

Modeling the Spread of Computer Viruses under the Effects of Infected External Computers and Removable Storage Media

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Abstract

This paper is intended to investigate the effects of infected external computers and removable storage media on the spread of computer viruses. To this end, a new dynamical model, which has four compartments, is proposed. The analysis of the model shows that the unique equilibrium is globally asymptotically stable. This result is well suited for numerical implementation. An illustration of the influences of infected external computers and removable storage media is also included. Furthermore, it is found that (1) infected external computers and removable storage media can both accelerate the viral spread; (2) infected removable storage media can pose a greater threat than infected external computers.

Keywords: Computer virus, Removable storage media, Dynamical model, Equilibrium, Global stability

1. Introduction

Computer viruses, ranging from host-dependent viruses and network worms to other malicious codes such as spyware and Trojans, have posed a serious threat to our daily life and work [1]. Even more serious, the rapidly popularized interconnected networks, ranging from the Internet to all sorts of social networks, provide the major channel for the fast spread of computer viruses. As a result, the issue of how to contain the virus diffusion on networks has received long and continuous attention from the network security community.

Noting the absorbing analogy between computer viruses and their biological counterparts, Cohen [2] and Murray [3] creatively suggested exploiting the techniques developed in epidemic dynamics of infectious disease to investigate the dynamics of computer viruses. Inspired by this idea, Kephart and White [4] established the first dynamical model for computer viruses. From then on, a multitude of dynamical models for computer viruses have been proposed [5-13].

All the previous models assume that latent computers have no infectivity. However, this assumption is in disagreement with the fact that, in general, a computer immediately possesses infectivity as soon as it is infected. In order to remedy this flaw, Yang *et al.* [14] proposed a three-compartment model for computer viruses which is known as the original SLBS (susceptible-latent-breaking-susceptible) model. By incorporating a called R (recovered)-compartment into the SLBS model, Yang *et al.* [15] proposed a four-compartment model which is known as the original SLBRS (susceptible-latent-breaking-recovered-susceptible) model. Subsequently, in order to investigate the impact of infected *external computers* (*i.e.*, a computer outside the Internet) on viral spread and based on the original SLBRS model, a dynamical model [16] has been proposed. However, this model does not consider the impact of *removable storage media* on virus diffusion.

Apart from the Internet as a channel for virus propagation, various removable storage media, including floppy disks, USB flash disks, portable hard disks and even smart phones, offer another channel for virus spreading. By incorporating infected removable

storage media into the original SLBRS model, Gan and Yang [17] suggested a dynamical model. Unfortunately, this model does not consider the impact of infected external computers on virus spreading.

To our knowledge, however, there is no dynamical model considering the combined effect of infected external computers and removable storage media on viral spread. Having this idea in mind and assuming that the Internet offers P2P service for every pair of computers on the network, a new four-compartment dynamical model, which incorporates the combined impact of infected external computers and removable storage media, is presented. Certainly, this model is more realistic than the previous SLBRS models. A qualitative analysis of the model is performed. There is a unique equilibrium, and it is globally asymptotically stable. Some numerical simulations are also conducted. Moreover, it is found that (a) infected external computers and removable storage media can both accelerate the viral spread; (b) infected removable storage media can pose a greater threat to *internal computers* (i.e., a computer on the Internet) than infected external computers.

The subsequent materials of this paper are organized as follows: Section 2 formulates the new model. Section 3 shows the existence of the unique equilibrium and examines its global stability. Some numerical experiments are given in Section 4. Finally, this work is summarized in Section 5.

2. Model Formulation

This section aims to introduce the new proposed model. It is assumed that the Internet offers P2P service for every pair of computers on the network. The model has four compartments: *S*-compartment (the set of all susceptible computers which are uninfected and have no immunity to new viruses on the Internet), *L*-compartment (the set of all latent computers on the Internet), *B*-compartment (the set of all breaking-out computers on the Internet), and *R*-compartment (the set of all recovered computers which are uninfected and have temporary immunity after the virus breaks out and is removed from the Internet). Every computer is assumed to be in one of the four possible states. Let $S(t)$, $L(t)$, $B(t)$ and $R(t)$ (abbreviated for S , L , B , R) denote, at time t , the average numbers of *S*-, *L*-, *B*- and *R*-compartments computers, respectively. Besides, the following basic assumptions are made.

- (A1) The leaving rate of every compartment is positive constant μ .
- (A2) The entering rates of the four compartments are b_1 , b_2 , b_3 and b_4 , respectively. b_1 , b_2 , b_3 and b_4 are positive constants.
- (A3) Every computer in *S*-compartment is infected by *L*- (or *B*-) compartment computers with probability $\beta_1 L$ (or $\beta_2 B$). β_1 and β_2 are positive constants.
- (A4) Every computer in *S*-compartment is infected by infected removable storage media with probability θ .
- (A5) Every computer in *L*-compartment becomes breaking-out with probability α .
- (A6) Every computer in *R*-compartment loses immunity with probability η .
- (A7) Due to installing and updating the antivirus software timely, every computer on the Internet becomes recovered with probability γ_1 .
- (A8) Due to reinstalling the operating system, every computer in *L*- (or *B*-) compartment becomes susceptible with probability γ_2 (or γ_3).

This collection of assumptions can be presented in the form of Figure 1. On this basis, the new model can be formulated as the following differential system:

$$\begin{cases} \dot{S} = b_1 + \gamma_2 L + \gamma_3 B + \eta R - \mu S - \gamma_1 S - \beta_1 LS - \beta_2 BS - \theta S, \\ \dot{L} = b_2 + \beta_1 LS + \beta_2 BS + \theta S - \gamma_1 L - \gamma_2 L - \mu L - \alpha L, \\ \dot{B} = b_3 + \alpha L - \mu B - \gamma_1 B - \gamma_3 B, \\ \dot{R} = b_4 + \gamma_1 S + \gamma_1 L + \gamma_1 B - \eta R - \mu R, \end{cases} \quad (1)$$

with initial condition $(S(0), L(0), B(0), R(0)) \in \mathbb{R}_+^4$.

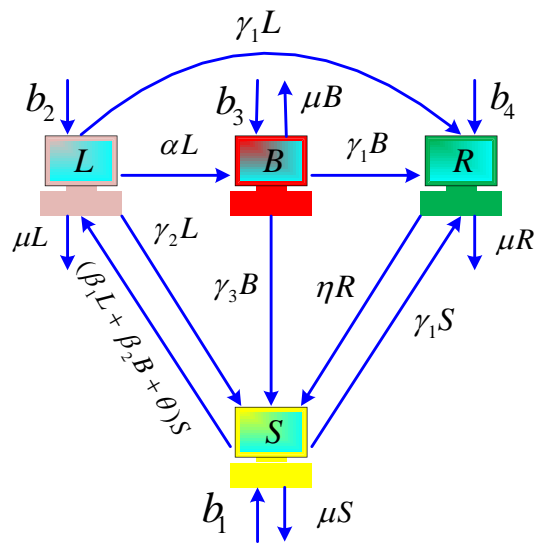


Figure 1. The Transfer Diagram of the New Proposed Model

3. Model Analysis

Let $N = S + L + B + R$, and $b = b_1 + b_2 + b_3 + b_4$. Adding up the four equations of system (1), we have $\lim_{t \rightarrow \infty} N = N^*$, and $\lim_{t \rightarrow \infty} R = R^*$, where

$$N^* = \frac{b}{\mu}, \quad R^* = \frac{\mu b_4 + b \gamma_1}{\mu(\gamma_1 + \eta + \mu)}.$$

Then, system (1) is equivalent to the following reduced limiting system [18].

$$\begin{cases} \dot{L} = b_2 + (\beta_1 L + \beta_2 B + \theta)(N^* - R^* - L - B) - (\gamma_1 + \gamma_2 + \mu + \alpha)L, \\ \dot{B} = b_3 + \alpha L - (\mu + \gamma_1 + \gamma_3)B, \end{cases} \quad (2)$$

with initial condition $(L(0), B(0)) \in \Omega$, where

$$\Omega = \{(L, B) \mid L \geq 0, B \geq 0, 0 \leq L + B \leq N^* - R^*\}.$$

It is easily verified that Ω is positively invariant for the system.

In what follows, it suffices to consider the dynamics of system (2).

Theorem 1: System (2) has a unique equilibrium $E^* = (L^*, B^*)$, where

$$L^* = \frac{a_0 a_2 - a_1 a_3 - a_4 + \sqrt{(a_0 a_2 - a_1 a_3 - a_4)^2 + 4 a_0 a_3 (b_2 + a_1 a_2)}}{2 a_0 a_3}, \quad (3)$$

$$B^* = \frac{b_3 + \alpha L^*}{\mu + \gamma_1 + \gamma_3}, \quad (4)$$

$$a_0 = \frac{\beta_1 (\mu + \gamma_1 + \gamma_3) + \alpha \beta_2}{\mu + \gamma_1 + \gamma_3}, \quad a_1 = \theta + \frac{\beta_2 b_3}{\mu + \gamma_1 + \gamma_3}, \quad a_2 = N^* - R^* - \frac{b_3}{\mu + \gamma_1 + \gamma_3} > 0,$$

$$a_3 = 1 + \frac{\alpha}{\mu + \gamma_1 + \gamma_3}, \quad a_4 = \mu + \alpha + \gamma_1 + \gamma_2.$$

Proof: Assume that (\bar{L}, \bar{B}) is an equilibrium of system (2). Then,

$$\begin{cases} b_2 + (\beta_1 \bar{L} + \beta_2 \bar{B} + \theta)(N^* - R^* - \bar{L} - \bar{B}) - (\gamma_1 + \gamma_2 + \mu + \alpha)\bar{L} = 0, \\ b_3 + \alpha \bar{L} - (\mu + \gamma_1 + \gamma_3)\bar{B} = 0. \end{cases} \quad (5)$$

Solving system (5), it is easy to get $\bar{B} = B^*$, and $\bar{L} = L^*$. Thus, the proof is complete.

Theorem 2: E^* is locally asymptotically stable with respect to Ω .

Proof: The characteristic equation of Jacobian matrix of system (2) evaluated at E^* is $\lambda^2 + k_1 \lambda + k_2 = 0$, (6)

where

$$\begin{aligned} k_1 &= \beta_1 L^* + \beta_2 B^* + \theta + (\gamma_1 + \gamma_2 + \mu + \alpha) - \beta_1 S^* + \gamma_1 + \gamma_3 + \mu, \\ k_2 &= (\gamma_1 + \gamma_3 + \mu) [\beta_1 L^* + \beta_2 B^* + \theta + (\gamma_1 + \gamma_2 + \mu + \alpha) - \beta_1 S^*] + \alpha (\beta_1 L^* + \beta_2 B^* + \theta - \beta_2 S^*), \\ S^* &= N^* - L^* - B^* - R^*. \end{aligned}$$

Noting that $b_2 + (\beta_1 L^* + \beta_2 B^* + \theta)S^* - (\gamma_1 + \gamma_2 + \mu + \alpha)L^* = 0$, we have

$$(\gamma_1 + \gamma_2 + \mu + \alpha) - \beta_1 S^* = \frac{b_2 + (\beta_2 B^* + \theta)S^*}{L^*} > 0.$$

Thus, $k_1 > 0$.

Note that

$$\begin{aligned} k_2 &= (\gamma_1 + \gamma_3 + \mu + \alpha)(\beta_1 L^* + \beta_2 B^* + \theta) + (\gamma_1 + \gamma_3 + \mu) \frac{b_2 + (\beta_2 B^* + \theta)S^*}{L^*} - \alpha \beta_2 S^* \\ &= (\gamma_1 + \gamma_3 + \mu + \alpha)(\beta_1 L^* + \beta_2 B^* + \theta) + \frac{(\gamma_1 + \gamma_3 + \mu)(b_2 + \theta S^*) + b_3 \beta_2 S^*}{L^*} > 0. \end{aligned}$$

From Hurwitz criterion [19], the two roots of equation (6) both have negative real parts. Thus, it follows from Lyapunov stability theorem [19] that E^* is locally asymptotically stable.

Lemma 1: System (2) admits no periodic orbit with respect to Ω .

Proof: Let

$$W_1(L, B) = b_2 + (\beta_1 L + \beta_2 B + \theta)(N^* - R^* - L - B) - (\gamma_1 + \gamma_2 + \mu + \alpha)L,$$

$$W_2(L, B) = b_3 + \alpha L - (\mu + \gamma_1 + \gamma_3)B,$$

$$D(L, B) = \frac{1}{LB}.$$

In the interior of Ω , we have

$$\frac{\partial(DW_1)}{\partial L} + \frac{\partial(DW_2)}{\partial B} = -\frac{b_2}{BL^2} - \frac{\beta_2(N^* - R^* - L - B)}{L^2} - \frac{\beta_1}{B} - \frac{\beta_2}{L} - \frac{\theta}{LB} - \frac{b_3}{LB^2} - \frac{\alpha}{B^2} < 0.$$

Thus, it follows from Bendixson-Dulac criterion [19] that system (2) has no periodic orbit in the interior of Ω .

Next, let us consider an arbitrary point (\tilde{L}, \tilde{B}) on the boundary of Ω . Then,

$$\text{Case 1: } 0 \leq \tilde{L} \leq N^* - R^*, \tilde{B} = 0. \text{ Then, } \left. \frac{dB}{dt} \right|_{(\tilde{L}, \tilde{B})} = b_3 + \alpha \tilde{L} > 0$$

$$\text{Case 2: } 0 \leq \tilde{B} \leq N^* - R^*, \tilde{L} = 0. \text{ Then, } \left. \frac{dL}{dt} \right|_{(\tilde{L}, \tilde{B})} = b_2 + (\beta_2 \tilde{B} + \theta)(N^* - R^* - \tilde{B}) > 0.$$

Case 3: $\tilde{L} + \tilde{B} = N^* - R^*$, $\tilde{L} \neq 0$, $\tilde{B} \neq 0$. Then,

$$\left. \frac{d(L+B)}{dt} \right|_{(\tilde{L}, \tilde{B})} = -\frac{\mu b_1(\mu + \gamma_1) + b\eta\gamma_1 + \mu\eta(b_1 + b_4)}{\mu(\mu + \gamma_1 + \eta)} - \gamma_2 \tilde{L} - \gamma_3 \tilde{B} < 0.$$

Hence, system (2) has no periodic orbit passing through the boundary of Ω . The claimed result follows.

Next, let us present the main result in this paper.

Theorem 3: E^* is globally asymptotically stable with respect to Ω .

Proof: Combining Theorems 1, 2 and Lemma 1, it follows from the generalized Poincare-Bendixson theorem [19] that E^* is globally asymptotically stable with respect to Ω .

4. Numerical Experiments

The theoretical analysis of the proposed model has been fully investigated in Section 3. To illustrate the main result of this paper and the combined impact of infected external computers and removable storage media on virus spreading, some numerical simulations are made in this section.

Example 1. Consider system (1) with $\mu=0.005$, $b_1=0.25$, $b_2=0.28$, $b_3=0.27$, $b_4=0.23$, $\beta_1=0.0043$, $\beta_2=0.0063$, $\theta=0.0038$, $\alpha=0.033$, $\eta=0.015$, $\gamma_1=0.021$, $\gamma_2=0.01$ and $\gamma_3=0.018$. Four initial conditions are $(S(0), L(0), B(0), R(0)) = (150, 25, 15, 5)$, $(S(0), L(0), B(0), R(0)) = (120, 35, 5, 5)$, $(S(0), L(0), B(0), R(0)) = (50, 55, 25, 1)$, $(S(0), L(0), B(0), R(0)) = (75, 25, 35, 1)$, respectively. The time plots of the system are shown in Figure 2, from which it can be seen that computer viruses would persist and converge to a stationary state, which accord with the main result.

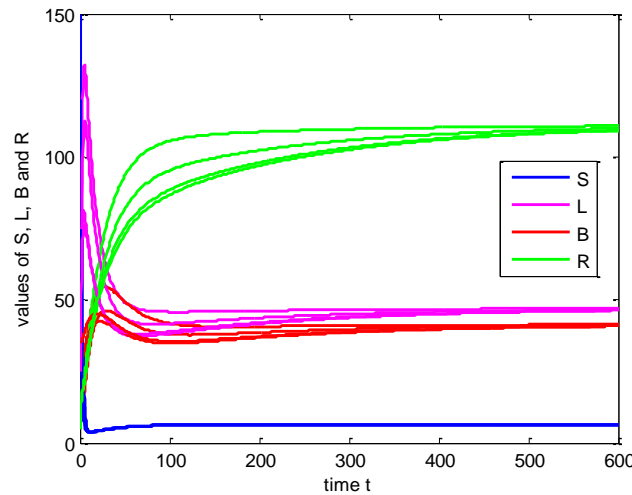


Figure 2. The Time Plots of the New Proposed Model with Different Initial Conditions

Example 2. Consider system (1) with $\mu=0.005$, $b_1=0.25$, $b_2=0.28$, $b_3=0.27$, $b_4=0.23$, $\beta_1=0.0043$, $\beta_2=0.0063$, $\theta=0.0038$, $\alpha=0.033$, $\eta=0.015$, $\gamma_1=0.021$, $\gamma_2=0.01$ and $\gamma_3=0.018$. Six initial conditions are $(S(0), L(0), B(0), R(0)) = (150, 25, 15, 5)$, $(S(0), L(0), B(0), R(0)) = (120, 35, 5, 5)$, $(S(0), L(0), B(0), R(0)) = (50, 55, 25, 1)$, $(S(0), L(0), B(0), R(0)) = (75, 25, 5, 2)$, $(S(0), L(0), B(0), R(0)) = (55, 20, 13, 2)$, $(S(0), L(0), B(0), R(0)) = (33, 15, 14, 1)$, respectively. Figure 3 displays the phase portrait of the system. From this figure,

one can see that computer viruses would persist and converge to a stationary state, which are consistent with the main result.

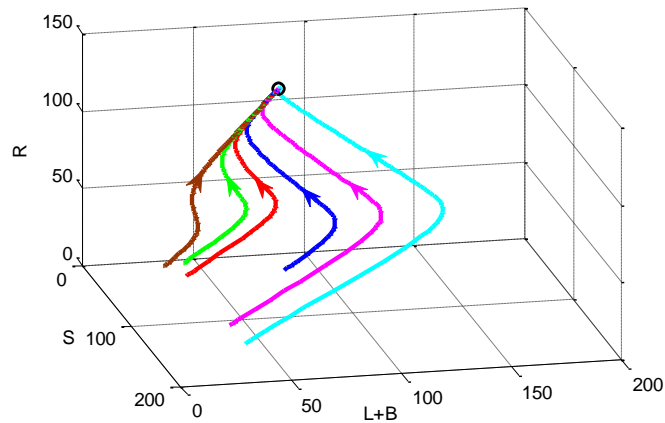


Figure 3. The Phase Portrait of the New Proposed Model with Different Initial Conditions

Example 3. Consider four groups of different parameters of system (1): (a) $\mu=0.005$, $b_1=0.45$, $b_2=0.28$, $b_3=0.17$, $b_4=0.1$, $\beta_1=0.0043$, $\beta_2=0.0053$, $\theta=0.0035$, $\alpha=0.032$, $\eta=0.011$, $\gamma_1=0.022$, $\gamma_2=0.012$ and $\gamma_3=0.015$; (b) $\mu=0.006$, $b_1=0.32$, $b_2=0.21$, $b_3=0.27$, $b_4=0.2$, $\beta_1=0.0043$, $\beta_2=0.0063$, $\theta=0.0045$, $\alpha=0.062$, $\eta=0.01$, $\gamma_1=0.011$, $\gamma_2=0.009$ and $\gamma_3=0.018$; (c) $\mu=0.006$, $b_1=0.28$, $b_2=0.35$, $b_3=0.27$, $b_4=0.15$, $\beta_1=0.0053$, $\beta_2=0.0056$, $\theta=0.0035$, $\alpha=0.032$, $\eta=0.014$, $\gamma_1=0.019$, $\gamma_2=0.015$ and $\gamma_3=0.018$; (d) $\mu=0.0055$, $b_1=0.55$, $b_2=0.25$, $b_3=0.15$, $b_4=0.12$, $\beta_1=0.0033$, $\beta_2=0.0063$, $\theta=0.0031$, $\alpha=0.012$, $\eta=0.009$, $\gamma_1=0.024$, $\gamma_2=0.011$ and $\gamma_3=0.012$. The common initial condition is $(S(0), L(0), B(0), R(0)) = (85, 25, 15, 10)$. The time series of the system are demonstrated in Figure 4, from which it can be seen that computer viruses would persist and converge to a stationary state, which are in accordance with the main result.

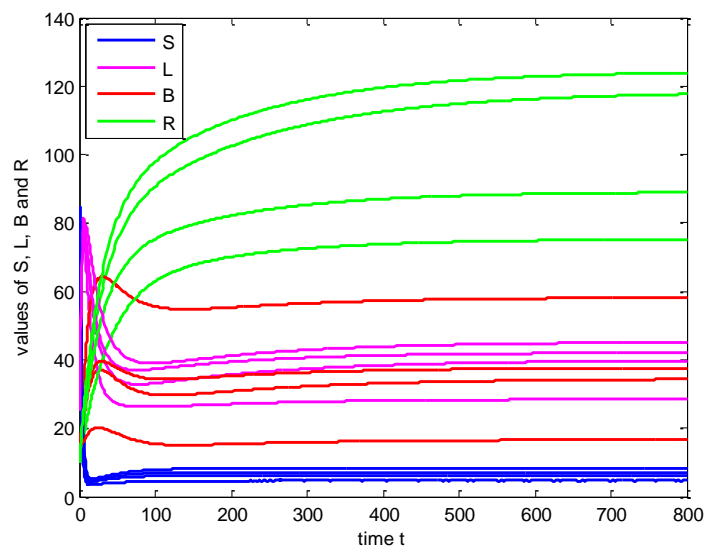


Figure 4. The Time Plots of the New Proposed Model with Different System Parameters

Example 4. Consider six groups of different parameters of system (1): (a) $\mu=0.005$, $b_1=0.25$, $b_2=0.28$, $b_3=0.27$, $b_4=0.23$, $\beta_1=0.0043$, $\beta_2=0.0063$, $\theta=0.0038$, $\alpha=0.033$, $\eta=0.015$, $\gamma_1=0.021$, $\gamma_2=0.01$ and $\gamma_3=0.018$; (b) $\mu=0.005$, $b_1=0.45$, $b_2=0.28$, $b_3=0.17$, $b_4=0.1$, $\beta_1=0.0043$, $\beta_2=0.0053$, $\theta=0.0035$, $\alpha=0.032$, $\eta=0.011$, $\gamma_1=0.022$, $\gamma_2=0.012$ and $\gamma_3=0.015$; (c) $\mu=0.006$, $b_1=0.32$, $b_2=0.21$, $b_3=0.27$, $b_4=0.2$, $\beta_1=0.0043$, $\beta_2=0.0063$, $\theta=0.0045$, $\alpha=0.062$, $\eta=0.01$, $\gamma_1=0.011$, $\gamma_2=0.009$ and $\gamma_3=0.018$; (d) $\mu=0.006$, $b_1=0.28$, $b_2=0.35$, $b_3=0.27$, $b_4=0.15$, $\beta_1=0.0053$, $\beta_2=0.0056$, $\theta=0.0035$, $\alpha=0.032$, $\eta=0.014$, $\gamma_1=0.019$, $\gamma_2=0.015$ and $\gamma_3=0.018$; (e) $\mu=0.0055$, $b_1=0.55$, $b_2=0.25$, $b_3=0.15$, $b_4=0.12$, $\beta_1=0.0033$, $\beta_2=0.0063$, $\theta=0.0031$, $\alpha=0.012$, $\eta=0.009$, $\gamma_1=0.024$, $\gamma_2=0.011$ and $\gamma_3=0.012$; (f) $\mu=0.0055$, $b_1=0.45$, $b_2=0.28$, $b_3=0.17$, $b_4=0.1$, $\beta_1=0.0023$, $\beta_2=0.0033$, $\theta=0.0035$, $\alpha=0.022$, $\eta=0.011$, $\gamma_1=0.012$, $\gamma_2=0.022$ and $\gamma_3=0.015$. The common initial condition is $(S(0),L(0),B(0),R(0))=(85,25,15,10)$. Figure 5 exhibits the phase portrait of the system, from which one can see that computer viruses would persist and converge to a stationary state, which fit the main result well.

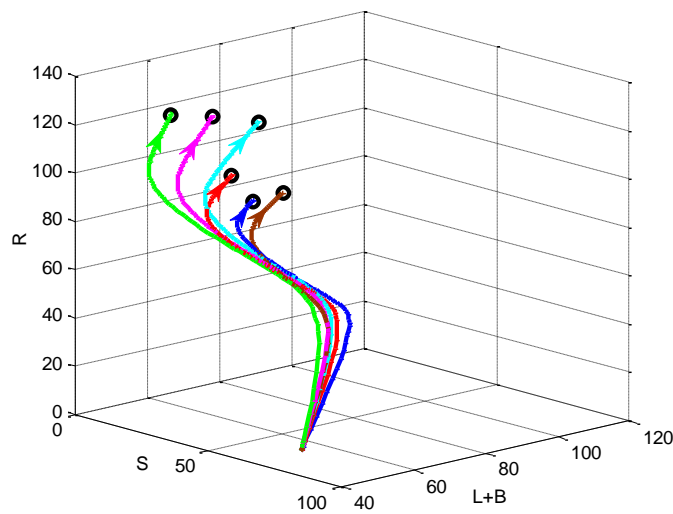


Figure 5. The Phase Portrait of the New Proposed Model with Different System Parameters

Example 5. Consider four groups of different parameters of system (1): (a) $\mu=0.005$, $b_1=0.45$, $b_2=0$, $b_3=0$, $b_4=0.1$, $\beta_1=0.0033$, $\beta_2=0.0043$, $\theta=0$, $\alpha=0.052$, $\eta=0.0011$, $\gamma_1=0.012$, $\gamma_2=0.015$ and $\gamma_3=0.025$; (b) $\mu=0.005$, $b_1=0.45$, $b_2=0.28$, $b_3=0.17$, $b_4=0.1$, $\beta_1=0.0033$, $\beta_2=0.0043$, $\theta=0$, $\alpha=0.052$, $\eta=0.0011$, $\gamma_1=0.012$, $\gamma_2=0.015$ and $\gamma_3=0.025$; (c) $\mu=0.005$, $b_1=0.45$, $b_2=0$, $b_3=0$, $b_4=0.1$, $\beta_1=0.0033$, $\beta_2=0.0043$, $\theta=0.15$, $\alpha=0.052$, $\eta=0.0011$, $\gamma_1=0.012$, $\gamma_2=0.015$ and $\gamma_3=0.025$; (d) $\mu=0.005$, $b_1=0.45$, $b_2=0.28$, $b_3=0.17$, $b_4=0.1$, $\beta_1=0.0033$, $\beta_2=0.0043$, $\theta=0.15$, $\alpha=0.052$, $\eta=0.0011$, $\gamma_1=0.012$,

$\gamma_2 = 0.015$ and $\gamma_3 = 0.025$. The common initial condition is $(S(0), L(0), B(0), R(0)) = (70, 18, 12, 8)$. Figure 6 depicts the combined effect of infected external computers and removable storage media on viral spread. Additionally, this figure shows that (1) infected external computers and removable storage media would both accelerate the viral spread; (2) infected removable storage media would pose a greater threat to internal computers than infected external computers.

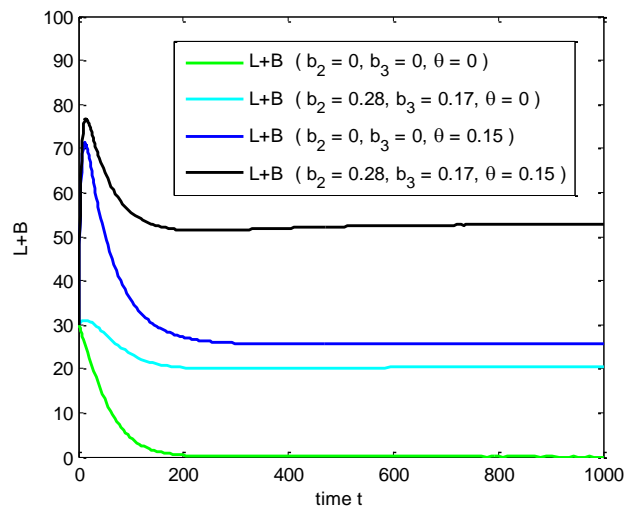


Figure 6. The Combined Effect of Infected External Computers and Removable Storage Media on Viral Spread

5. Conclusion

To better understand the combined effect of infected external computers and removable storage media on viral spread, this paper has proposed a new four-compartment model. An exhaustive model analysis has shown that the unique equilibrium is globally asymptotically stable. Additionally, this main result and the combined impact of infected external computers and removable storage media are illustrated by some numerical simulations, from which one can see that (1) infected external computers and removable storage media can both speed up the virus diffusion; (2) infected removable storage media can cause more terrible results than infected external computers.

In my opinion, the next work could be made as follows. First, one can extend the new proposed model on complex networks [20]. Second, some optimal control strategies [21, 22] may be considered. Finally, the next work is to study the impact of the network topology on virus diffusion [23, 24].

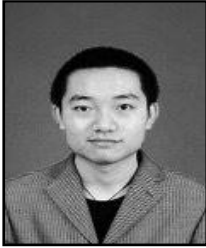
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