

Novel Quantum-Inspired Co-evolutionary Algorithm

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Abstract

Co-evolutionary mechanism is now used into evolutionary algorithms and provides these algorithms the power to promote the convergence. In order to promote the performance of the traditional quantum-inspired evolutionary algorithm (QEA), we proposed a novel quantum-inspired co-evolutionary algorithm (NQCEA), in this paper. The quantum state population is firstly divided into multiple sub-populations, which complete the evolution processes independently. In each evolution cycle, every sub-population will produce an elitist individual, which is then selected to construct an elite library. Subsequently, these individuals in this elite library can help the poor sub-population to find the global optimal solution or near-optimal solution. In addition, a diversity indicator is defined for every sub-population and is used to measure the diversity of the corresponding sub-population. As for the sub-population with poor diversity, the mutation strategies are implemented in order to expand its diversity and improve its global search ability. Finally, the NQCEA is compared with the traditional QEA to test their performance. Experiments are performed on the global numerical optimization functions and the simulation results indicate that this new algorithm has the characteristics of good global search capability and more stable performance than the traditional QEA.

Keywords: *Quantum Evolutionary Algorithm, Co-evolution, Global optimization, Elitist individual*

1. Introduction

The traditional intelligent optimization algorithms (TIOAs), based on the Genetic algorithms (GAs), usually applied the Darwin's theory of evolution, such as the survival of the fittest, natural selection and competition. In the actual application, these algorithms have quick convergence, robustness and high efficiency capability of searching the optimal solution within the whole defined space, which make them be widely used in many fields [1]. However, these traditional intelligent optimization algorithms also have some demerits when they are in the process of evolution. For example, these algorithms are often easy to fall into local optimum, and the corresponding populations have poor diversity at the end of iteration. Recently, the co-evolution based intelligent optimization algorithm, as a new evolutionary algorithm framework, has been a hot research topic of computational intelligence. As for the co-evolution, it is inspired by the practical ecosystem, and had firstly proposed by Ehrlich and Raven [2]. The co-evolution emphasizes the mutual influence or co-ordination among the different populations, populations and the environments. On the basis of this, the co-evolutionary algorithms (CEAs) are also focusing on the interaction and mutual influence among different populations, then to achieve the co-evolution, and to improve their optimization performance.

From the view of operation mechanisms, there are a lot of differences between TIOAs and CEAs. Based on the neural network, genetic algorithm, and particle swarm optimization algorithm [3] [4], the TIOAs start their evolution process from generating a random variable population, and then taking the evolutionary strategy of "survival of the fittest" to solve global optimization problems. However, the co-evolution is broadly defined as a fitness based on population density, the population itself and the evolution of genetic composition between interacting populations. Comparing with the TIOAs, CEAs have strong search capabilities and progressive learning abilities to overcome the premature convergence, which is the usual disadvantage of the TIOAs.

As for the co-evolutionary individuals, they are often affected by the following three factors when they are in the evolutionary process. The first is the individual fitness, the second is their located environment and the last is the competition with each other. By coordinating these three factors, they can effectively solve the singleness problem of the evolutionary model such as the genetic algorithm, thus can maintain the diversity of the population better, and avoid premature convergence and slow convergence issues [5]. Meanwhile, K. H. Han et al. was firstly proposed Genetic Quantum Algorithm (GQA) in Proc. 2000 Congress on Evolutionary Computation, which is also considered to be one of the earliest model of quantum evolutionary algorithm (QEA), and soon afterward, they expanded the GQA and presented quantum-inspired evolutionary algorithm, and it utilizes the concepts of quantum bit (Q-bit), superposition of states and collapse of states on the basis of GQA [6]. Compared to the traditional evolutionary algorithms, QEA has a number of other advantages, for example, QEA uses Q-bit individual which can describe a linear superposition of states in search space probabilistically, thus the Q-bit representation has a better characteristic to maintain population diversity than any other representation; at the same time, QEA applies quantum rotation gate as Q-gate to guiding the searching direction into the optimal area and accelerating the algorithm's convergence speed. In recent years, many scholars had tried to improve the quantum evolutionary algorithm, and proved its performance was better than traditional evolutionary algorithms. But quantum evolutionary algorithm is also easy to fall into local optimum, especially for complex global numerical optimization problems. Therefore, how to ensure QEA overcome above disadvantage have been a hotspot but also a difficulty [7].

Nowadays, some researchers began to combine co-evolutionary mechanism with QEA and then proposed some improved QEAs. Gu et al. proposed a novel competitive co-evolutionary quantum genetic algorithm (CCQGA) which included three new strategies named as competitive hunter, cooperative surviving and the big fish eating small fish. This algorithm could not only increase the diversity of genes and avoid premature convergence, but also accelerate the convergence. The experiment results achieved by CCQGA are compared with quantum-inspired genetic algorithm (QGA) and standard genetic algorithm (GA), which shows that CCQGA has better feasibility and effectiveness [8]. Xiong, Gui and et al. proposed a double population co-evolution algorithm based on the quantum evolution algorithm and difference evolution algorithm. In their new algorithm, these two populations complete different evolutionary process, and one population is responsible for global searching, the other is for partial search [9]. Zhang proposed an elite collaborative quantum evolution algorithm. His new algorithm divided the entire population into several sub-populations, and then two sub-populations were selected randomly. Subsequently, the corresponding elite individuals from the above two sub-populations would construct a mutually beneficial relationship in order to

implement co-evolutionary operation, and guide the individual populations evolve toward the optimal solution [10].

In general, the existing improved QEAs on the co-evolutionary mechanism often include the following two main design ideas:

- (1) Focusing on the co-evolution between the two sub-populations;
- (2) Using the elite individual to guide the entire population toward the global optima.

Actually, these above two improvement strategies based on the multiple sub-populations are just attempting to find the current global optimal individual in each generation, and then apply the global optima to help these sub-populations. However, they ignore the evolutionary characteristics of individual sub-population, which may include some useful information for the optimization process.

In this paper, the co-evolutionary mechanisms are introduced into QEA in order to propose a novel quantum-inspired co-evolutionary algorithm (NQCEA). This NQCEA will construct its own elite library which includes multiple elite individuals from the different sub-populations. In the course of evolution, the elite individual will guide not only the corresponding sub-population itself, but also the selected sub-population. In addition, the NQCEA will extract the characteristic information of every sub-population in order to get a better way of evolution for the corresponding sub-population.

The remainder of this paper is organized as following. Section 2 describes the basic theory of quantum evolutionary algorithm and co-evolutionary mechanism; section 3 gives the details of the proposed algorithm; section 4 gives the experimental simulation in order to evaluate the novel algorithm; finally, some conclusions are drawn in section 5.

2. Co-Evolutionary Algorithm and Quantum Evolutionary Algorithm

2.1 Co-Evolutionary Algorithm

In general, a co-evolutionary algorithm satisfies the following conditions [11]:

- (1) This algorithm can maintain multiple sub-populations simultaneously;
- (2) As for the individuals in this algorithm, their fitness values depend on the individuals in the other sub-populations;
- (3) As for the individuals in this algorithm, the evolutionary operations of individuals (including insert, delete, and update) will lead to the fitness landscape of other sub-populations fitness change.

For example, suppose the genetic algorithm is using co-evolutionary strategy to improve its optimizing performance, and its essence of co-evolution is to change the evaluation method of individual viability. At this time, the fitness of every individual is not only with itself, but also with the other individuals which is in the other sub-populations. When the population is divided into two sub-populations, its co-evolutionary process can be described as following [12].

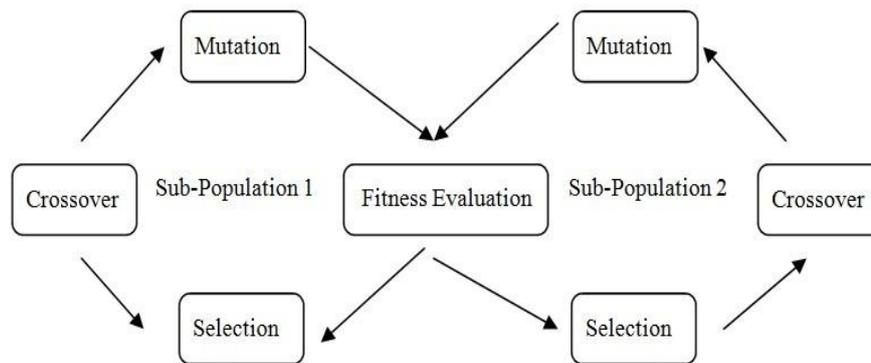


Figure 1. A Diagram of the Co-evolutionary Process between two Sub-populations

Now there are two structural forms for CEAs to implement evolution, one is Competitive Co-evolutionary Algorithm (Com-CEA) and the other is Cooperative Co-evolutionary Algorithm (Coo-CEA) [13][14]. In Coo-CEA, the fitness of individual is not evaluated in isolation; it is firstly in accordance with the priori knowledge of solving problem, subsequently, it combines the "representatives" individuals from the other sub-populations in order to construct a solution vector of solving problem and then to evaluate the fitness of individuals. This process is called "cooperation", because the ultimate evaluation result of the individual is decided by the following two conditions:

- (1) Whether it helps to solve the problem;
- (2) What extent it helps to solve the problem.

In addition, as for Com-CEA, the individual fitness in a sub-population is determined by the competition results which include a series of competition with the individuals in other sub-population. At this time, these two sub-populations always take the role of "Host" population and "Parasite" population in order. The evolutionary operations will be based on these in order to produce new sub-populations. With the development of the evolution, these different sub-populations will show different genetic characteristics.

2.2 Quantum Evolutionary Algorithm

Quantum Evolutionary Algorithm (QEA) has the advantages of quantum computing and evolutionary computation; it is also characterized by representation of the individual, population, the evaluation function and the evolutionary process. However, QEA uses Q-bit as a probabilistic representation in order to improve the diversity of population. Meanwhile, quantum rotation gate is used as the update operation instead of select, crossover and mutation operations in GA, which can guide the search direction of individual to the optimal area, and increase the algorithm's convergence speed. Han K. pointed out that QEA could avoid the phenomena of stagnation and premature convergence compared with traditional evolutionary algorithms [15].

As for the QEA, Q-bit may be in the "1" state, "0" state, or any superposition of these two states. It can be represented as following:

$$|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

(1)

Where α and β are complex numbers that specify the corresponding probability amplitudes of the states “0” and “1”. Normalization of the two states to unity always guarantees:

$$|\alpha|^2 + |\beta|^2 = 1$$

(2)

So a Q-bit in QEA is defined with a pair of number $\begin{bmatrix} \alpha \\ \beta \end{bmatrix}$.

In generally, a quantum individual q is defined as a string of Q-bits,

$$q_i = \begin{bmatrix} \alpha_{i1} & \alpha_{i2} & \dots & \alpha_{im} \\ \beta_{i1} & \beta_{i2} & \dots & \beta_{im} \end{bmatrix}, i = 1, 2, \dots, n$$

(3)

The population $Q(t) = \{q_1, q_2, \dots, q_n\}$, where i is the size of population, and m is the number of Q-bits. Since the Q-bits representation is able to express as a linear superposition of states probabilistically, it is profitable for generating diversity in the evolutionary process, which is also a major advantage of QEA. The further details about QEA can be seen in papers [6] [16].

3. Novel Quantum-Inspired Co-Evolutionary Algorithm

This paper combines the advantage of quantum-inspired evolutionary algorithm and co-evolution strategy, and then proposes a novel quantum-inspired co-evolutionary algorithm (NQCEA). At the first, the initial population will be divided into multiple sub-populations, and then every sub-population will take a standard QEA evolutionary process; subsequently, this NQCEA will complete the following two new evolutionary strategies in order to improve its global optimization ability.

(1) Evolutionary strategy 1

Elite individual generally corresponds to the individual with a higher fitness of the individual and contains corresponding characteristic information which will play a very important role on the process of evolution.

In this case, every sub-population will correspond with an elite individual which actually contains the characteristics information of current sub-population in the implementation process of algorithm. Thus, these elite individuals can be collected to construct an elite library. Accompanied by the evolution of the whole population, this elite library is also constantly updated. Subsequently, this new elite library can be used to judge the evolution of each sub-population between superiority and inferiority to a certain extent.

On the basis of the above, this NQCEA would rely on the individuals in elite library as one of the standards of evaluating the corresponding sub-populations. Here the best individual in the elite library will be used to guide the worst sub-population instead of the whole.

(2) Evolutionary Strategy 2

The current experience had shown that the evolution population should maintain its diversity in order to keep its ability to produce good offspring and effectively avoid the premature convergence and stagnation [17].

Normally, when the optimization algorithms use the co-evolutionary strategy, its sub-populations are relatively independent. At the end of each generation, if the diversity of sub-populations is not identified, it would be gradually reduced in the later iterative process of algorithm. This would be resulting in a gene decrease for effective individuals, affecting the optimization process of the algorithm and finally causing a premature convergence. Therefore, it is necessary to design a mechanism to maintain the diversity of the population.

In this paper, a modified indicator of degree of population diversity is introduced to measure the diversity of sub-populations based on the reference [18]. Here this modified indicator will include the characteristic information of every individual in each sub-population and it can also be as a parameter involved in the evolution of the population.

When the Q-bits are applied to optimization problems, they will converge to the corresponding binary encoding space, which is $\{0,1\}^L$, L is the length of binary code, the population size is n . A group of individuals in this population can be described: $Q = \{q_1, q_2, \dots, q_n\}$, where $q_j = \{q_{1j}, q_{2j}, \dots, q_{Lj}\}$, and $j = 1, 2, \dots, n$, then the degree of population diversity is defined as following formula (3-1):

$$div(Q) = \exp\left(-\frac{\left|\sum_{l=1}^L \sum_{j=1}^n q_{lj} - \sum_{l=1}^L \sum_{j=1}^n (1-q_{lj})\right|}{sizepop}\right) \quad (4)$$

Where $sizepop$ is the scale of individuals in population, and $sizepop = L \times n / 2$, $\sum_{j=1}^n q_{lj}$ represent the numbers of binary code "1" of all the individuals in their l th bit

in this population; while $\sum_{j=1}^n (1-q_{lj})$ represent the numbers of binary code "0" of all the individuals in their l th bit in this population. And the calculation result of

$\left|\sum_{l=1}^L \sum_{j=1}^n q_{lj} - \sum_{l=1}^L \sum_{j=1}^n (1-q_{lj})\right|$ indicates the distribution situation of binary code "0" and "1" of all the individuals in this population. In each iteration, we could see that the greater the result, the more unbalanced distribution between the binary code "1" and "0", which also means to such an extent that the population diversity is in reducing.

Finally, when the result of $div(Q)$ becomes less which shows that the diversity of this population is getting worse.

As mentioned above formula (4), when the NQCEA in this paper divides the population into many sub-populations, every sub-population will has its own $div(Sub_Q)$, and here firstly $div(Sub_Q)$ should be used to measure its degree of diversity during the entire evolutionary process.

In addition, mutation operator can produce a random disturbance for the candidate solution in order to get the new generations. In general, the efficiency of evolutionary algorithm relies heavily on the performance of mutation operation [19]. In this paper, the NQCEA will implement mutation operation for the worst sub-population with smallest div in order to improve its diversity. It should be noted that the changes of div will be related to the probability of mutation. Therefore, when the div becomes less which means the diversity of population is worse, while the probability of mutation will be increasing. Through the mutation operation, it can maintain the diversity of population better, and let the population improve their ability to expand the solution space.

4. Experimental Results

In order to illustrate the effectiveness of NQCEA, it is compared with the traditional QEA [20]. The maximum iteration times of these two algorithms are 100, and the initial population size is 200. As for NQCEA, this paper divided it into four sub-populations which mean the sub-population size is 50. In addition, the solving of numerical optimization function problems do not require specialized knowledge in a particular field of study, and then can also reflect the actual performance of the algorithms. Therefore, this section using some existing numerical optimization benchmark functions to evaluate the performance of the above two algorithms and four benchmark functions are selected from the literature [21].

Four test functions are selected as follows:

Function 1: Shaffer's F6 Function

$$f_1(x) = 0.5 - \frac{\sin^2 \sqrt{x_1^2 + x_2^2} - 0.5}{[1 + 0.001(x_1^2 + x_2^2)]^2}, \quad x_i \in [-10, 10]$$

Function 1 has unlimited local maximum points. Of these, only (0, 0) is the global maximum point, and the maximum value is 1. Here the range of the independent variables is restricted to (-100, 100). When the optimized result is greater than 0.995, it can be considered that the algorithm converges globally.

Function 2 : Goldstein-Price Function

$$f_2(x, y) = [1 + (x + y + 1)^2(19 - 14x + 3x^2 - 14y + 6xy + 3y^2)] \times [30 + (2x - 3y)^2(18 - 32x + 12x^2 + 48y - 36xy + 27y^2)]$$

Function 2 has four local minimum points: (1.2, 0.8), (1.2, 0.2), (-0.6, -0.4), (0, -1). And its global minimum point is (0, -1), which corresponding value is 3. Here the range of the independent variables is restricted to (-2, 2). When the optimized result is less than 3.005, it can be considered that the algorithm converges globally.

Function 3 : Shubert Function

$$f_3(x, y) = \left\{ \sum_{i=1}^5 i \cos[(i+1)x + i] \right\} \left\{ \sum_{i=1}^5 i \cos[(i+1)y + i] \right\} + 0.5[(x + 1.42513)^2 + (y + 0.80032)^2]$$

Function 3 has 760 local minimum points, only (-1.42513, -0.80032) is the global minimum point, which corresponding value is -186.73090882259. Here the range of the independent variables is restricted to (-10, 10). As for function 3, it is particularly easy to fall into the local minimum value -186.34. When the optimized

result is less than -186.34, it can be considered that the algorithm converges globally.

Function 5 : Multi-peaks function 2

$$f_5(x, y) = -x \sin \sqrt{|y+1-x|} \cos \sqrt{|y+1+x|} - (y+1) \cos \sqrt{|y+1-x|} \sin \sqrt{|y+1+x|}$$

Function 5 has unlimited local maximum points. Of these, only one 511.7319 is the global maximum value. Here the range of the independent variables is restricted to (-512,512). When the optimized result is greater than 511, it can be considered that the algorithm converges globally.

It should be noted that some of the above numerical optimization benchmark functions are to solve the minimum, and this paper will reverse these optimization functions into solving the maximum in order to handling uniformity. Here the simulation environment is based primarily on using MATLAB in implementing these algorithms and the corresponding computer programming environment is on windows XP operating system with Pentium Dual-Core CPU 2.80 GHz, 2.00 GB RAM. Each algorithm runs 10 times independently. The statistical results including the best optimal result, the average result, the worst results, and the standard deviations are described as following tables. Additionally, we also select the average elapsed time of these algorithms to indicate their operation complexity.

The results are shown in Table 1, Table 2, Table 3, and Table 4 respectively. It can be seen that the NQCEA is superior to QEA from the view of quality of solutions when these two optimization algorithms perform the same functions. The statistical results show that the NQCEA is more efficient in finding the global optimal solution and robustness than QEA. Such as the multi-peaks function 2, NQCEA also gives a good optimization result. In addition, the statistical results based on standard deviation indicate that NQCEA have a good stability and widespread adaptability. At the same time, the result of the average elapsed time shows that NQCEA has a little longer running time in the testing efficiency, and this is due to the NQCEA needs to split the population into many sub-populations, and it implements frequently the evolution operation across multiple sub-populations.

Table 1. Statistical Results of QEA and NQCEA for Function 1 (10 Experiments)

Performance	Algorithm	
	QEA	NQCEA
Best	1	1
Worst	0.99028	0.99028
Average	0.996108	0.998056
Convergence times	6	8
Standard Deviation	0.0047585	0.003888
Average Elapsed time	15.9737	18.0761

Table 2. Statistical Results of QEA and NQCEA for Function 2 (10 Experiments)

Performance	Algorithm	
	QEA	NQCEA
Best	-3	-3
Worst	-3.2316	-3
Average	-3.04632	-3
Convergence times	8	10
Standard Deviation	0.09264	0
Average Elapsed time	15.3439	17.5968

Table 3. Statistical Results of QEA and NQCEA for Function 3 (10 Experiments)

Performance	Algorithm	
	QEA	NQCEA
Best	186.7309	186.7309
Worst	185.9291	186.3391
Average	186.45270	186.61283
Convergence times	6	8
Standard Deviation	0.25504	0.17887
Average Elapsed time	14.6235	17.7534

Table 4. Statistical Results of QEA and NQCEA for Function 5 (10 Experiments)

Performance	Algorithm	
	QEA	NQCEA
Best	511.732	511.732
Worst	500.7907	509.8916
Average	509.7967	511.0962
Convergence times	3	7
Standard Deviation	3.0814	0.56053
Average Elapsed time	14.2245	17.0235

Meanwhile, the simulation results of NQCEA are briefly shown in Figure 2, Figure 3, Figure 4 and Figure 5. Although each sub-population is evolving independently, these four sub-populations will become synchronized toward convergence in the latter iterative process, and these results also present the following two conclusions. Firstly, the evolutionary strategy of constructing elite library is effective for the Co-evolutionary Quantum-Inspired Evolutionary Algorithm to a certain degree; on the other hand, these elite individuals just guide the selected sub-population instead of entire population toward the global optimum, which is enough to complete the optimize search and can improve the operating efficiency of the algorithm to some extent.

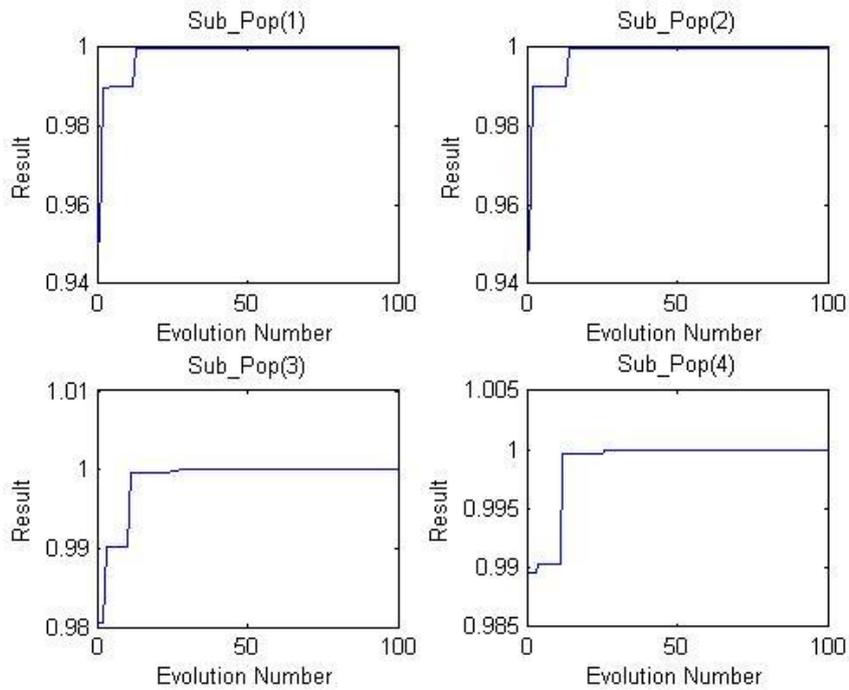


Figure 2. Evolutionary Results for Function 1 on the Basis of NQCEA

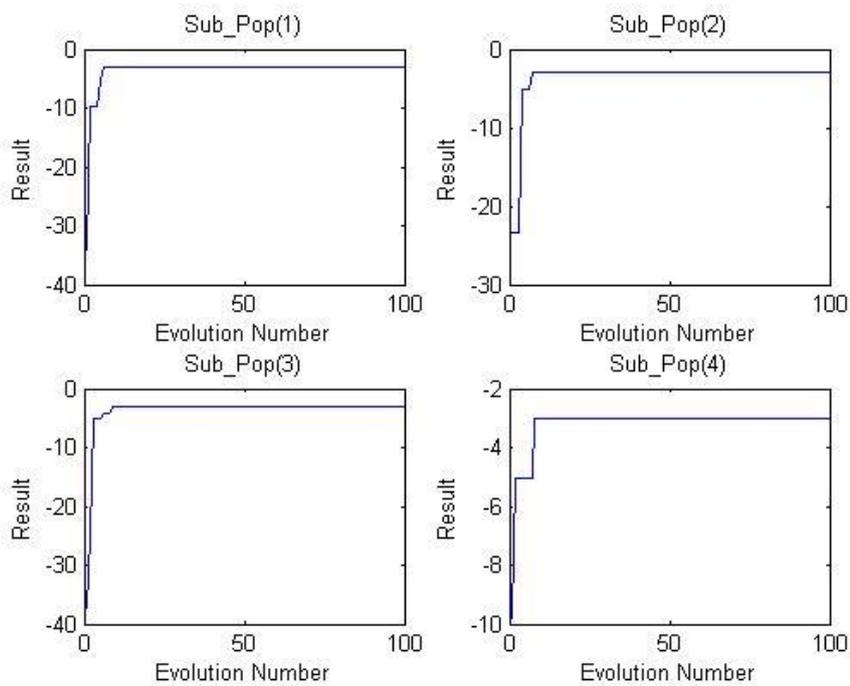


Figure 3. Evolutionary Results for Function 2 on the Basis of NQCEA

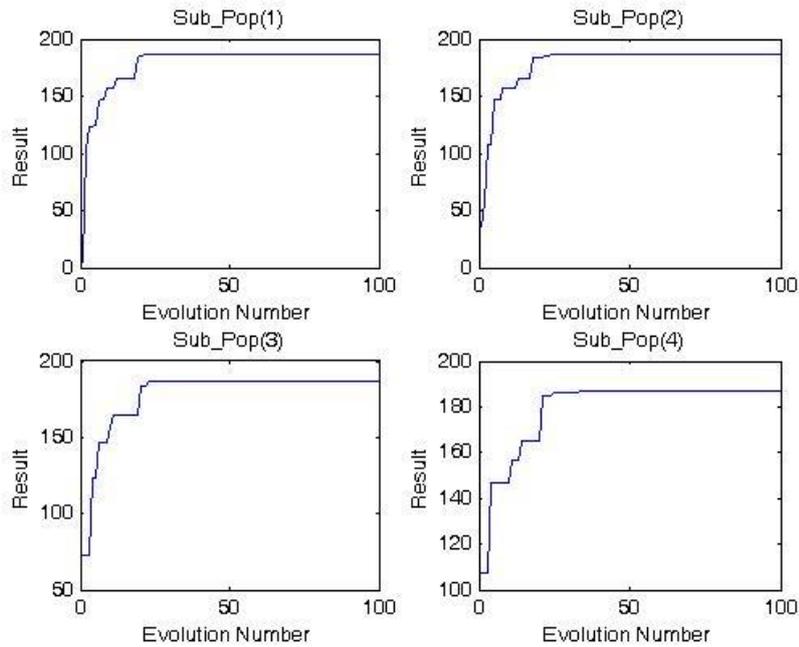


Figure 4. Evolutionary Results for Function 3 on the Basis of NQCEA

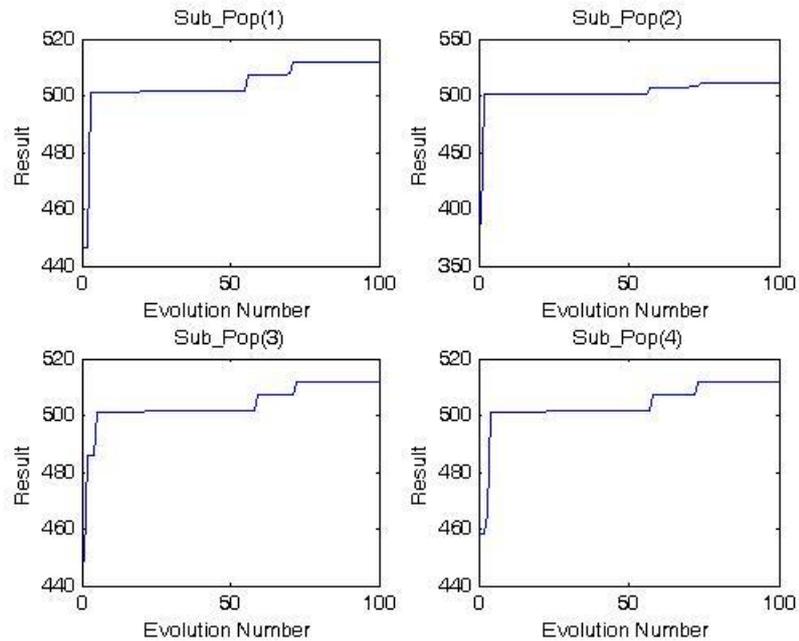


Figure 5. Evolutionary Results for Function 5 on the Basis of NQCEA

5. Conclusions

In order to promote the performance of QEA, we proposed the novel quantum-inspired co-evolutionary algorithm (NQCEA). This new NQCEA firstly divided the whole population into multiple sub-populations, which completed the evolution process independently, and then made full use of the characteristic information of each sub-population to implement the evolution process. The NQCEA is compared with the QEA using the benchmarks functions. It is observed that the NQCEA has

similar convergence speeds to the traditional QEA. However, the convergence performance for the NQCEA is superior to the QEA.

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