

# Landing Risk Evaluation of Carrier-Based Aircraft Based on Mean-Variance Statistics Error Multi-Attribute Dynamic Decision Making

Li Hui

*School of Computer and Information Engineering  
Harbin University of Commerce  
hrbcu\_lh@163.com*

## **Abstract**

*To induce the landing risk of carrier-based aircraft, and improve the safety of pilots and carrier-based aircraft, this paper presents an improved landing risk evaluation of carrier-based aircraft based on Mean-Variance statistics error multi-attribute dynamic decision making. Attribute values is shown as triangular fuzzy numbers, and ordering vectors are achieved through possibility-degree and ordering expression. In accordance with ordering wave from ordering values at different times to desire rank, condition analysis should be conducted. Finally, the rank ordering is acquired. The model simulation results indicate the better performance of the new method in comparison with the traditional controller with more accuracy and practicability.*

**Keywords:** *Mean-Variance Statistics Error; Multi-Attribute Dynamic Decision Making; Landing Risk Evaluation; Carrier-Based Aircraft*

## **1. Introduction**

Landing risk of carrier-based aircraft has always existed in every point during landing process. To ensure landing safely and quickly, there are lots of studies for researching. [1] had established the Landing Signal Officer (LSO) instruction associated with operation guide system to control hazard. Asymmetric variable universe adaptive landing fuzzy controller for carrier-based aircraft was designed to control the landing voyage in [2]. Integrated evaluation technology of LSO should be presented for assessing landing risk in [3].

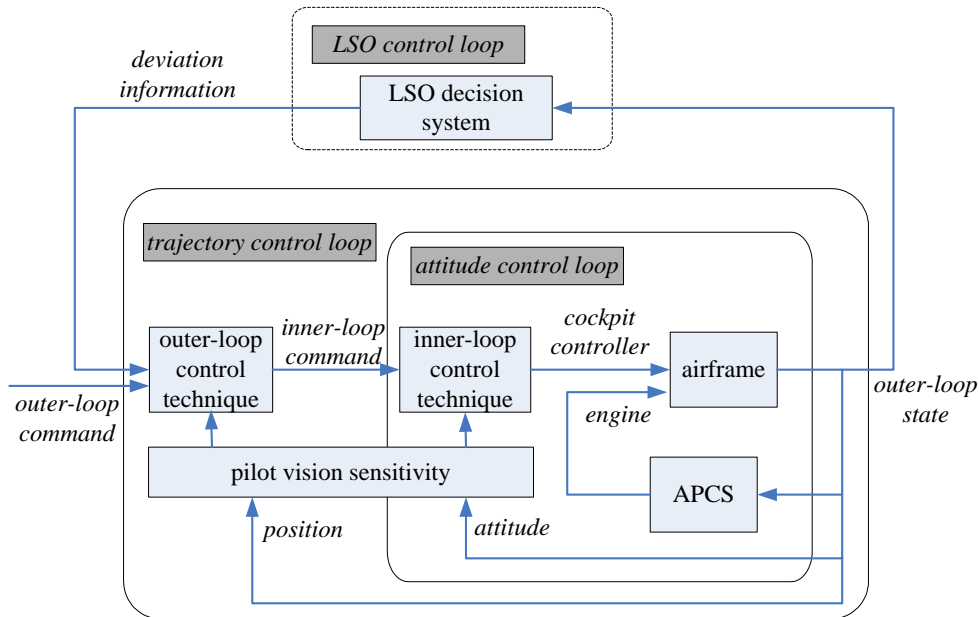
Dynamic Multi-Attribute Decision Making (DMADM) based on advantage retention degree is described in [4]. An improved VIKOR should be presented for researching triangular fuzzy numbers in [5]. Landing risk assessment based on fuzzy multi-attribute decision making should be discussed in [6]. It is used concentrated factors to amalgamate the decision making information at different time points for the traditional dynamic decision problems, as that we call the dynamic decision making above is another Group Multi-Attribute Decision Making (GMADM).

Considering landing of carrier-based aircraft is a process from the start position to the desire point, every decision time has connection with each other. Landing risk evaluation of carrier-based aircraft based on Mean-Variance statistics error multi-attribute decision making should be proposed in this paper.

The rest of this paper is structured as follows: next section we first analyze the landing risk evaluation problem. Section 3 designs the landing risk evaluation of carrier-based aircraft based on Mean-Variance statistics error multi-attribute dynamic decision making. Multiple landing simulations of carrier-based aircraft should be shown in Section 4.

## 2. Problem Description

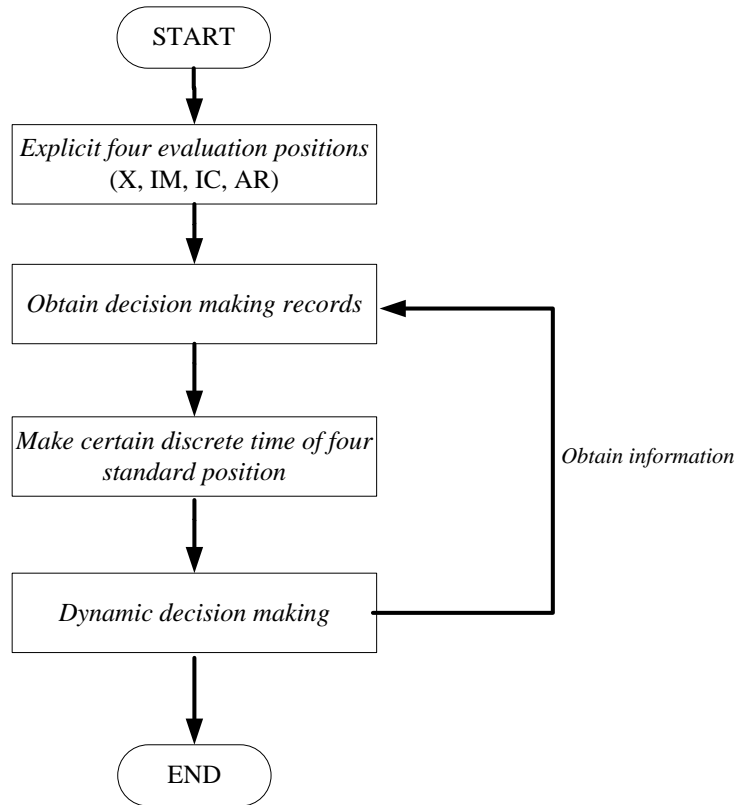
Generally speaking, for landing system of carrier-based aircraft which is shown in Figure. 1, landing risk evaluation should be realized as Figure. 2 by four steps [9-11]:



**Figure 1. Landing system of Carrier-Based Aircraft**

- 1) Explicit four evaluation positions: start position(X), middle position(IM), close position(IC) and ramp position(AR).
- 2) Obtain decision making records of flight status information at different positions.
- 3) Make certain discrete time of four standard positions.
- 4) Dynamic decision making with decision information using appropriate ways.

Dynamic decision making for carrier-based aircraft is the correlative problem in allusion to the discrete time points. Flight status will be represented as nine criterion description, and time moments information should be interacted for final alternatives ranking order.



**Figure 2. Landing Risk Evaluation Steps**

### 3. Landing Risk Evaluation of Carrier-Based Aircraft Based on Mean-Variance Statistics Error Multi-Attribute Dynamic Decision Making

Landing risk evaluation is an complicated process. For one dynamic decision making problem, outset let  $X = \{x_1, x_2, \dots, x_m\}$  be a discrete set of  $m$  decision feasible alternatives,  $U = \{u_1, u_2, \dots, u_n\}$  is a finite set of attributes, and  $T = \{t_1, t_2, \dots, t_l\}$  is a set of decision time points. Construct  $R^k = (r_{ij}^{(k)})_{m \times n}$  a decision matrix as shown in (1) [12-15].

$$\begin{matrix}
 u_1 & u_2 & \dots & u_n \\
 \begin{matrix}
 r_{11}^{(k)} & r_{12}^{(k)} & \dots & r_{1n}^{(k)} \\
 r_{21}^{(k)} & r_{22}^{(k)} & \dots & r_{2n}^{(k)} \\
 \dots & \dots & \dots & \dots \\
 r_{m1}^{(k)} & r_{m2}^{(k)} & \dots & r_{mn}^{(k)}
 \end{matrix}
 \end{matrix}
 \begin{matrix}
 x_1 \\
 x_2 \\
 \dots \\
 x_m
 \end{matrix}
 \quad (1)$$

The steps of landing risk evaluation based on Mean-Variance statistics error multi-attribute decision making are:

**Step 1:** Explicit attributes  $\omega = (\omega_1, \omega_2, \dots, \omega_m)^T$ . As landing assessment hierarchy in [4], there are three influencing indicator during lading loop: glideslope deviation, speed and rate of decent (ROD). As we know, glideslope deviation and speed are main risk indicator, the other one is lesser. Let attributes are defined as (2):

$$\omega_j = \begin{cases} \frac{1-\lambda}{n} + \frac{\lambda}{n-1}, & j \neq n \\ \frac{1-\lambda}{n}, & j = n \end{cases} \quad (2)$$

where  $n=3$ ,  $\lambda = 0.2$  and  $\omega = (0.4, 0.4, 0.2)^T$ .

**Step 2:** Calculate comprehensive attribute evaluation value at  $t_k$  moment. The  $i$  ( $i \in M$ )<sup>th</sup> column data of matrix  $R^k$  should be concentrated using TFLWA operator, and comprehensive attribute evaluation value of  $x_i$  at  $t_k$  moment is:

$$z_i^k = \sum_{j=1}^n \omega_j r_{ij}^k \quad (3)$$

**Step 3:** Install possibility-degree matrix at every moment as possibility-degree formula of triangle fuzzy numbers [16-18].

$$p_{ij}^k = p(z_i^k \geq z_j^k) (i, j \in M) \quad (4)$$

**Step 4:** Acquire ordering vector  $\omega^k = (\omega_1, \omega_2, \dots, \omega_m)$  at  $t_k$  point by ordering formula [19-22].

$$\omega_i = \frac{1}{n(n-1)} \left( \sum_{j=1}^n p_{ij} + \frac{n}{2} - 1 \right) \quad (5)$$

**Step 5:** Confirm attribute  $\varpi = (\varpi_1, \varpi_2, \dots, \varpi_l)^T$  of time with four glideslope standard positions as dynamic decision making time. From [6], we know that the farther to carrier, the larger for safe flight area, and the less risk for pilots. So the relative attribute weight relationship of standard positions is shown as (6).

$$\varpi_X < \varpi_{IM} < \varpi_{IC} < \varpi_{AR} \quad (6)$$

where we can describe as (7).

$$\varpi_i = \frac{1}{z_{p_i}^{up} - z_{p_i}^{down}} \quad (i=X, IM, IC \text{ or } AR) \quad (7)$$

Finally,

$$\varpi = (\varpi_X, \varpi_{IM}, \varpi_{IC}, \varpi_{AR}) = (0.0403, 0.0638, 0.1231, 0.7728)^T \quad (8)$$

**Step 6:** Calculate expected rank value of  $x_i$ . The expected value should be gained by rank value of  $x_i$  at different time points, and expected rank value of  $x_i$  is shown as (9).

$$E_{x_i} = \sum_{k=1}^l \varpi_k w_i^k \quad (9)$$

**Step 7:** Ascertain discrete deviation of  $x_i$ . Because of various kinds of elements, there is certain deviation between rank value of  $x_i$  at different moments and expected rank  $E_{x_i}$  value of  $x_i$ , and variance  $\sigma_i^2(k) = (w_i^k - E_{x_i})^2$  should be used to express the deviation at definite moment.  $\sigma_i^2(k)$  is the influence of expected rank value of  $x_i$  by rank value at  $k$  moment, and the risk is larger with  $\sigma_i^2(k)$  is more.

To indicate general fluctuation of the entire alternative  $x_i$ , the rank values are related to seek the general deviation of expected  $x_i$  at different moments, recording discrete deviation of  $x_i$ .

$$\sigma_i^2 = \sum_{k=1}^l \sigma_i^2(k) = \sum_{k=1}^l (w_i^k - E_i)^2 \quad (10)$$

With  $\sigma_i^2$  is more, the landing risk is larger, and the stability is less relatively. Finally flight evaluation effect is minor of voyages  $x_i$ .

**Step 8:** Decision analysis based on Mean-Variance statistics error. Rank the  $m$  expected rank value and discrete deviations of alternatives according value. The sequence of mean is positive, and the ordering position is frontier with mean larger, from 1 to  $m$ . On the contrary, the sequence of variance is reverse, and the ordering position is frontier with variance less, from 1 to  $m$  similarly. We can execute rank estimation as below situations.

1) If the ordering result of mean is identical of variance's, no difference, and we establish the alternatives sequence as the ordering result of mean and variance.

2) If the ordering result of mean is different of variance's, we should establish the alternatives sequence as (11).

$$E_i - E_j \{ \geq \text{or} < \} \xi \cdot \frac{1}{m} (E_i > E_j) \quad (11)$$

where  $\xi$  is the accuracy of alternatives estimation, and  $\frac{1}{m}$  is the equivalent intermission span of alternatives.

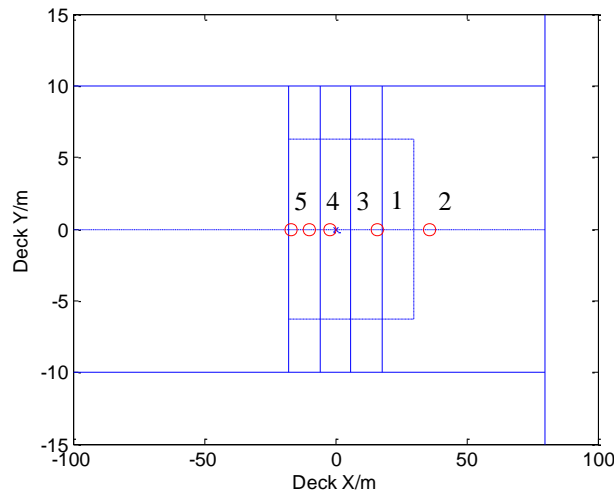
If  $E_i - E_j > \xi \cdot \frac{1}{m}$ , we install the sequence as the ordering result of mean;

If  $E_i - E_j < \xi \cdot \frac{1}{m}$ , we install the sequence as the order result of variance.

#### 4. Model Simulation

Five times landing processes of carrier-based aircraft are simulated for analysis, and final distributions of touchdown points are shown in Figure. 3. Landing risk decision making is realized as follow.

There are five flight voyages as alternatives  $X = [x_1, x_2, x_3, x_4, x_5]$ , and risk index of linguistic loop as attributes  $U = [\text{glideslope deviation, velocity, rate of descend}] = [u_1, u_2, u_3]$ , four standard positions of landing process as discrete decision making time  $T = [t_1, t_2, t_3, t_4]$ , respectively, the decision matrices  $R^k = (r_{ij}^k)_{m \times n}$  ( $k = 1, 2, 3, 4; i = 1, 2, 3, 4, 5; j = 1, 2, 3$ ) here as listed in Tables 1-4.



**Figure 3. Distributions of Touchdown Points**

**Table 1. Evaluation Matrix of Decision Moment t1 (X point)**

Pilot\flight state	$u_1$	$u_2$	$u_3$
$x_1$	[0.2,0.3,0.4]	[0.1,0.2,0.3]	[0.1,0.2,0.3]
$x_2$	[0.1,0.2,0.3]	[0.3,0.4,0.5]	[0.2,0.3,0.4]
$x_3$	[0.5,0.6,0.7]	[0.6,0.7,0.8]	[0.5,0.6,0.7]
$x_4$	[0.3,0.4,0.5]	[0.4,0.5,0.6]	[0.3,0.4,0.5]
$x_5$	[0.3,0.4,0.5]	[0.1,0.2,0.3]	[0.1,0.2,0.3]

**Table 2. Evaluation Matrix of Decision Moment t2 (IM point)**

Pilot\flight state	$u_1$	$u_2$	$u_3$
$x_1$	[0.3,0.4,0.5]	[0.3,0.4,0.5]	[0.2,0.3,0.4]
$x_2$	[0.2,0.3,0.4]	[0.3,0.4,0.5]	[0.3,0.4,0.5]
$x_3$	[0.6,0.7,0.8]	[0.7,0.8,0.9]	[0.6,0.7,0.8]
$x_4$	[0.4,0.5,0.6]	[0.4,0.5,0.6]	[0.4,0.5,0.6]
$x_5$	[0.4,0.5,0.6]	[0.1,0.2,0.3]	[0.2,0.3,0.4]

**Table 3. Evaluation Matrix of Decision Moment t3 (IC point)**

Pilot\flight state	$u_1$	$u_2$	$u_3$
$x_1$	[0.4,0.5,0.6]	[0.3,0.4,0.5]	[0.3,0.4,0.5]
$x_2$	[0.3,0.4,0.5]	[0.4,0.5,0.6]	[0.3,0.4,0.5]
$x_3$	[0.7,0.8,0.9]	[0.7,0.8,0.9]	[0.7,0.8,0.9]
$x_4$	[0.5,0.6,0.7]	[0.5,0.6,0.7]	[0.6,0.7,0.8]
$x_5$	[0.5,0.6,0.7]	[0.3,0.4,0.5]	[0.3,0.4,0.5]

**Table 4. Evaluation Matrix of Decision Moment t4 (AR point)**

Pilot\flight state	$u_1$	$u_2$	$u_3$
$x_1$	[0.5,0.6,0.7]	[0.5,0.6,0.7]	[0.4,0.5,0.6]
$x_2$	[0.4,0.5,0.6]	[0.5,0.6,0.7]	[0.4,0.5,0.6]
$x_3$	[0.8,0.9,1.0]	[0.8,0.9,1.0]	[0.7,0.8,0.9]
$x_4$	[0.7,0.8,0.9]	[0.6,0.7,0.8]	[0.7,0.8,0.9]
$x_5$	[0.6,0.7,0.8]	[0.4,0.5,0.6]	[0.4,0.5,0.6]

(1) Use the TFLWA operator to aggregate the  $i(i \in M)$ th column data of matrix  $R^1$ , and compute the comprehensive attribute value at  $t_1$  moment:

$$z_1^1=[0.14,0.24,0.34], z_2^1=[0.2,0.3,0.4], z_3^1=[0.54,0.64,0.74],$$

$$z_4^1=[0.34,0.44,0.54], z_5^1=[0.18,0.28,0.38].$$

(2) Calculate the possibility-degree at  $t_1$  moment, and construct the possibility-degree matrix  $P^1 = (p^1)_{m \times m}$ .

$$P^1 = \begin{bmatrix} 0.5 & 0.245 & 0 & 0 & 0.32 \\ 0.755 & 0.5 & 0 & 0.045 & 0.595 \\ 1 & 1 & 0.5 & 1 & 1 \\ 1 & 0.955 & 0 & 0.5 & 0.98 \\ 0.68 & 0.405 & 0 & 0.02 & 0.5 \end{bmatrix}$$

(3) Compute ordering vector  $\omega^1$  at  $t_1$  moment.

$$\omega^1=(0.1283,0.1697,0.3,0.2468,0.1553)^T$$

Similarly, repeat (1)-(3), we can obtain ordering vectors at other moments.

$$\omega^2=(0.1678,0.1523,0.3,0.2428,0.1373)^T$$

$$\omega^3=(0.1613,0.1323,0.2998,0.2455,0.1613)^T$$

$$\omega^4=(0.1772,0.1250,0.2960,0.2515,0.1502)^T$$

(4) Receive time attribute  $\varpi=(0.0403,0.0638,0.1231,0.7728)^T$  based on (3), count mean  $E_i = \sum_{k=1}^4 w_i^k \varpi_k$  and variance  $\sigma_i^2 = \sum_{k=1}^4 (w_i^k - E_i)^2$  of alternative  $x_i$ .

$$E_1=0.1727, E_2=0.1294, E_3=0.2969, E_4=0.25, E_5=0.1509.$$

$$\sigma_1^2=0.00215, \sigma_2^2=0.00218, \sigma_3^2=0.00003, \sigma_4^2=0.00008, \sigma_5^2=0.00031.$$

(5) Rank mean and variance of five voyages.

Ordering with mean:

$$E_3 > E_4 > E_1 > E_5 > E_2$$

Ordering with variance:

$$\sigma_3^2 < \sigma_4^2 < \sigma_5^2 < \sigma_1^2 < \sigma_2^2$$

(6) Judge and analyze, confirm ordering result of alternatives.

There are some ordering position differences between mean and variance of  $x_1$  and  $x_5$ .

Based on decision maker's experiences, let  $\xi = 0.1$ ,  $\xi \cdot \frac{1}{m} = 0.1 \times \frac{1}{5} = 0.02$ , and  $E_1 - E_5 = 0.0218 > 0.02$ . Finally, we choose ordering result with mean as ultimate result.

(7) Rank all the alternatives  $r_i$ , and we can see that the ranking order of five flight voyages is:

$$x_3 > x_4 > x_1 > x_5 > x_2$$

Above all, it's difficult to judge some alternatives where means are similar in traditional dynamic decision making. In order to distinguish stand or fall, it introduces Mean-Variance statistics error multi-attribute dynamic decision making for resolving the intricate situation, and enhances the accuracy of decision result.

As shown in Figure. 3, the touchdown point of third voyage locates between the 2<sup>nd</sup> cable and the 3<sup>rd</sup> one, and we would call it optimal touchdown point. Other voyages touchdown points distributions are nearer the desired one, and the landing risk is decreased.

## 6. Conclusion

This paper has introduced an improved decision making approach based on Mean-Variance statistics error multi-attribute dynamic decision making. Mean and variance would be synthesized to solve the shortage of traditional dynamic decision making, and the decision making problem whose rank alike especially. At last Mean-Variance statistics error multi-attribute dynamic decision making has an application in landing risk assessment of carrier-based aircraft. Simulation results show that the reasonable and application of the new algorithm.

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## Authors



**Hui Li**, He received a D.E. degree in Control Theory and Control Engineering from Harbin Engineering University, Harbin, China, 2013. He is the member of council of the Operations Research Society of China. His recent research interests are in intelligent control, Multi-attribute decision making, fuzzy decision making.

