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Modified Sliding Mode Control for Mismatched Uncertain Large-Scale Systems

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Abstract

In this paper, a modified sliding mode control is proposed to stabilize a class of large-scale systems with mismatched uncertainties, unknown exogenous disturbances and without the measurements of the states. Based on the Lyapunov method, appropriate linear matrix inequality stability conditions are derived to guarantee the stability of the system. Then, a modified adaptive sliding mode control law is proposed to guarantee the finite time reachability of the system states by using output feedback only. Final, a double-inverted pendulums system connected by a spring is simulated to demonstrate the efficacy of the proposed method.

Keywords*: Large-scale systems, Output feedback controller, Linear matrix inequalities (LMI), Sliding mode control (SMC).*

1. Introduction

With the development of engineering systems, nowadays the systems are getting more and more complex and large. To control these systems, it is essential to solve not only conventional problems, such as nonlinearity, but also additional problems such as: high dimensionality, unmodeled dynamics, and uncertain or unknown disturbances. Because of these problems, a decentralized control technique is more feasible than the centralized control one. The advantage of decentralized control is to use local signals at the level of each subsystem in the controller implementation for large-scale interconnected systems, and this therefore overcomes the limitations of the traditional centralized control. Therefore, many decentralised adaptive control schemes have been proposed for various large-scale systems [1-10]. The works in [1] investigated stabilization problems for interconnected systems without a priori knowledge of subsystems' high- frequency gain signs. The authors of [2] proposed a decentralised adaptive output-feedback control scheme for large-scale systems with three types of uncertainties, including parametric uncertainties, nonlinear uncertain interactions and stochastic inverse dynamics. In [3] a new totally decentralized adaptive output tracking scheme was developed for a class of mismatched uncertain interconnected systems by using a backstepping approach. In [4], the problem of decentralised output-feedback stabilization was presented for a class of large-scale high-order stochastic non-linear systems. The problem of robust decentralized adaptive output feedback stabilization for a class of interconnected systems with dynamic input and output interactions was considered in [5]. In [6], the decentralised adaptive identifier was proposed to design the decentralised L_1 adaptive control scheme for a class of interconnected systems. In order to stabilize a class of nonlinear interconnected systems with matched uncertainties, the authors [7] proposed a new decentralized dynamic output feedback based linear controller. In [8], a decentralized adaptive approximation design was presented for the fault tolerant control of a class interconnected feedback linearizable nonlinear systems. More recently, the controller design for a class

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of stochastic interconnected systems with both parametric uncertainties and unknown nonlinear interactions was presented in [9]. In [10], based on fuzzy system, a decentralized fuzzy control problem was developed for asymptotic stabilization of a class of nonlinear large scale systems that used an observer-based output-feedback scheme. As a result, the closed-loop of large-scale systems driven by the proposed decentralized adaptive control scheme is asymptotical stable. However, it should be noted that most of the existing results for large-scale systems are required to have a special structure [1-5]. In addition, these approaches given in [6-10] cannot be directly applied for large scale systems with mismatched parameter uncertainties in the state matrix of each isolated subsystem and unknown exogenous disturbance.

In this paper, a modified sliding mode control is proposed to stabilize a class of large-scale systems with mismatched uncertainties, unknown exogenous disturbance and without the measurements of the states. The first, sufficient conditions in the term of LMI are derived such that the system in the sliding mode dynamics is asymptotically stable. Then, a modified adaptive sliding mode control law is proposed to guarantee the finite time reachability of the system states by using output feedback only. Finally, a numerical example is used to demonstrate the efficacy on the method

2. Problem Formulation

In this paper, we investigate a class of interconnected systems consisting of L interconnected subsystems modelled as below

$$\dot{x}_i = (A_i + \Delta A_i)x_i + B_i(u_i + G_i(t,x_i)) + \sum_{\substack{j=1 \\ j\neq i}}^{L} H_{ij}x_j \tag{1}$$

$$y_i = C_i x_i$$

where $x_i \in R^{n_i}$, $u_i \in R^{m_i}$ and $y_i \in R^{p_i}$ represent the state vector, control input to be designed and outputs of the ith subsystem, respectively; the triples $(A_i,\ B_i,\ C_i)$ and H_{ij} comprises constant matrices of appropriate dimensions with B_i and C_i both being of full rank; $\Delta A_i x_i$ and $G_i(t,x_i)$ are the mismatched and matched uncertainties, respectively. In this paper, only the output variables y_i are assumed to be available for measurements.

First, the following basic assumptions are made on the system(1):

Assumption 1: ΔA_i is of the form $D_i F_i(x_i,t) E_i$ where $F_i(x_i,t)$ is unknown but bounded as $\|F_i(x_i,t)\| \le 1$, and $D_i,\ E_i$ are known matrices of appropriate dimensions.

Assumption 2: $rank(C_i B_i) = m_i$.

Assumption 3: The exogenous disturbance $G_i(t,x_i)$ is assumed to be bounded by an r-order polynomial of the norm of the output variables

$$\|G_i(t,x_i)\| \le b_{i1} + b_{i2}\|y_i\| + b_{i3}\|y_i\|^2 + \ldots, b_{ir}\|y_i\|^{r-1}$$

where the scalars $b_{i1},\ b_{i2},\ b_{i3},\ \ldots,\ b_{ir}$ are unknown bounds, which are not easily obtained due to the complicated structure of the uncertainties in practical control systems.

Second, we will need the following lemmas.

Lemma 1 [11]: Let X, Y and F are real matrices of suitable dimension with $F^T F \le I$ then, for any scalar $\varphi > 0$, the following matrix inequality holds:

$$XFY + Y^T F^T X^T \le \varphi^{-1} X X^T + \varphi Y^T Y.$$

Lemma 2 [12]: The following matrix inequality:

$$\begin{bmatrix} Q(x) & \Pi(x) \\ \Pi(x)^T & R(x) \end{bmatrix} > 0$$

where $Q(x) = Q(x)^T$, $R(x) = R(x)^T$ and $\Pi(x)$ depend affinity on x, is equivalent to $R(x) > 0$, $Q(x) - \Pi(x)R(x)^{-1}\Pi(x)^T > 0.$

Lemma 3 [13]: Let X and Y are real matrices of suitable dimension then, for any scalar $\mu > 0$, the following matrix inequality holds:

$$X^T Y + Y^T X \le \mu X^T X + \mu^{-1} Y^T Y.$$

3. Stability Analysis of Sliding Mode Dynamics

Under Assumption 2, it follows from equations (11), (12) and (13) of paper [14] that there exists a coordinate transformation $z_i = T_i x_i$ such that the system (1) has following form

$$\dot{z}_i = \left(\begin{bmatrix} A_{i1} & A_{i2} \\ A_{i3} & A_{i4} \end{bmatrix} + \begin{bmatrix} D_{i1} \\ D_{i2} \end{bmatrix} \Delta F_i \begin{bmatrix} E_{i1} & E_{i2} \end{bmatrix} \right) z_i + \begin{bmatrix} 0 \\ B_{i2} \end{bmatrix} [u_i + G_i(t)] + \sum_{\substack{j=1 \\ j \ne i}}^{L} \begin{bmatrix} H_{ij1} & H_{ij2} \\ H_{ij3} & H_{ij4} \end{bmatrix} z_j \qquad (2)$$

$$y_i = \begin{bmatrix} 0 & C_{i2} \end{bmatrix} z_i$$

where $T_i A_i T_i^{-1} = \begin{bmatrix} A_{i1} & A_{i2} \\ A_{i3} & A_{i4} \end{bmatrix}$, $T_i B_i = \begin{bmatrix} 0 \\ B_{i2} \end{bmatrix}$, $C_i T_i^{-1} = \begin{bmatrix} 0 & C_{i2} \end{bmatrix}$, $T_i H_{ij} T_j^{-1} = \begin{bmatrix} H_{ij1} & H_{ij2} \\ H_{ij3} & H_{ij4} \end{bmatrix}$

$T_i D_i \Delta F_i E_i T_i^{-1} = \begin{bmatrix} D_{i1} \\ D_{i2} \end{bmatrix} \Delta F_i \begin{bmatrix} E_{i1} & E_{i2} \end{bmatrix}$, $\quad T_i D_{ij} \Delta F_{ij} E_{ij} T_j^{-1} = \begin{bmatrix} D_{ij1} \\ D_{ij2} \end{bmatrix} \Delta F_{ij} \begin{bmatrix} E_{ij1} & E_{ij2} \end{bmatrix}$.

The matrices $B_{i2} \in R^{m_i \times m_i}$ and $C_{i2} \in R^{p_i \times p_i}$ are non-singular.

Now, in order to fully use the available structure characteristics, partition $z_i = \begin{bmatrix} z_{i1} \\ z_{i2} \end{bmatrix}$

where $z_{i1} \in R^{n_i - m_i}$, $z_{i2} \in R^{m_i}$ then the first equation of (2) can be rewritten as

$$\dot{z}_{i1} = (A_{i1} + D_{i1}\Delta F_i E_{i1}) z_{i1} + (A_{i2} + D_{i1}\Delta F_i E_{i2}) z_{i2} + \sum_{\substack{j=1 \\ j \ne i}}^{L} (H_{ij1} z_{j1} + H_{ij2} z_{j2}) \qquad (3)$$

$$\dot{z}_{i2} = (A_{i3} + D_{i2}\Delta F_i E_{i1}) z_{i1} + (A_{i4} + D_{i2}\Delta F_i E_{i2}) z_{i2} + B_{i2}[u_i + G_i(t, T_i^{-1} z_i)] + \sum_{\substack{j=1 \\ j \ne i}}^{L} (H_{ij3} z_{j1} + H_{ij4} z_{j2}). \qquad (4)$$

Obviously, the system (3) represents the sliding-motion dynamic of the system (2), and hence, the composite sliding surface for the interconnected systems (2) can be chosen as follows

$$\sigma_i(x_i) = F_i y_i = F_i C_i x_i = F_i \begin{bmatrix} 0 & \mathrm{C}_{i2} \end{bmatrix} z_i = F_{i2} z_{i2} = 0 \qquad (5)$$

where $F_i \in R^{m_i \times p_i}$ are obtained from the algorithm given in [15] and $F_{i2} \in R^{m_i \times m_i}$ is non-singular. From the nonsingularity of $F_{i2} \in R^{m_i \times m_i}$, in sliding mode $\sigma_i(x_i) = 0$ and $\dot{\sigma}_i(x_i) = 0$, we have $z_{i2} = 0$. Then, from the structure of system (3)–(4), the sliding mode dynamics of system (1) associated with the sliding surface (5) is described by

$$\dot{z}_{i1} = (A_{i1} + D_{i1}\Delta F_i E_{i1})z_{i1} + \sum_{\substack{j=1 \\ j \neq i}}^{L} H_{ij1} z_{j1}. \tag{6}$$

Obviously, (15) is a reduced order interconnected system composed of L subsystems with dimension $n_i - m_i$. Next, a stability result will be presented for the interconnected system (6).

Theorem 1: Consider the sliding mode dynamics given by equation (6). Under assumptions 1– 3, the sliding motion is asymptotically stable if there exists a matrix $P_i > 0$ satisfying the following LMI

$$\begin{bmatrix} \Psi_i & P_i D_{i1} & E_{i1}^T & P_i \\ D_{i1}^T P_i & -\varphi_i I_{m_i} & 0 & 0 \\ E_{i1} & 0 & -\varphi_i^{-1} I_{m_i} & 0 \\ P_i & 0 & 0 & -\frac{\hat{\varphi}_i}{L-1} I_{(n_i - m_i)} \end{bmatrix} < 0, \quad i = 1, 2, ..., L. \tag{7}$$

for some constants $\varphi_i > 0,\ \bar{\varphi}_i > 0$ and $\hat{\varphi}_i > 0$, where $\Psi_i = A_{i1}^T P_i + P_i A_{i1} + \sum_{\substack{j=1 \\ j \neq i}}^{L} \hat{\varphi}_j H_{ji1}^T H_{ji1}$.

Proof: Now we are going to prove the sliding mode dynamics given by equation (6) is asymptotically stable. For system (6), consider the Lyapunov function candidate

$$V = \sum_{i=1}^{L} z_{i1}^T P_i z_{i1} \tag{8}$$

where the matrix $P_i \in R^{(n_i - m_i)\times(n_i - m_i)} > 0$ is defined in LMI (7). Then, taking the time derivative of V along the state trajectories of the sliding mode dynamics (6), we can obtain that

$$\begin{aligned} \dot{V} = &\sum_{i=1}^{L} z_{i1}^T (A_{i1}^T P_i + P_i A_{i1} + E_{i1}^T \Delta F_i^T D_{i1}^T P_i + P_i D_{i1} \Delta F_i E_{i1}) z_{i1} \\ &+ \sum_{i=1}^{L} \sum_{\substack{j=1 \\ j \neq i}}^{L} (z_{j1}^T H_{ij1}^T P_i z_{i1} + z_{i1}^T P_i H_{ij1} z_{j1}). \end{aligned} \tag{9}$$

Applying Lemma 1 to equation (9), it is easy to get that

$$\begin{aligned} \dot{V} \le &\sum_{i=1}^{L} z_{i1}^T (A_{i1}^T P_i + P_i A_{i1} + \varphi_i^{-1} P_i D_{i1} D_{i1}^T P_i + \varphi_i E_{i1}^T E_{i1}) z_{i1} \\ &+ \sum_{i=1}^{L} \sum_{\substack{j=1 \\ j \neq i}}^{L} (z_{j1}^T H_{ij1}^T P_i z_{i1} + z_{i1}^T P_i H_{ij1} z_{j1}) \end{aligned} \tag{10}$$

where the scalar $\varphi_i > 0$. By Lemma 3, it follows that for any $\hat{\varphi}_i > 0$

$$\sum_{i=1}^{L} \sum_{\substack{j=1 \\ j \neq i}}^{L} (z_{j1}^T H_{ij1}^T P_i z_{i1} + z_{i1}^T P_i H_{ij1} z_{j1}) \le \sum_{i=1}^{L} \sum_{\substack{j=1 \\ j \neq i}}^{L} (\hat{\varphi}_i z_{j1}^T H_{ij1}^T H_{ij1} z_{j1} + \frac{1}{\hat{\varphi}_i} z_{i1}^T P_i P_i z_{i1}) \tag{11}$$

According to equations (10) and (11), it is obviously that

$$\dot{V} \le \sum_{i=1}^{L} z_{i1}^T (A_{i1}^T P_i + P_i A_{i1} + \varphi_i^{-1} P_i D_{i1} D_{i1}^T P_i + \varphi_i E_{i1}^T E_{i1}) z_{i1} + \sum_{i=1}^{L} \sum_{\substack{j=1 \\ j \ne i}}^{L} (\hat{\varphi}_i z_{j1}^T H_{ij1}^T H_{ij1} z_{j1} + \frac{1}{\hat{\varphi}_i} z_{i1}^T P_i P_i z_{i1}) \tag{12}$$

From property $\sum_{i=1}^{L} \sum_{\substack{j=1 \\ j \ne i}}^{L} \hat{\varphi}_i z_{j1}^T H_{ij1}{}^T H_{ij1} z_{j1} = \sum_{i=1}^{L} \sum_{\substack{j=1 \\ j \ne i}}^{L} \hat{\varphi}_j z_{i1}^T H_{ji1}{}^T H_{ji1} z_{i1}$, it generates

$$\begin{aligned} \dot{V} &\le \sum_{i=1}^{L} z_{i1}^T (A_{i1}^T P_i + P_i A_{i1} + \varphi_i^{-1} P_i D_{i1} D_{i1}^T P_i + \varphi_i E_{i1}^T E_{i1}) z_{i1} \\ &\quad + \sum_{i=1}^{L} \sum_{\substack{j=1 \\ j \ne i}}^{L} (\hat{\varphi}_j z_{i1}^T H_{ji1}^T H_{ji1} z_{i1} + \frac{1}{\hat{\varphi}_i} z_{i1}^T P_i P_i z_{i1}) \\ &= \sum_{i=1}^{L} z_{i1}^T [A_{i1}^T P_i + P_i A_{i1} + \varphi_i^{-1} P_i D_{i1} D_{i1}^T P_i + \varphi_i E_{i1}^T E_{i1} + \frac{L-1}{\hat{\varphi}_i} P_i P_i + \sum_{\substack{j=1 \\ j \ne i}}^{L} (\hat{\varphi}_j H_{ji1}^T H_{ji1})] z_{i1} \end{aligned} \tag{13}$$

In addition, using Lemma 2, the LMI (7) is equivalent to the following inequalities

$$\begin{bmatrix} \Psi_i & P_i D_{i1} \\ D_{i1}^T P & -\varphi_i I_{m_i} \end{bmatrix} + \varphi_i \begin{bmatrix} E_{i1}{}^T \\ 0 \end{bmatrix} \begin{bmatrix} E_{i1} & 0 \end{bmatrix} + \frac{L-1}{\hat{\varphi}_i} \begin{bmatrix} P_i \\ 0 \end{bmatrix} \begin{bmatrix} P_i & 0 \end{bmatrix} < 0 \tag{14}$$

and

$$A_{i1}^T P_i + P_i A_{i1} + \varphi_i E_{i1}^T E_{i1} + \varphi_i^{-1} P_i D_{i1} D_{i1}^T P_i + \frac{L-1}{\hat{\varphi}_i} P_i P_i + \sum_{\substack{j=1 \\ j \ne i}}^{L} (\hat{\varphi}_j H_{ji1}^T H_{ji1}) < 0. \tag{15}$$

It follows from equations (13) and (15) that

$$\dot{V} < 0. \tag{16}$$

The inequality (16) shows that if LMI (7) holds, which further implies that the sliding motion (6) is asymptotically stable. □

Remark 1: Theorem 1 provides a new existence condition of the sliding surface in terms of strict LMI, which can be easily worked out using LMI Toolbox in Matlab.

4. Decentralized Adaptive Output Feedback Sliding Mode Control

The objective is now to design a decentralized output feedback sliding mode control such that that the state trajectories of system (1) reach the sliding surface (5) in finite time and stay on it thereafter. In order to satisfy the above aims, the modified sliding mode controller is selected to be

$$u_i(t) = -(F_{i2} B_{i2})^{-1} (\kappa_i \eta_i + \bar{\kappa}_i \|y_i\| + \zeta_i(t) + \alpha_i) \frac{\sigma_i}{\|\sigma_i\|}, \ i = 1 \ldots L \tag{17}$$

where the scalars $\alpha_i > 0$, $\eta_i > 0$ and $\kappa_i = \|F_{i2}\| (\|A_{i3}\| + \|D_{i2}\| \|E_{i1}\|) + \sum_{\substack{j=1 \\ j \ne i}}^{L} \|F_{j2}\| \|H_{ji3}\|$,

$\bar{\kappa}_i = [\|F_{i2}\| (\|A_{i4}\| + \|D_{i2}\| \|E_{i2}\|) + \sum_{\substack{j=1 \\ j \ne i}}^{L} \|F_{j2}\| \|H_{ji4}\|] \|F_{i2}^{-1}\| \|F_i\|$. The adaptive law is defined as:

$$\zeta_i(t) \geq \sum_{k=1}^{r} \hat{b}_{ik}(t)\|F_{i2}\|\|B_{i2}\|\|y_i\|^{k-1} \tag{18}$$

where $\hat{b}_{ik}$ is the solution of the following equations

$$\dot{\hat{b}}_{ik}(t) = \bar{q}_{ik}\|F_{i2}\|\|B_{i2}\|\|y_i\|^{k\text{-}1},\ k = 1\ldots r \tag{19}$$

in which scalars $\bar{q}_{ik} > 0,\ k = 1\ldots r$. It should be pointed out that the controller (17) uses only outputs variables.

Then, we derive the following theorem to prove the state trajectories of system (3)-(4) reach the sliding surface (5) in finite time and stay on it thereafter.

Theorem 2: Consider the mismatched uncertain interconnected system (3)-(4).Under assumptions 1– 3, the decentralized output feedback sliding mode control (17) drives the system (3)-(4) to the composite sliding surface (5) and maintains a sliding motion for all $\|z_{i1}\| \leq \eta_i$.

Proof. We consider the following positive definite function

$$V = \sum_{i=1}^{L}(\|\sigma_i\| + \sum_{k=1}^{r}\frac{1}{2\bar{q}_{ik}}\tilde{b}_{ik}^2(t)). \tag{20}$$

where $\tilde{b}_{ik}(t) = b_{ik} - \hat{b}_{ik}(t)$, $k = 1, \ldots, r$. By differentiating equation (20) along the trajectories of (5), we can obtain that

$$\dot{V} = \sum_{i=1}^{L}(\frac{\sigma_i^T}{\|\sigma_i\|}\dot{\sigma}_i - \sum_{k=1}^{r}\frac{1}{\bar{q}_{ik}}\tilde{b}_{ik}\dot{\hat{b}}_{ik}) = \sum_{i=1}^{L}(\frac{\sigma_i^T}{\|\sigma_i\|}F_{i2}\dot{z}_{i2} - \sum_{k=1}^{r}\frac{1}{\bar{q}_{ik}}\tilde{b}_{ik}\dot{\hat{b}}_{ik}). \tag{21}$$

Substituting equation (4) into equation (21), we have

$$\begin{aligned}\dot{V} = &\sum_{i=1}^{L}\frac{\sigma_i^T}{\|\sigma_i\|}F_{i2}[(A_{i3} + D_{i2}\Delta F_i E_{i1})z_{i1} + (A_{i4} + D_{i2}\Delta F_i E_{i2})z_{i2}] - \sum_{i=1}^{L}\sum_{k=1}^{r}\frac{1}{\bar{q}_{ik}}\tilde{b}_{ik}\dot{\hat{b}}_{ik} \\ &+ \sum_{i=1}^{L}\frac{\sigma_i^T}{\|\sigma_i\|}F_{i2}B_{i2}(u_i + G_i(t)) + \sum_{i=1}^{L}\sum_{\substack{j=1\\ j\neq i}}^{L}\frac{\sigma_i^T}{\|\sigma_i\|}F_{i2}(H_{ij3}z_{j1} + H_{ij4}z_{j2}).\end{aligned} \tag{22}$$

Using (22) and property $\|AB\| \leq \|A\|\|B\|$, it generates

$$\begin{aligned}\dot{V} \leq &\sum_{i=1}^{L}\|F_{i2}\|[(\|A_{i3}\| + \|D_{i2}\|\|E_{i1}\|)\|z_{i1}\| + (\|A_{i4}\| + \|D_{i2}\|\|E_{i2}\|)\|z_{i2}\|] + \sum_{i=1}^{L}\frac{\sigma_i^T}{\|\sigma_i\|}F_{i2}B_{i2}u_i \\ &+ \sum_{i=1}^{L}\sum_{\substack{j=1\\ j\neq i}}^{L}\|F_{i2}\|(\|H_{ij3}\|\|z_{j1}\| + \|H_{ij4}\|\|z_{j2}\|) + \sum_{i=1}^{L}\|F_{i2}\|\|B_{i2}\|\|G_i\| - \sum_{i=1}^{L}\sum_{k=1}^{r}\frac{1}{\bar{q}_{ik}}\tilde{b}_{ik}\dot{\hat{b}}_{ik}.\end{aligned} \tag{23}$$

Since $\|G_i\| \leq \sum_{k=1}^{r} b_{ik}\|y_i\|^{k-1}$, we have

$$\dot{V} \le \sum_{i=1}^{L} \|F_{i2}\| [(\|A_{i3}\| + \|D_{i2}\|\|E_{i1}\|)\|z_{i1}\| + (\|A_{i4}\| + \|D_{i2}\|\|E_{i2}\|)\|z_{i2}\|] + \sum_{i=1}^{L} \frac{\sigma_i^T}{\|\sigma_i\|} F_{i2} B_{i2} u_i$$
$$+ \sum_{i=1}^{L} \sum_{\substack{j=1 \\ j \ne i}}^{L} \|F_{i2}\| (\|H_{ij3}\|\|z_{j1}\| + \|H_{ij4}\|\|z_{j2}\|) - \sum_{i=1}^{L}\sum_{k=1}^{r} \frac{1}{\bar{q}_{ik}} \tilde{b}_{ik} \dot{\hat{b}}_{ik} + \sum_{i=1}^{L}\sum_{k=1}^{r} b_{ik} \|F_{i2}\|\|B_{i2}\|\|y_i\|^{k-1} \tag{24}$$

From the fact that $\sum_{i=1}^{L} \sum_{\substack{j=1 \\ j \ne i}}^{L} \|F_{i2}\|\|H_{ij3}\|\|z_{j1}\| = \sum_{i=1}^{L} \sum_{\substack{j=1 \\ j \ne i}}^{L} \|F_{j2}\|\|H_{ji3}\|\|z_{i1}\|$ and $\sum_{i=1}^{L} \sum_{\substack{j=1 \\ j \ne i}}^{L} \|F_{i2}\|\|H_{ij4}\|\|z_{j2}\| = \sum_{i=1}^{L} \sum_{\substack{j=1 \\ j \ne i}}^{L} \|F_{j2}\|\|H_{ji4}\|\|z_{i2}\|$, we obtain

$$\dot{V} \le \sum_{i=1}^{L} \|F_{i2}\| [(\|A_{i3}\| + \|D_{i2}\|\|E_{i1}\|)\|z_{i1}\| + (\|A_{i4}\| + \|D_{i2}\|\|E_{i2}\|)\|z_{i2}\|] + \sum_{i=1}^{L} \frac{\sigma_i^T}{\|\sigma_i\|} F_{i2} B_{i2} u_i$$
$$+ \sum_{i=1}^{L} \sum_{\substack{j=1 \\ j \ne i}}^{L} \|F_{j2}\| (\|H_{ji3}\|\|z_{i1}\| + \|H_{ji4}\|\|z_{i2}\|) - \sum_{i=1}^{L}\sum_{k=1}^{r} \frac{1}{\bar{q}_{ik}} \tilde{b}_{ik} \dot{\hat{b}}_{ik} + \sum_{i=1}^{L}\sum_{k=1}^{r} b_{ik} \|F_{i2}\|\|B_{i2}\|\|y_i\|^{k-1} \tag{25}$$

In addition, the equation (6) implies that

$$\|z_{i2}\| = \|F_{i2}^{-1}\|\|F_i\|\|y_i\| \tag{26}$$

According to equations (25) and (26), we can get that

$$\dot{V} \le \sum_{i=1}^{L} \|F_{i2}\| [(\|A_{i3}\| + \|D_{i2}\|\|E_{i1}\|)\eta_i + (\|A_{i4}\| + \|D_{i2}\|\|E_{i2}\|)\|F_{i2}^{-1}\|\|F_i\|\|y_i\|]$$
$$+ \sum_{i=1}^{L} \sum_{\substack{j=1 \\ j \ne i}}^{L} \|F_{j2}\| (\|H_{ji3}\|\eta_i + \|H_{ji4}\|\|F_{i2}^{-1}\|\|F_i\|\|y_i\|) + \sum_{i=1}^{L} \frac{\sigma_i^T}{\|\sigma_i\|} F_{i2} B_{i2} u_i \tag{27}$$
$$- \sum_{i=1}^{L}\sum_{k=1}^{r} \frac{1}{\bar{q}_{ik}} \tilde{b}_{ik} \dot{\hat{b}}_{ik} + \sum_{i=1}^{L}\sum_{k=1}^{r} b_{ik} \|F_{i2}\|\|B_{i2}\|\|y_i\|^{k-1}$$

Substituting equations (17)-(19) into equation (27), it is clearly that

$$\dot{V} \le \sum_{i=1}^{L}\sum_{k=1}^{r} \hat{b}_{ik} \|F_{i2}\|\|B_{i2}\|\|y_i\|^{k-1} - \sum_{i=1}^{L}\sum_{k=1}^{r} \hat{b}_{ik} \|F_{i2}\|\|B_{i2}\|\|y_i\|^{k-1} - \sum_{i=1}^{L} \alpha_i . \tag{28}$$

Clearly, we have

$$\dot{V} \le -\sum_{i=1}^{L} \alpha_i < 0. \tag{29}$$

The above inequality implies that the state trajectories of system (3)-(4) reach the sliding surface (5) in finite time and stay on it thereafter □

5. Numerical Example

To verify the effectiveness of the proposed decentralized adaptive SMC law, we consider a double-inverted pendulums system connected by a spring [10] and composed of two subsystems:

The first subsystem's dynamics are given as

$$\begin{aligned} \dot{x}_1 &= (A_1 + \Delta A_1)x_1 + B_1(u_1 + G_1(x_1,t)) + H_{12}x_2 \\ y_1 &= C_1 x_1 \end{aligned} \tag{30}$$

where $x_1 = \begin{bmatrix} x_{11} \\ x_{12} \end{bmatrix} \in R^2$, x_{11} is the angular displacement of the first pendulum from the vertical reference; a torque input $u_1 \in R^1$ applied by a servomotor at its base to position the first pendulum; $y_1 \in R^1$, $A_1 = \begin{bmatrix} 0 & 1 \\ \frac{m_1 gl}{J_1} - \frac{k}{J_1} & 0 \end{bmatrix}$, $B_1 = \begin{bmatrix} 0 \\ \frac{1}{J_1} \end{bmatrix}$, $C_1 = \begin{bmatrix} 0 & 1 \end{bmatrix}$, $H_{12} = \begin{bmatrix} \frac{k}{J_1} & 0 \\ 0 & 0 \end{bmatrix}$; $m_1 = 2$ is the end masses of the first pendulum; the moment of inertia is $J_1 = 2(kg)$; the constant of the connecting torsional spring is $k = 2(N.m/rad)$; $l = 1(m)$ is the length of the pendulum; the gravitational acceleration is $g = 9.81(m/s^2)$. The mismatched parameter uncertainties in the state matrix of the first subsystem is $\Delta A_1 = D_1 \Delta F_1 E_1$ with $D_1 = \begin{bmatrix} 1 & 1 \end{bmatrix}^T$, $E_1 = \begin{bmatrix} 1 & 1 \end{bmatrix}$ and $\Delta F_1 = \sin(x_{12})$. The exogenous disturbance in the first subsystem is $\|G_1(x_1,t))\| \le 0.1 + 1\|y_1\| + 0.1\|y_1\|^2$.

The second subsystem's dynamics are given as

$$\begin{aligned} \dot{x}_2 &= (A_2 + \Delta A_2)x_2 + B_2(u_2 + G_2(x_2,t)) + H_{21}x_1 \\ y_2 &= C_2 x_2 \end{aligned} \tag{31}$$

where $x_2 = \begin{bmatrix} x_{21} \\ x_{22} \end{bmatrix} \in R^2$, x_{21} is the angular displacement of the second pendulum from the vertical reference; a torque input $u_2 \in R^1$ applied by a servomotor at its base to position the second pendulum; $y_2 \in R^1$, $A_2 = \begin{bmatrix} 0 & 1 \\ \frac{m_2 gl}{J_2} - \frac{k}{J_2} & 0 \end{bmatrix}$, $B_2 = \begin{bmatrix} 0 \\ \frac{1}{J_2} \end{bmatrix}$, $C_2 = \begin{bmatrix} 0 & 1 \end{bmatrix}$, $H_{21} = \begin{bmatrix} \frac{k}{J_2} & 0 \\ 0 & 0 \end{bmatrix}$; $m_2 = 2.5$ is the end masses of the second pendulum; the moment of inertia is $J_2 = 2.5(kg)$. The mismatched parameter uncertainties in the state matrix of the second subsystem is $\Delta A_2 = D_2 \Delta F_2 E_2$ with $D_2 = \begin{bmatrix} 1 & 1 \end{bmatrix}^T$, $E_2 = \begin{bmatrix} 1 & 1 \end{bmatrix}$ and $\Delta F_2 = \cos(x_{21})$. The exogenous disturbance in the second subsystem is $\|G_2(x_2,t))\| \le 0.1 + 1\|y_2\| + 0.1\|y_2\|^2$.

It is easy to check that assumption 2 hold. The coordinate transformation matrices T_i are chosen as $T_1 = T_2 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$. By solving LMI (7), it is easy to verify that the condition in Theorem 1 are satisfied with positive matrix $P_1 = 0.1223$ and $P_2 = 0.4095$. The sliding matrices are given by $F_1 = F_2 = 1$, and the sliding functions are $\sigma_1 = y_1$ and $\sigma_2 = y_2$. So from Theorem 1, the sliding motion associated with the sliding surface σ_1 and σ_2 is globally asymptotically stable.

From Theorem 2, the decentralized adaptive output feedback controller for two subsystems are as

$$u_1(t) = -1.33(53.3 + \zeta_1(t) + 4.5\|y_1\|)\frac{\sigma_1}{\|\sigma_1\|} \tag{32}$$

and

$$u_2(t) = -2.5(46.65 + \zeta_2(t) + 3\|y_2\|)\frac{\sigma_2}{\|\sigma_2\|} \tag{33}$$

where $\zeta_1 \geq 0.75\hat{b}_{11}(t) + 0.75\hat{b}_{12}(t)\|y_1\| + 0.75\hat{b}_{13}(t)\|y_1\|^2$, $\zeta_2 \geq 0.4\hat{b}_{21}(t) + 0.4\hat{b}_{22}(t)\|y_2\| + 0.4\hat{b}_{23}(t)\|y_2\|^2$, $\dot{\hat{b}}_{1k} = 75\|y_1\|^{k\text{-}1}$, $k = 1 \ldots 3$ and $\dot{\hat{b}}_{2k} = 40\|y_2\|^{k\text{-}1}$, $k = 1 \ldots 3$.

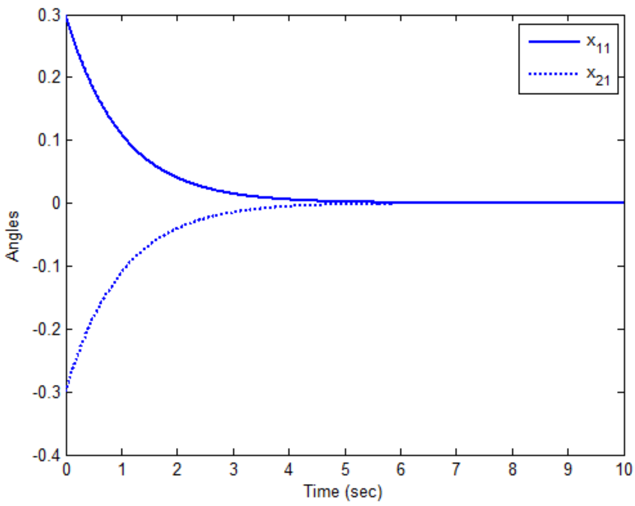

Figure 1. Time Responses of States of Two Inverted Pendulum Systems: x_{11} (Slid), x_{21} (Dashed)

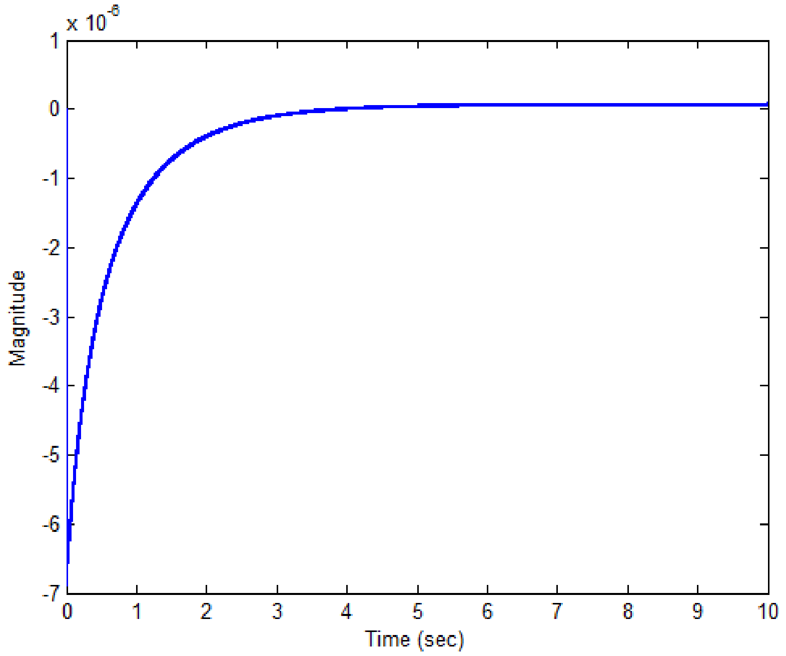

Figure 2. Time Responses of the Sliding Function σ_1 of the Subsystem 1

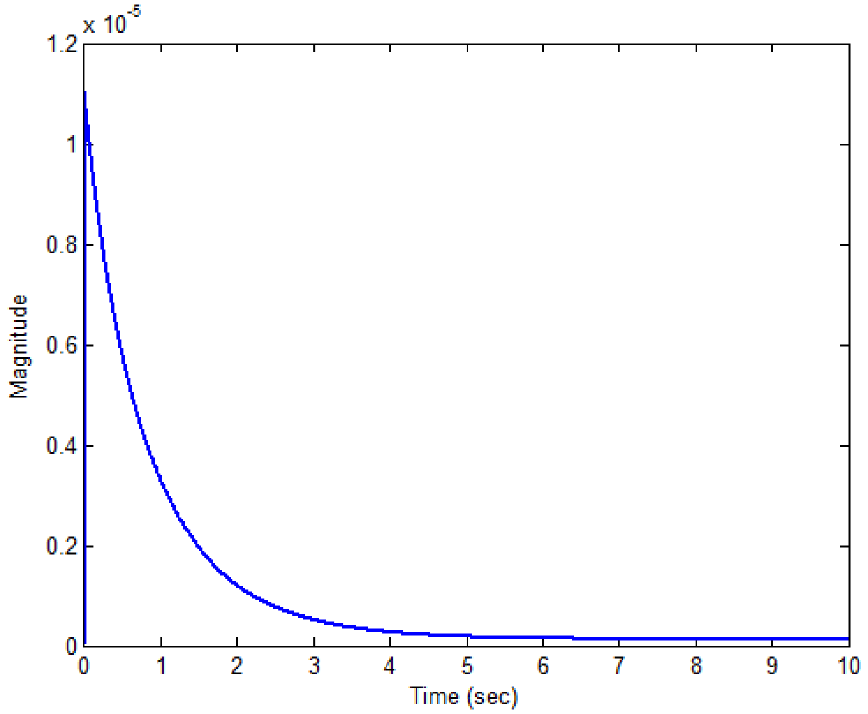

Figure 3. Time Responses of the Sliding Function σ_2 of the Subsystem 2

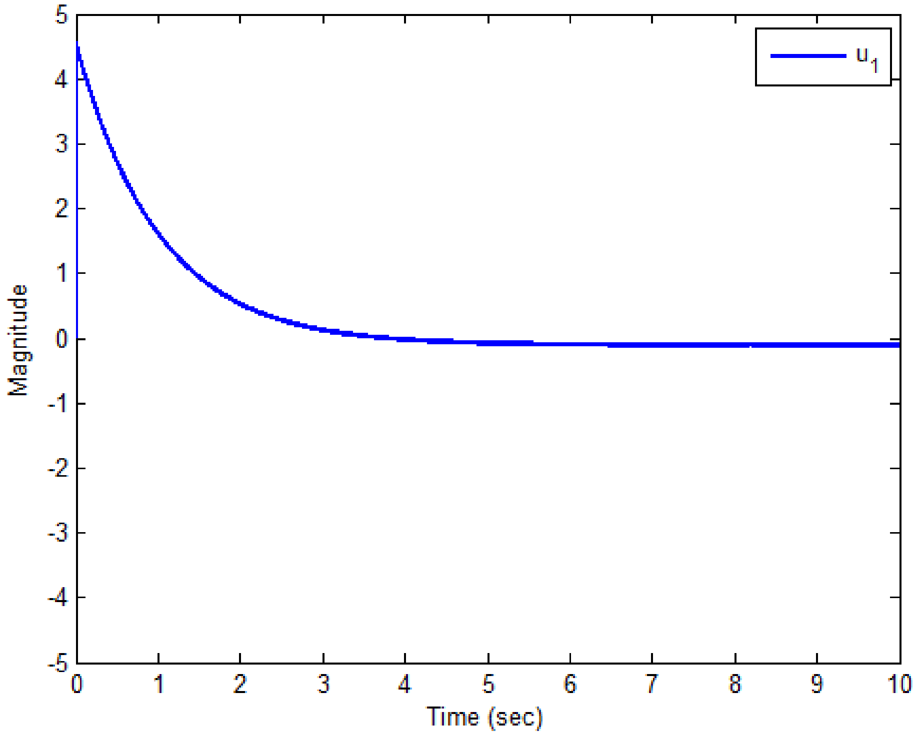

Figure 4. Time Responses of the Control Input u_1 of the Subsystem 1

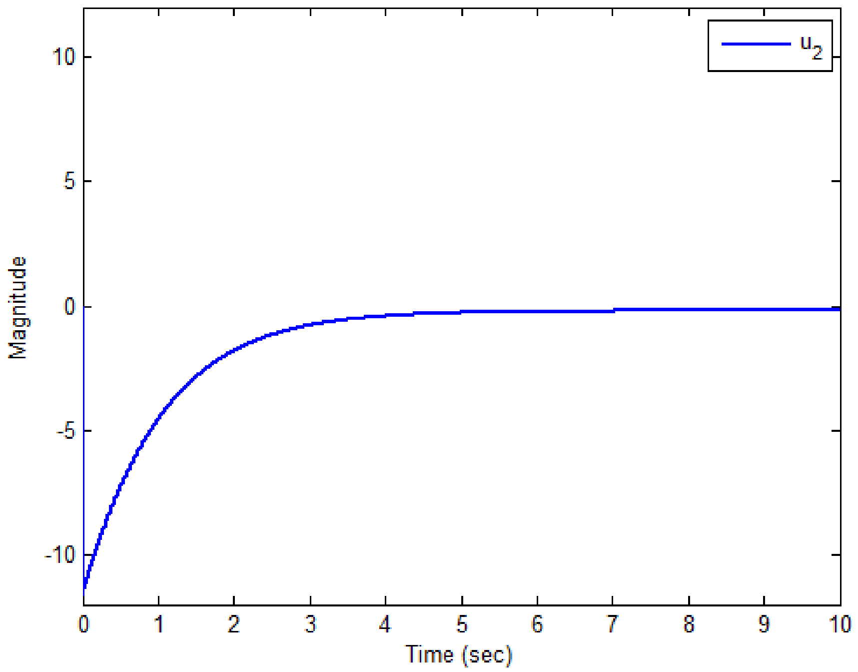

Figure 5. Time Responses of the Control Input u_2 of the Subsystem 2

The initial conditions for two subsystems are selected to be $x_1(0) = [0.3 \;\; 0]^T$ and $x_2(0) = [-0.3 \;\; 0]^T$, respectively. By Figure 1 to Figure 5, it is easy to see that the proposed controller has a good performance and is effective in dealing with matched and mismatched uncertainties.

6. Conclusion

In this paper, a modified decentralized adaptive output feedback sliding mode control law is proposed to stabilize a class of large-scale systems with unknown exogenous disturbances, mismatched uncertainties and without the measurements of the states. Here the disturbances are very large and more general form than these approaches given in [6-10]. The modified adaptive law has been proposed to solve this problem. Moreover, the sliding mode controller guarantees the finite time reachability of the system states and the system in the sliding mode is asymptotically stable under certain conditions.

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