Then (10) is substituted into (11) with an assumption that there is no correlation between data symbols and AWGN.

$$\begin{aligned} &D_{i,n}H\tilde{F}_{2P}E\left\{\left(x - \hat{x}_{i,n}\right)\left(x - \hat{x}_{i,n}\right)^H\right\}\tilde{F}_{2P}H^H + \\ &D_{i,n}E\left\{ZZ^H\right\} - E\left\{x_i(n)\left(x - \hat{x}_{i,n}\right)\right\}\tilde{F}_{2P}^H H^H = 0 \end{aligned} \quad \textbf{(13)}$$

Meanwhile it's assumed that the symbols are independent, and the priori information about the desired symbol $x_i(n)$ should not be used in the evaluation, then we have,

$$\begin{aligned} &E\left\{x_i(n)\left(x - \hat{x}_{i,n}\right)^H\right\} = \Phi_{i,n} \\ &= \left[\underbrace{0, \cdots, 0}_{(i-1)N=n-1}, \sigma_S^2, \underbrace{0, \cdots, 0}_{(2P-i+1)N-n}\right] = \sigma_S^2 e_{i,n}^H \end{aligned} \quad \textbf{(14)}$$

$$E\left\{\left(x-\hat{x}_{i,n}\right)\left(x-\hat{x}_{i,n}\right)^{H}\right\}=\Gamma_{i,n}=diag\left\{\Upsilon_{1,}\Upsilon_{2,}\cdots,\Upsilon_{i-1,}\Upsilon_{i,}\Upsilon_{i+1,}\Upsilon_{2P}\right\} \quad \textbf{(15)}$$

where $\Upsilon_{j}=diag\left\{\Upsilon_{j}^{2}(1)_{,}\ \Upsilon_{j}^{2}(2)_{,}\cdots,\ \Upsilon_{j}^{2}(N)\right\}(j\neq i)$,and $\Upsilon_{j}^{2}(m)$ is variance of symbol x_{j} on the basis of prior information from decoder.

$\tau_{i}=diag\left\{\Upsilon_{i}^{2}(1)_{,}\ \Upsilon_{i}^{2}(2)_{,}\cdots,\ \Upsilon_{i}^{2}(n-1)_{,}\ \sigma_{S}^{2},\Upsilon_{i}^{2}(n+1)_{,}\cdots\Upsilon_{i}^{2}(N)\right\}$ and σ_{S}^{2} is the symbol energy.

$$\Upsilon_{j}^{2}(m)=\mathrm{cov}\left(x_{j}^{m},x_{j}^{m}\right) \quad \textbf{(16)}$$

Finally, the optimal symbol-wise frequency domain MMSE filtering weight coefficients are obtained,

$$D_{i,n}=\Phi_{i,n}\tilde{F}_{2P}^{H}H^{H}\left(H\tilde{F}_{2P}\Gamma_{i,n}\tilde{F}_{2P}^{H}H^{H}+\sigma_{S}^{2}I_{2Q*N}\right)^{-1} \quad \textbf{(17)}$$

C. Extrinsic LLR Calculation

After equalization, the estimate of time domain symbol $\mathrm{x}_{\mathrm{i}}(\mathrm{n})$ can be obtained by IFFT,

$$\begin{aligned}\hat{x}_{i}(n)&=D_{i,n}Y_{i,n}=D_{i,n}\left(H\tilde{F}_{2P}\left(x-\hat{x}_{i,n}\right)+Z\right)\\&=D_{i,n}\left(H\tilde{F}_{2P}\left(e_{i,n}x_{i}(n)+\hat{x}_{i,n}'-\hat{x}_{i,n}\right)\right)D_{i,n}Z\end{aligned} \quad \textbf{(18)}$$

In (18), we can see that the first term is the expected symbol multiplied by a factor, the second term is the residual interference from other antennas and symbols, the third term is AWGN. As the iteration continue, the prior information becomes more and more exact. So an assumption is made in [9] that the output of MMSE equalizer has undergone a equivalent Gaussian channel,

$$\hat{x}_{i}(n)=\phi_{i}(n)x_{i}(n)+\lambda_{i}(n) \quad \textbf{(19)}$$

where $\phi_{i}(n)$ is the equivalent amplitude of the signal at the output and $\lambda_{i}(n)$ is a complex white Gaussian noise with zero mean and variance $\sigma_{i}^{2}(n)$.

Then the soft-input soft-output decoder can utilize extrinsic information from equalizer which is treated as the prior information to calculate extrinsic LLR according to the expectation and variance of equalized data symbol. As described in [11], expectation $\mu_{i}(n)\ x_{i}(n)$ can be computed,

$$\mu_{i}(n)=D_{i,n}H\tilde{F}_{2P}e_{i,n}\sigma_{S}^{2} \quad \textbf{(20)}$$

Variance $\sigma_{i}^{2}(n)$ is written as [9],

$$\sigma_{i}^{2}(n)=\left|renew\left(\hat{x}_{i}(n)\right)\mu_{i}(n)-\hat{x}_{i}(n)\right|^{2} \quad \textbf{(21)}$$

The function of renew(.) is to renew $x_i(n)$ as a original modulated symbol. Then extrinsic LLR for the symbol can be obtained,

$$Le\left(x_i(n)\right) = \frac{2\hat{x}_i(n)\mu_i(n)}{\sigma_i^2(n)} \qquad \textbf{(22)}$$

D. Low Complexity Implementation

As the equalization is processed based on symbol-wise, it's hard to implement due to the complexity. Considering that the diagonal elements of the frequency domain covariance matrix $\Theta_{i,n} = \tilde{F}_{2P}\Gamma_{i,n}\tilde{F}_{2P}^H$ in (17) are constant with the same value,

$$\omega_j = \frac{1}{N}\left[\sum_{n=1}^{N}\gamma_j^2(n)\right](j \neq i) \qquad \textbf{(23)}$$

The off-diagonal elements can be ignored, because the diagonal elements is larger than the off-diagonal elements. Therefore, we approximate,

$$\omega_i = \frac{1}{N}\left[\sum_{n=1}^{N}\gamma_i^2(n)\right] \qquad \textbf{(24)}$$

Then

$$\nu = \frac{1}{N}\left[(N-1)\omega_i + \sigma_s^2\right] \qquad \textbf{(25)}$$

$$\Theta_{i,n} \approx \Theta_i = diag\left\{\omega_1 I_N, \cdots \omega_{i-1} I_N, \nu I_N, \omega_{i+1} I_N, \cdots \omega_N I_N\right\} \qquad \textbf{(26)}$$

Accordingly equalizer coefficients are given by,

$$D_{i,n} = \Phi_{i,n}\tilde{F}_{2P}^H H^H \left(H\Theta_i H^H + \sigma_s^2 I_{2Q*N}\right)^{-1} \qquad \textbf{(27)}$$

4. Simulation Results

In this section, the performance of the proposed Turbo FDE is evaluated by computer simulation for QPSK signals transmitted over frequency selective MIMO channel. The FEC code is a 1/2 rate convolutional code with generator polynomials (13, 15octal). The random inter-leaver length is 64. Let the length of CP be 16. The total average transmit power is log-MAP algorithm for channel coding. Eb/N0 is defined as the average transmit energy per bit and the Bit Error Rate(BER) is plotted as a function of Eb/N0.

The BER performance of our proposed turbo equalization algorithm for STBC-MIMO system is showed in Figure 3. It is obvious that our proposed iterative equalizer achieves significant performance compared with the traditional non-iterative ones, especially under well channel condition. As it's seen from Figure 4, as the iteration times increases, the performance of the proposed system is better, but the iterative gains become comparatively smaller, especially after 3 iterations.

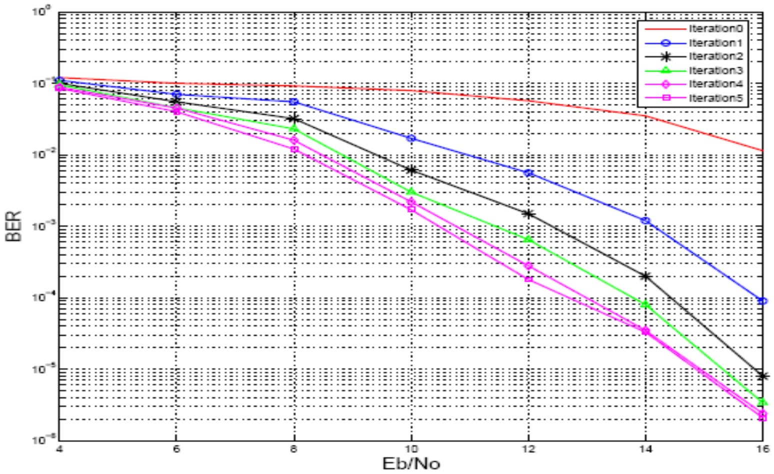

Figure 3. BER performance of our proposed equalizer for STBC-MIMO

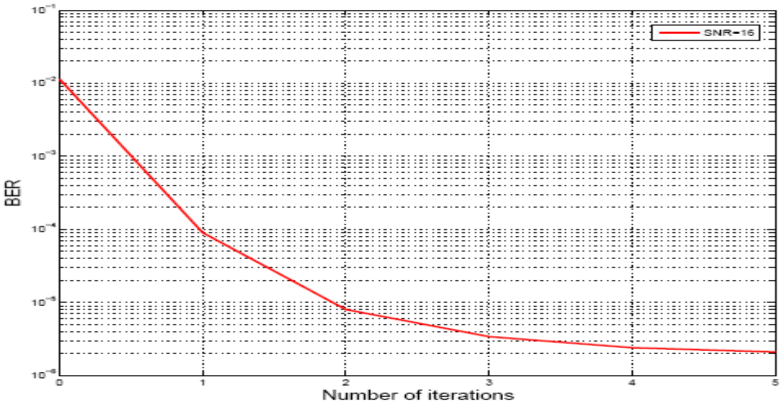

Figure 4. BER performance versus the number of iterations for SNR=16dB

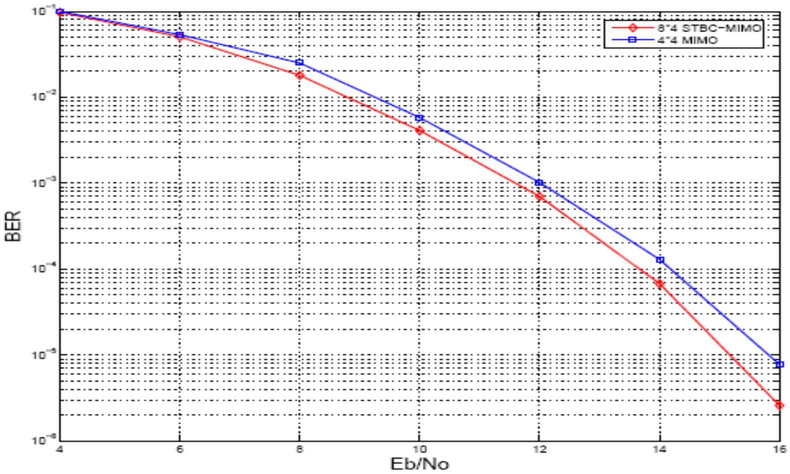

Figure 5. BER performance comparison for SNR=16dB

Figure 5 presents the BER performance comparison of our equalization algorithm for general MIMO system and STBCMIMO system with 2P×Q(p =4,Q = 4) antennas structure. From BER curves, STBC scheme can obtain visible gains compared to general MIMO system, specically about 2dB gains for SNR=16dB.

5. Conclusion

In this paper, we propose a novel Turbo FDE based on symbol-wise detection for single carrier STBC-MIMO system. The transmitter antennas double to get diversity gain without increasing receiving antennas. This algorithm can effectively utilize inter-antenna interference (IAI) and inter-symbol interference (IAI) followed by frequency domain equalization to process soft interference cancellation (SIC). Simulation results have shown that our proposed algorithm achieves better BER performance compared to both the traditional, non-iterative ones, and ones with general MIMO system.

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