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Subgrade Settlement Prediction Based on Least Square Support Vector Regession and Real-coded Quantum Evolutionary Algorithm

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Abstract

Due to the normal forecasting methods for subgrade settlement using observation data have different applicabilities, and the predicting results has bigger volatility and lower accuracy. In view of the above problems, a method based on least square support vector regression (LSTSVR) and real-coded quantum evolutionary algorithm (RQEA) is proposed. Firstly, the LSTSVR parameter is chosen as a combinatorial optimization problem, and determining the objective function of the combinatorial optimization problem, then, using RQEA to solve the combinatorial optimization problem and optimize the LSTSVR parameters, Finaly, LSTSVR-RQEA is used to sovle the prediction of subgrade settlement. The simulation results show that RQEA is an effective method to select LSTSVR parameters, and has excellent performance when applied to the prediction of subgrade settlement.

Keywords*: Subgrade Settlement prediction, Combination forecast model, Least Square twin support vector regession, Real number encoding quantum evolutionary algorithm*

1. Introduction

The development of subgrade settlement has complex characteristics, such as non-linear, non-stabilization, and including numerous uncertain information, so it is very difficult to forecast the subgrade settlement accurately[1]. At present, the methods of subgrade settlement prediction mainly include layer-wise summation method (LWSM), numerical analysis method (NAM) and modeling method based on the observed data (MMBD) [1].

LWSM and NAM demand the precise geotechnical parameters and model of elasto plastic constitutive relation, but the geotechnical parameters and the model of elasto plastic constitutive relation is difficult to be acquired, so their application is limited. The observed data of subgrade settlement is an integrated reaction of all kinds of factors, and contains abundant information, then MMBD is paid more attention. MMBD includes experience formula method (EFM) (such as hyperbolic method, exponential curve method, growth-curve approach and parabolic method), system analysis and control theory method (grey system method and nerve network method). In a word, each forecasting model has their own advantages and their scopes of application, but there are some shortcomings too[2,3].

Least square support vector machine integrates the idea of least square and double support vector regression machine, a new modeling tool to build up, Least squares support vector regression machine only needs to solve two smaller scale linear equations to be able to get the regression function, rather than the dual

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support vector regression machine requires solving two two planning problems, it greatly reduces the computational complexity, and it has been widely used[3,4]. But the performance of least square support vector regression is influenced by the parameters of the least square support vector machine[5,6]. RQEA is a random search algorithm, which is based on the correlation theory and concept of quantum computation, it has the characteristics of fast convergence, strong global search ability and good stability[7,8]. In this paper, RQEA is used to select the relevant parameters of LSTSVR, forming LSTSVR-RQEA. Firstly, the selection of LSTSVR related parameters is considered as a combinatorial optimization problem of related parameters, the objective function of the optimization problem is established, then, RQEA is applied to solve the combinatorial optimization problem, and then optimize the relevant parameters of LSTSVR. It combines with the specific engineering examples, using LSTSVR-RQEA in the prediction of subgrade settlement. Simulation experiment shows that using LSTSVR-RQEA to solve the prediction of subgrade settlement, it has high accuracy.

2. Least Square Twin Support Vector Regession

Considering linear regression, the original optimization problem of Least Square twin support vector regession is:

$$\min \frac{1}{2}\left(Y-\left(Xw_1+eb_1\right)\right)^{\mathrm{T}}\left(Y-\left(Xw_1+eb_1\right)\right)+\frac{C_1}{2}\xi^{\mathrm{T}}\xi \tag{1}$$

$$\text{s.t.}\, Y-\left(Xw_1+eb_1\right)=-e\varepsilon_1-\xi \tag{2}$$

$$\min \frac{1}{2}\left(Y-\left(Xw_2+eb_2\right)\right)^{\mathrm{T}}\left(Y-\left(Xw_2+eb_2\right)\right)+\frac{C_2}{2}\eta^{\mathrm{T}}\eta \tag{3}$$

$$\text{s.t.}\, Y-\left(Xw_2+eb_2\right)=-e\varepsilon_2-\eta \tag{4}$$

Among formulas: $C_1>0$, $C_2>0$ are penalty parameters, $\varepsilon_1>0$, $\varepsilon_2>0$ are the parameters of insensitive loss function, ξ, η are relaxation factors. The aim of Least Square twin support vector regession is to find $f_1(x)=w_1^T x+b_1$ and $f_2(x)=w_2^T x+b_2$ to construct the final regression function. Geometric interpretation of Least Square twin support vector regession is shown as Figure 1.

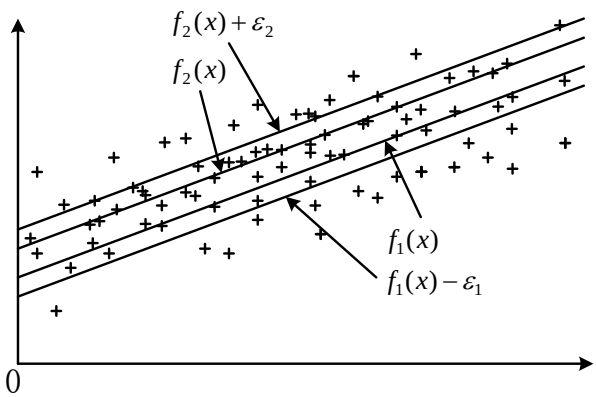

Figure 1. Geometric Interpretation of Least Square Twin Support Vector Regession

In order to solve the problems of (1) and (2), taking the constraint conditions into the objective function to obtain Lagrange function as follows:

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$$L=\frac{1}{2}\left\|Y-\left(Xw_1+eb_1\right)\right\|^2+\frac{C_1}{2}\left\|\left(Xw_1+eb_1\right)-Y-e\varepsilon_1\right\|^2 \tag{5}$$

Partial derivatives are as follows:

$$\frac{\partial L}{\partial w_1}=-X^T\left(Y-\left(Xw_1+eb_1\right)\right)+C_1X^T\left(\left(Xw_1+eb_1\right)-Y-e\varepsilon_1\right) \tag{6}$$

$$\frac{\partial L}{\partial b_1}=-e^T\left(Y-\left(Xw_1+eb_1\right)\right)+C_1e^T\left(\left(Xw_1+eb_1\right)-Y-e\varepsilon_1\right) \tag{7}$$

Making the formulas (6) and (7) equal to zero, we could obtain:

$$\begin{bmatrix} X^TX+C_1X^TX & X^Te+C_1X^Te \\ e^TX+C_1e^TX & e^Te+C_1e^Te \end{bmatrix}\begin{bmatrix} w_1 \\ b_1 \end{bmatrix}=\begin{bmatrix} X^T+C_1X^T & C_1X^Te \\ e^T+C_1e^T & C_1e^Te \end{bmatrix}\begin{bmatrix} Y \\ \varepsilon_1 \end{bmatrix} \tag{8}$$

$$\begin{bmatrix} w_1 \\ b_1 \end{bmatrix}=\left(1+C_1\right)\left(G^TG\right)^{-1}G^T\left[\left(1+C_1\right)Y+C_1\varepsilon_1e\right] \tag{9}$$

Among it, $G=\left[X,e\right]$.

Similarly, we also could get:

$$\begin{bmatrix} w_2 \\ b_2 \end{bmatrix}=\left(1+C_2\right)\left(G^TG\right)^{-1}G^T\left[\left(1+C_2\right)Y+C_2\varepsilon_2e\right] \tag{10}$$

The final regression function is:

$$f\left(x\right)=\frac{1}{2}\left(f_1\left(x\right)+f_2\left(x\right)\right)=\frac{1}{2}\left(w_1+w_2\right)^Tx+\frac{1}{2}\left(b_1+b_2\right) \tag{11}$$

Linear regression solving process of Least Square twin support vector regession is shown as Figure 2.

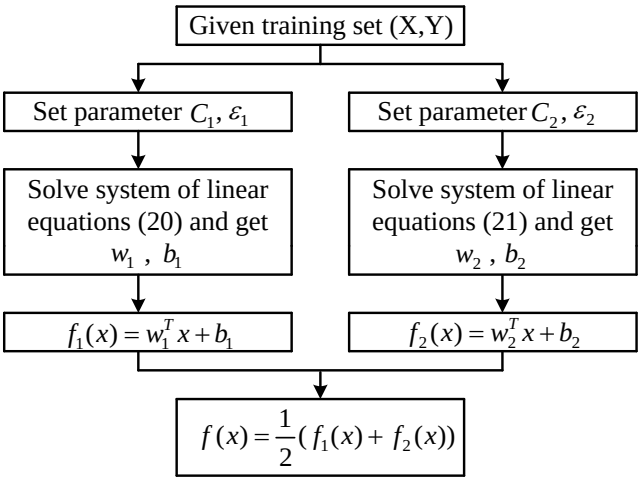

Figure 2. Flow Chart of Least Square Twin Support Vector Regession

Considering nonlinear regression, the task is to solve two nonlinear hyper planes $f_1\left(x\right)=u_1^TK\left(X,x\right)+\gamma_1$ and $f_2\left(x\right)=u_2^TK\left(X,x\right)+\gamma_2$. In order to get these two hyper planes, two optimization problems are established:

$$\min\frac{1}{2}\left(Y-\left(K\left(X,X^{\mathrm{T}}\right)u_1+e\gamma_1\right)\right)^{\mathrm{T}}\left(Y-\left(K\left(X,X^{\mathrm{T}}\right)u_1+e\gamma_1\right)\right)+\frac{C_1}{2}\xi^{\mathrm{T}}\xi \tag{12}$$

$$\text{s.t.}\,Y-\left(K\left(X,X^{\mathrm{T}}\right)u_1+e\gamma_1\right)=-e\varepsilon_1-\xi \tag{13}$$

$$\min\frac{1}{2}\left(Y-\left(K\left(X,X^{\mathrm{T}}\right)u_2+e\gamma_2\right)\right)^{\mathrm{T}}\left(Y-\left(K\left(X,X^{\mathrm{T}}\right)u_2+e\gamma_2\right)\right)+\frac{C_2}{2}\eta^{\mathrm{T}}\eta \tag{14}$$

$$\text{s.t.} Y-\left(K\left(X,X^{\mathrm{T}}\right)u_2+e\gamma_2\right)=-e\varepsilon_2-\eta \tag{15}$$

Among formulas: $C_1>0$, $C_2>0$ are penalty parameters, $\varepsilon_1>0$, $\varepsilon_2>0$ are the parameters of insensitive loss function, ξ, η are relaxation factors.

The final regression function is:

$$f(x)=\frac{1}{2}\left(f_1(x)+f_2(x)\right)=\frac{1}{2}\left(u_1+u_2\right)^T K(X,x)+\frac{1}{2}\left(\gamma_1+\gamma_2\right) \tag{16}$$

3. Real Number Encoding Quantum Evolutionary Algorithm

In Ref.[7,8], a real-coded chromosome, whose allele is composed of one component x_i of variable vector X and probability amplitudes $(\alpha_i,\beta_i)^T$ of one qubit, $i=1,2,\cdots,n$, in RQEA is represented as:

$$q=\begin{bmatrix} x_1 & x_2 & \cdots & x_n \\ \alpha_1 & \alpha_2 & \cdots & \alpha_n \\ \beta_1 & \beta_2 & \cdots & \beta_n \end{bmatrix} \tag{17}$$

Where n is the length of chromosome, and lies on the dimensions of variable vector X.

The single-gene mutation is adopted to update population at each iteration in RQEA. Assume that RQEA maintains a population $p(t)=\left\{p_j^t\right\}$ at the *t*-th iteration, where $j=1,2,\cdots,N$, N is the population size. Select the *i*-th gene $(x_{ji}^t,\alpha_{ji}^t,\beta_{ji}^t)^T$ of p_j^t, and update the value of $x_{j,i}^t$ using Gaussian mutation, which is expressed as:

$$x_{j,i}^{t+1,k}=x_{ji}^t+(x_{i,\max}-x_{i,\min})N(0,(\sigma_{j,i}^k)^2) \tag{18}$$

where $k\in\{\alpha,\beta\},(\sigma_{j,i}^k)^2$ denotes Gaussian distribution variance, and its value is designed as:

$$(\sigma_{j,i}^k)^2=\begin{cases} \left|\alpha_{j,i}^t\right|^2, & k=\alpha \\ \left|\alpha_{j,i}^t\right|^2/5, & k=\beta \end{cases} \tag{19}$$

To avid generation the infeasible solution, the value of $x_{j,i}^{t+1,k}$ is clipped according to Eq.(12). Until the value of $x_{j,i}^{t+1,k}$ lies in the feasible solution space, Eq.(5) has to be performed repeatedly.

$$\begin{cases} x_{j,i}^{t+1,k}=2x_{i,\max}-x_{j,i}^{t+1,k},\ x_{j,i}^{t+1,k}>x_{i,\max} \\ x_{j,i}^{t+1,k}=2x_{i,\min}-x_{j,i}^{t+1,k},\ x_{j,i}^{t+1,k}<x_{i,\min} \end{cases} \tag{20}$$

If the feasible solution derived from Eq.(10)~(12) $(x_{j,1}^t,\cdots,x_{j,i}^{t+1,k},\cdots x_{j,n}^t)^T)$ is superior to the feasible solution $(x_{j,1}^t,\cdots,x_{j,i}^t,\cdots x_{j,n}^t)^T$, then the valid evolution is carried out, and $x_{j,i}^t=x_{j,i}^{t+1,k}$, $\alpha_{j,i}^{t+1}=\alpha_{j,i}^t$, $\beta_{j,i}^{t+1}=\beta_{j,i}^t$. Otherwise ,the invalid

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evolution is done ,the feasible solution $(x_{j,1}^t,\cdots,x_{j,i}^t,\cdots x_{j,n}^t)^T$ is retained ,and $(\alpha_{ji}^t,\beta_{ji}^t)^T$ is updated by quantum rotation gates as:

$$\begin{bmatrix}\alpha_{j,i}^{t+1}\\ \beta_{j,i}^{t+1}\end{bmatrix}=\begin{bmatrix}\cos\left(\Delta\theta_{j,i}^t\right) & -\sin\left(\Delta\theta_{j,i}^t\right)\\ \sin\left(\Delta\theta_{j,i}^t\right) & \cos\left(\Delta\theta_{j,i}^t\right)\end{bmatrix}\begin{bmatrix}\alpha_{j,i}^{t+1}\\ \beta_{j,i}^{t+1}\end{bmatrix} \tag{21}$$

where $\Delta\theta_{j,i}^t$ is the rotation angle,and the value of $\Delta\theta_{j,i}^t$ is design as:

$$\Delta\theta_{j,i}^t=\operatorname{sgn}(\alpha_{j,i}^t\beta_{j,i}^t)\theta_0\exp(-\frac{\left|\beta_{j,i}^t\right|}{\left|\alpha_{j,i}^t\right|+\gamma}) \tag{22}$$

where $\operatorname{sgn}(\cdot)$ is the sign function and deternimes the direction of $\Delta\theta_{j,i}^t$, θ_0 is the initial rotation angle; γ is evolutionary scale. θ_0, γ and $(\alpha_{j,i}^t\beta_{j,i}^t)^T$ decide the size of $\Delta\theta_{j,i}^t$ together , control further the convergence rate.

In RQEA, discrete crossover is performed at period τ_c. Select individual p_u^t and p_v^t , $u\neq v$, $u=1,2,\cdots,N$, $v=1,2,\cdots,N$, at random in population, let p_u^t and p_v^t as parents,exchange ever corresponding gene of them by 0.5 probability, and generate new individual c^t.

4. Optimizing LSTSVR Parameters based on RQEA

LSTSVR parameters include loss function ($\varepsilon_1,\varepsilon_2$), penalty factor ($C_1$, C_2), in the case of selecting the Gauss kernel function, it also includes width parameter σ of kernel function. Under normal conditions, there are 3 parameters. LSTSVR parameter selection problem can be seen as combinatorial optimization problem of parameter C, σ, ε, or optimal matching problem, mean square error can be obtained through fitting value $\hat{y}_i$ and sample value y_i as:

$$MSE=\left(\frac{1}{l}\sum_{i=1}^{l}\left(\hat{y}_i-y_i\right)^2\right)^{\frac{1}{2}} \tag{23}$$

It can regard as LSTSVR performance evaluation index, so LSTSVR parameter selection problem can transform into the continuous function optimization problem which generated through C, σ and ε on R^3 space.

$$\begin{aligned}&\min f\left(\varepsilon,C,\sigma\right)=\min MSE\\ &\text{s.t. } \varepsilon_{\min}\le\varepsilon\le\varepsilon_{\max}\\ &\quad C_{\min}\le C\le C_{\max}\\ &\quad \sigma_{\min}\le\sigma\le\sigma_{\max}\end{aligned} \tag{24}$$

For a set of given parameter $\left(C,\sigma,\varepsilon\right)$, we could solve quadratic programming problem to train sample data to establish LSTSVR model, then calculating output estimate that corresponding to the sample input according to established LSTSVR model, and to calculate MSE. If choosing MSE as the fitness function, we could use RQEA to solve optimization problem (19), which has the characteristics of fast convergence, strong global search ability and good stability, to optimize LSTSVR parameters, and forming LSTSVR –RQEA model.

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5. Engineering Example

In this paper, choose Ning-Hang highway subgrade NH standard K095+520 section as observation point, select 15 group settlement observation data from 2001-08-09 to 2002-12-14 as research object. Using Usher model, Logistic model, Gompertz model and combined forecasting model to model and forecast, the first 11 data are used to model and after 4 data are used to forecast.

Forecast results and relative errors of each single forecasting model and combined forecasting model are shown in table 1. Forecast results contrast of each single forecasting model and combined forecasting model is shown in figure 3, relative error contrast is shown in figure 4.

Table 1. Four Kinds of Forecast Methods Comparison of Forecast Results

NO.	Measured values (cm)	Usher		Logistics		Gompertz		LSTSVR-RQEA	
		Predicted values (cm)	Relative error (%)	Predicted values (cm)	Relative error (%)	Predicted values (cm)	Relative error (%)	Predicted values (cm)	Relative error (%)
1	5.35	5.5590	-3.9067	5.6292	-5.2187	5.6878	-6.3139	5.4121	-1.1607
2	5.88	6.1583	-4.7323	6.2051	-5.5293	6.2218	-5.8124	5.7932	1.4761
3	6.56	6.3039	3.9047	6.3465	3.2549	6.3518	3.1736	6.5487	0.1722
4	7.48	6.9992	6.4282	7.0282	6.0012	6.9780	6.7119	7.5321	-0.6965
5	7.82	7.6306	2.4221	7.6540	2.1229	7.5595	3.3312	7.7667	0.6815
6	8.05	7.9754	0.9271	7.9968	0.6614	7.8854	2.0452	8.02900	0.2608
7	8.38	8.1905	2.2608	8.2105	2.0227	8.0927	3.4282	8.4387	-0.7004
8	8.59	8.6772	-1.0156	8.6915	-1.1821	8.5763	0.1599	8.5801	0.1152
9	8.84	9.0932	-2.8643	9.0973	-2.9103	9.0107	-1.9309	8.8371	0.0328
10	9.01	9.2409	-2.5629	9.2394	-2.5464	9.1711	-1.7880	9.0465	-0.4051
11	9.18	9.4438	-2.8735	9.4324	-2.7499	9.3979	-2.3735	9.2186	-0.4204
12	9.81	9.7359	0.7554	9.7046	1.0744	9.7407	0.7069	9.7865	0.2395
13	10.04	9.9770	0.6274	9.9225	1.1699	10.0423	-0.0234	10.0187	0.2121
14	10.06	10.0198	0.3999	9.9604	0.9899	10.0981	-0.3784	10.0945	-0.3429
15	10.11	10.1745	-0.6382	10.0951	0.1472	10.3066	-1.9443	10.1246	-0.1444

From table 1, we can see that the relative error maximum of fitting value of LSTSVR model is 1.4761%, it is better than one of Usher model which is 6.4282%, Logistics model which is 6.0012% and Gompertz model which is 6.7119%, the elative error maximum of predicting value of LSTSVR model is 0.3429%, it is lower than the one of Usher model is 0.7554%, Logistic model is 1.0744% and Gompertz model is 1.9443%. So the fitting and forecasting precision of LSTSVR model is more superior to each single forecasting model.

In figure 3, the settlement development curves from each forecasting model are describes repectively. We could know that LSTSVR model can reflects the characteristics and law of settlement development curve in both fitting period and forecasting period, but each single forecasting model has good performance at some time.

In figure 4, the error curves of each forecasting model are illustrates repectively. We could see that the relative error of single forecasting model appears larger fluctuations at the beginning of the model period, and the relative error is also larger during the forecast period, but the relative error of predicted results of LSTSVR model is relatively stable, it reduces the risk of forecasting.

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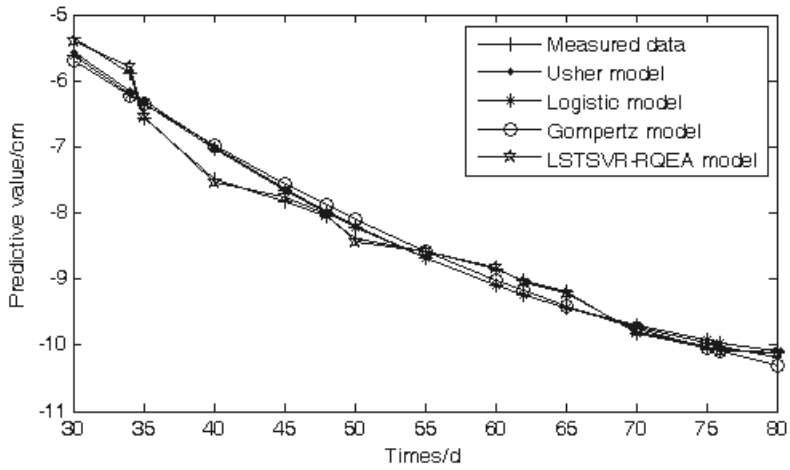

Figure 3. Predictive Effect of Four Kinds of Forecasting Models

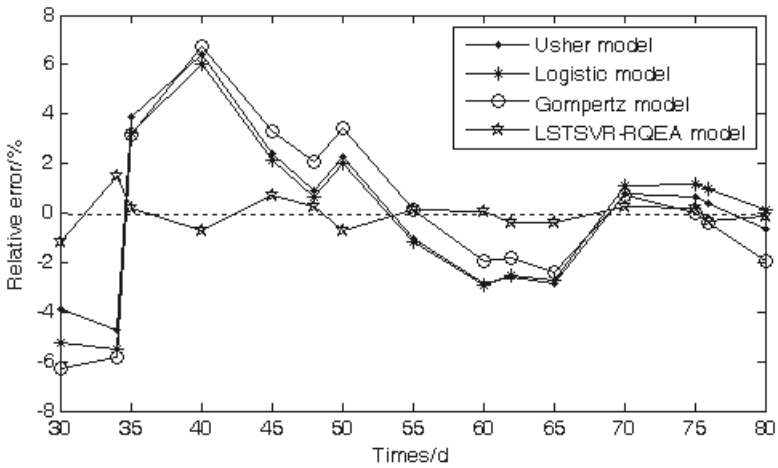

Figure 4. Relative Error of Four Kinds of Forecasting Model

6. Conclusion

In view of the selection of LSTSVR related parameters directly affect the performance of LSTSVR, in this paper, the selection of LSTSVR related parameters is considered as a combinatorial optimization problem which is composed of LSTSVR related parameters, and RQEA is used to solve the problem, and then optimize the relevant parameters of LSTSVR, forming LSTSVR-RQEA, it provides an effective tool for the selection of LSTSVR related parameters, it also expands the application field of LSTSVR. The effectiveness of the method is proved by the simulation test, the application of LSTSVR-RQEA to solve the prediction problem of subgrade settlement has a high precision, it has certain practical value.

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