

Electron Escaping in the Magnetically Confined Electron Cloud

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Abstract

The morphology of magnetic equipotential lines in axisymmetric magnetic mirror field was analyzed. Based on the magnetic fluid theory of electrons, an electron cloud model formed by magnetic mirror field confined electrons was built in this paper. And two-dimension differential potential equations were solved by using Runge-Kutta method. The calculation gave the Boltzmann distribution and potential distribution of electron density. The principle of both electron escape and confinement were analyzed in magnetic mirror field environment, the runaway electron current situation of the electronic cloud boundary was analyzed. Based on the leakage current of electron boundary, the balance of ionization electrons and escape electrons was used to derive the confined region leakage current factor relationship equation in magnetic mirror environment. The relationship between the runaway electron current and the runaway factor, the effects electron temperature, electron velocity, and magnetic field lines on runaway factor were given. Furthermore, the minimum confined magnetic field needed to build electron cloud in magnetic mirror field was also given in this paper.

Keywords: *Electromagnetic Field; Runaway Electron Current; Runaway Factor; Magnetically Confined Factor*

1. Introduction

The former Soviet Union scholar of Tamm and Sakharov proposed the concept of magnetic confinement in the 1950s [1, 2]. Artsimovich from the former Soviet Union Scientific Centre “Kurchatov Institute” completed the first magnetic confinement device named Tokamak in 1954 according to this idea. Tokamak is a typical representative of magnetic confinement device[3-6]. The ultimate goal of the magnetic confinement is to realize nuclear fusion. The cyclotron radius R is inversely proportional to the magnetic induction B when a charged particle does spiral motion in a magnetic field. Charged particle is confined in a small area around the magnetic field line in high-intensity magnetic field, and the center of spiral orbit can only move along but cannot cross the magnetic field line. A large number of electrons do spiral motion around the magnetic field line of force adopt the radial electric field and the axial magnetic field, and electrons drift along the magnetic field lines in the magnetic field [7-13]. The total energy remains constant, and the particle's vertical velocity becomes larger and parallel velocity reduces in the strong magnetic field; the particle's vertical velocity reduces and parallel velocity becomes larger in the weak magnetic field [14]. Some particles with low parallel velocity will lose kinetic energy in somewhere and move to the opposite direction and then be reflected by the magnetic field, and will be reflected when they arrive at the symmetric position, so the particles move back and forth in the magnetic field, become confined particle, and thus the magnetically confined electron cloud is formed [15]. Some particles with high parallel velocity through the magnetic field line and become escape particle. When the magnetic field is weaker, the magnetically confined factor become smaller and the confined ability becomes weaker, and then it cannot confine electron.

2. The Formation of Electron Cloud

Assuming that the magnetic mirror field is rotational symmetry, in cylindrical coordinate system, magnetic vector potential is expressed as below[16]:

$$A_\theta(r, z) = \frac{1}{2} B_0 r - \left(B_0 \frac{\alpha}{k} \right) \cos(kz) I_1(kr) \quad (1)$$

where $\alpha = \frac{R-1}{R+1}$; R is mirror ratio; $k = \frac{2\pi}{L}$; L is magnetic mirror length; r is magnetic field

radius; $B_0 = \frac{B_{\max} - B_{\min}}{2\alpha}$. So the distribution of the magnetic mirror induction internal as below:

$$\begin{cases} B_z = B_0 - \alpha B_0 \cos(kz) I_0(kr) \\ B_r = -\frac{1}{r} \alpha B_0 \sin(kz) I_1(kr) \end{cases} \quad (2)$$

where I_0 and I_1 are modified Bessel function of the second kind. Distribution of the magnetic equipotential lines in axisymmetric magnetic mirror field is shown in Figure 1.

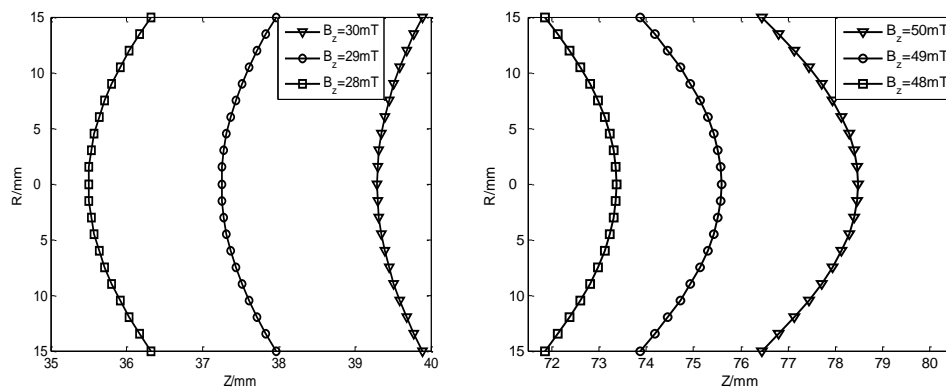


Figure 1. The Magnetic Equipotential Line Distribution in the Magnetic Mirror Field

The equations of the electron magnetic fluid motion in the cross electromagnetic field is shown as below:

$$\begin{cases} \nabla \cdot (nv) = 0 \\ (v \cdot \nabla)v = \frac{e}{m} (\nabla U - v \times B) - \frac{1}{nm} \nabla (n\kappa T) - vv \\ v \cdot \nabla \left(eU - \frac{1}{2} m v^2 - \frac{5}{2} \kappa T \right) = 0 \\ \nabla^2 U = \frac{en}{\epsilon_0} \end{cases} \quad (3)$$

The above equations are respectively the continuity equation, momentum equation, energy equation and Electrostatic field equation of electron. ϵ_0 is Vacuum permittivity, v is electron velocity; m is mass of the electron; e is electron charge; T is electron temperature; n is electron density; v is electronic Potentiometer; B is magnetic

indensity; ν is collision frequency; κ is Boltzmann constant. And we can get below from formula (3):

$$n = n_0 \exp\left(\frac{eU}{\kappa T_e}\right) \quad (4)$$

This is the Boltzmann relationship. n_0 is the electron density of magnetic mirror center. Assuming the electric potential of the beam axis center is 0, the Electric field of electron offset the axial component of the space charge field, so $\frac{\partial U}{\partial z} \approx 0$. We can simplify the Poisson formula through the Runge Kutta method to solve the electric potential of two order differential equations, and get the distribution of electron density and electric potential Along the radial direction as below[17,18]:

$$\begin{cases} n(r) = n_0 \exp\left(\frac{eU(r)}{\kappa T} - \alpha r^2\right) \\ \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial U}{\partial r} \right) = \frac{en(r)}{\epsilon_0} \end{cases} \quad (5)$$

And $\alpha = \frac{m}{2\kappa T}(\Omega\omega - \omega^2)$, $\Omega = \frac{eB_z(r)}{m}$, Assuming $n_0 = 2 \times 10^{14} \text{ m}^{-3}$, $\kappa T = 120 \text{ eV}$, $\omega = 2 \times 10^8 \text{ s}^{-1}$,

and we can get the distribution of electron density and electric potential of electron cloud in the space charge lens:

The electron density distribution is uneven under the influence of radial electric field of the electron cloud. The maximum electron density distribution exists around the central axis, and the electron density decreases as the radial distance increases. The decreasing speed accelerates at first, then decreases and finally becomes zero. Electric potential increases as the radial distance increases, the increasing speed accelerates at first, then decreases and finally tends to be zero. The electron density distribution and electric potential distribution are the parameters closely related to each other, the higher electron density regional represents a lower electronic potential and the less electron density regional represents a higher electronic potential.

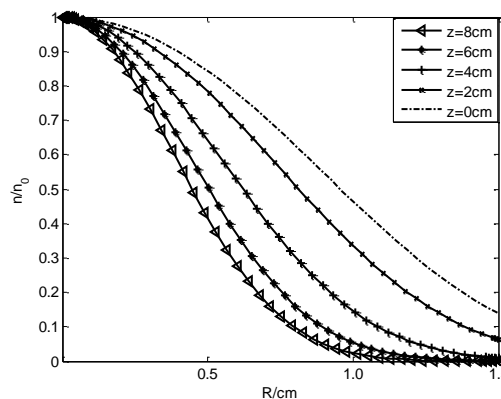


Figure 2. The Distribution of Electron Density in the Electron Cloud

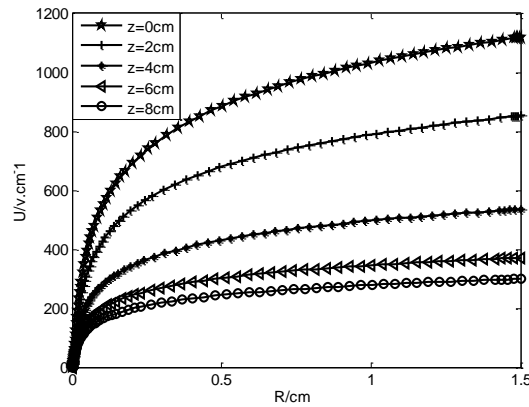


Figure 3. The Electric Potential in the Electron Cloud

3. The Electron Escaping Loss

Some particles can be confined and some particles can escape in the magnetic mirror field. We assume the magnetic induction of the magnetic mirror field coil center is B_{max} and the medium surface between the magnetic coils is B_{min} , the vertical, parallel and Sum velocity of the Particles are v_{\perp} , v_{\parallel} and v_0 at B_{min} . We can reason below due to the magnetic moment invariance of the reflected particles at B_{min} :

$$\frac{v_{\perp}^2}{B_{min}} = \frac{v_0^2}{B_{max}} \quad (6)$$

So the reflected condition at B_{max} is $\sin^2 \theta_m = \frac{v_{\perp}^2}{v_0^2} = \frac{B_{min}}{B_{max}} = \frac{1}{R}$, namely the electron is

confined when $\sin \theta_m > \sqrt{\frac{1}{R}}$. θ_m is the critical angle value of particle velocity and the magnetic field line on the medium surface. R is the mirror ratio. The particles with the angle less than θ_m has a greatly parallel velocity and they still remain some parallel kinetic energy when arrive B_{max} , so they can escape through the coil; the particles with the angle more than θ_m has a low parallel velocity and they lose all the parallel kinetic energy before arriving B_{max} , so they are confined by the magnetic field, which is irrelevant with the mass and quantity of electricity itself why the particle confined, only related to the ratio of the maximum and minimum value of magnetic induction. A loss cone whose vertex angle is $2\theta_m$ formed by the particles confined by the magnetic mirror field appears, and the particles in the loss cone all escape and the particles out of the loss cone all confined.

A magnetic surface formed by a magnetic line's rotation around the central axis. A closed boundary surface is formed by drawing along the inner wall of the magnetic confinement device. A magnetic boundary surface is formed by the intersecting of the two ends of each magnetic surfaces, and the boundary surface is the confined region. We assume that the electron cannot return to the confined region once escape. The electron angular motion has no effect on the electron current density j, and can be ignored.

Because the electron diffusion velocity is small, we can consider the velocity of each electron follows the Maxwell distribution approximately, the distribution function as follows [19]:

$$f(v) = n \left(\frac{m}{2\pi\kappa T} \right)^{\frac{3}{2}} \exp\left(-\frac{mv^2}{2\kappa T} \right) \quad (7)$$

When the boundary is perpendicular to the magnetic field lines, the electron escape longitudinally, the current can be expressed as:

$$j = 2\pi \int_0^{\frac{\pi}{2}} \sin^2 \theta \cos \theta d\theta \int_0^{\infty} v^3 f(v) dv = n \sqrt{\frac{\kappa T}{2\pi m}} \quad (8)$$

When the magnetic field lines parallel to the border, the electrons escape laterally, in this case electronics need to rely on the collision to cross the border. θ is the angle of the electron velocity and magnetic field lines. Due to the collision, the electronic guide center will move outwardly in a direction perpendicular to the magnetic field lines in unit time, the average of the amount of movement is $\frac{v}{Q^2} \overline{Q \times v}$, therefore.

$$j = \frac{nv}{Q} \overline{\sin \theta} = \frac{2\pi v}{Q} \int_0^{\frac{\pi}{2}} \sin^2 \theta d\theta \int_0^{\infty} v^3 f(v) d\theta = \frac{nv}{2Q} \sqrt{\frac{\pi\kappa T}{2m}} \quad (9)$$

It is easier for electrons to move along the magnetic field lines than to move perpendicularly to the magnetic field lines. And most of electrons escape along the edge of magnetic field lines. When the angle between magnetic confined boundary and the magnetic field line is θ , the centripetal velocity along the magnetic boundary surface is shown as below:

$$v_N = \sqrt{\frac{\kappa T}{2\pi m}} \sin \theta \quad (10)$$

So the electronics' leakage current as below:

$$j = n(r) \sqrt{\frac{\kappa T}{2\pi m}} \sin \theta \quad (11)$$

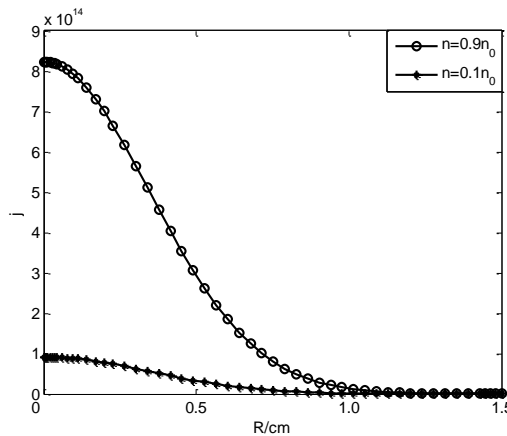


Figure 4. The Leakage Current of Electron Cloud Boundary

The Figure 4 is a comparing graph of the runaway electron current from electron cloud boundary and boundary of electron cloud center. In electron cloud boundary, the electron density is small, the number of electron is less, the probability of collisions between electrons is smaller and the runaway electron current is smaller as well. In the boundary

of electron cloud center, the electron density is large, the number of electron is large, the probability of collisions between electrons is larger and the runaway electron current is larger.

According to the balance relationship between electron and escape electron produced by ionization, get a relationship as below[20]:

$$N\bar{Z} = \iiint j dA \quad (12)$$

Among them, N is the total number of electrons of electron cloud, \bar{Z} is the average ionization frequency of the electron cloud, dA is the element area of integration, define

$v_e = \frac{N}{n_0}$ is the equivalent volume of the electron cloud, $\sigma = \frac{j}{n_0}$ is runaway factor on the

boundary surface, the equation above can be rewrite as $v_e = \frac{1}{Z} \iiint \sigma dA$, so the runaway factor of the electron cloud in equilibrium is shown as below:

$$\sigma = \sqrt{\frac{\kappa T}{2\pi m}} \exp\left(\frac{eU(r)}{\kappa T} - \alpha r^2\right) \sin \theta \quad (13)$$

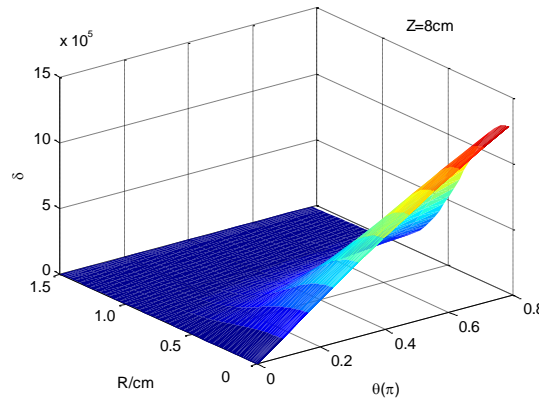


Figure 5. The Boundary Plane Leakage Factor with Z=8cm

The runaway electron current is linearly proportional to the runaway factor and the proportion coefficient is the central axial electron density. We can get the distribution map of the plane runaway factor with Z=8cm according to the electron Boltzmann distribution. The runaway factor is commonly used in engineering to express the leakage characteristics of sealing. We assume that the number of confined electrons increases when the confined range becomes larger and leakage factor decreases correspondingly. The electric potential is lower and the runaway factor is smaller where it is closer to the symmetrical axis. The electric potential is higher and the runaway factor is larger where close to the Magnetic boundary. When the angle between the magnetic field line and boundary surface becomes larger, the axial component of the magnetic field becomes smaller, and the magnetic ability of confinement reduces. Meanwhile the electron cyclotron radius increases, probability of collision increases, and the number of escape electrons increases, and as a result, leakage current increases and runaway factor increases correspondingly. There are some other reasons as well why the runaway electron current is produced: the electrons with high axial velocity traverse and escape directly; the collision rate of electron movement increases even result in the avalanche for its high density and temperature which cause a large number of electrons escape; the loop voltage produced by electron itself also contribute to the electron escape.

It shows in Figure 6 that under the same external environment and the same beam parameter, the electron temperature is high, the electron motion is active, the collision probability is increased, on the other hand, the magnetic field restriction ability is declined, and the factor of runaway electric current is large. It shows in Figure 7 that under the same external environment and the same beam parameter, when the included angle between the electron velocity and magnetic line is big, it means that the magnetic field in the parallel direction of particle movement is decreased, the magnetic field restriction ability is declined, the runaway electron is increased, the runaway factor is large.

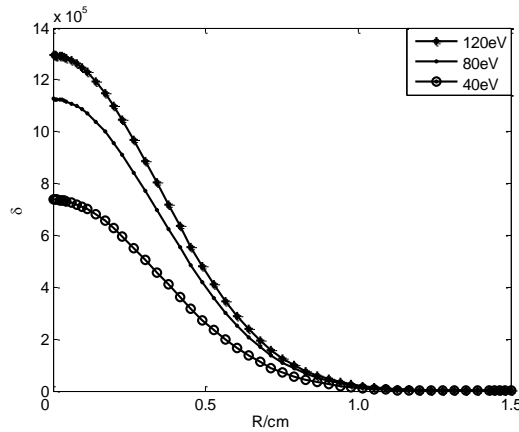


Figure 6. The Influence of the Electron Temperature Effect on the Leakage Factor

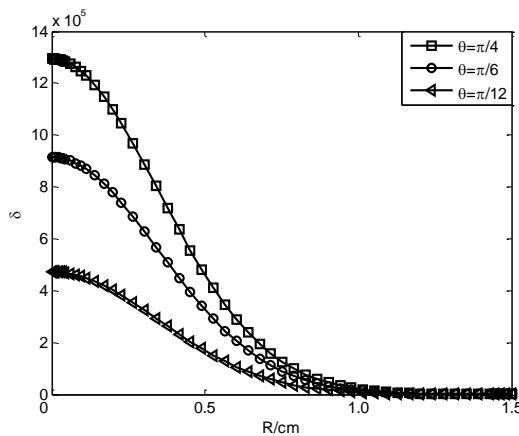


Figure 7. The Influence of the Angle of V and Magnetic Line Effect on the Leakage Factor

4. The Minimum Magnetic Field Confined Electron

Introducing the magnetic confined factor $\gamma = \frac{2 \varepsilon_0 \omega}{e n(r)} (B_z - \frac{m \omega}{e}) - 1$, ε_0 is vacuum; B_z is

Axial magnetic induction. Magnetic confinement factor, Magnetic confinement factor is a parameter to characterize the confinement capacity of magnetic field; magnetic confinement factor is larger and the confinement capacity of magnetic field is stronger. Expanding the formula above we get:

$$\omega^2 - \frac{e B_z}{m} \omega + \frac{e^2 n(r) (\gamma + 1)}{2 m \varepsilon_0} = 0 \quad (15)$$

And it must be satisfied that $B_z \geq \sqrt{\frac{2mn(r)}{\epsilon_0}}(\gamma + 1)$ to ensure ω has it's the real

root. B_z is the minimum magnetic field needed to confine electrons, which can also be called the critical magnetic field. γ is between 0 and 0.01 in engineering fields .Further simplifying the formula above we can get the formula below and the minimum magnetic field needed to confine electron is proportional to its square root of electron density:

$$B_z \geq 4.55 \times 10^{-10} \sqrt{n(r)} \quad (16)$$

5. Summary

In this paper, we simulated the distribution of the magnetic equipotential lines in axisymmetric magnetic mirror field .We calculate the distribution of electron density and electric potential in this magnetic field restriction of electron cloud in the axisymmetric magnetic mirror field by numerical calculations. The electron density distribution is uneven in the electron cloud, the maximum electron density distributes around the central axis, and the electron density decreases as the radial distance increases. The electric potential in the electron cloud increases with the radial distance increases, the increasing trend changes from fast to slow. The confined electron condition in the magnetic mirror field is $\sin \theta_m > \sqrt{\frac{1}{R}}$. The runaway electron current is produced by electron escaping vertically. Near the center area of the electron cloud, the electric current formed by runaway electron is strong. On the contrary, the boundary region of the electron cloud, the electric current is low. And the runaway electron current is linear proportional to the runaway factor. Assume that the number of confined electrons increases when the confined range becomes larger and runaway factor decreases correspondingly. When the electron temperature is high, the runaway factor is greater .The electric potential is lower and the runaway factor is smaller where it is closer to the symmetrical axis. The electric potential is higher and the runaway factor is larger where it is closer to the Magnetic boundary. The angle between the magnetic field line and boundary surface becomes larger and the axial component of the magnetic field becomes smaller. The magnetic confinement ability reduces, and as a result the Runaway electron current increases and the runaway factor increases correspondingly. There is a minimum critical magnetic field needed to confine the electrons and it cannot confine electrons suitably when the magnetic field is less than that the minimum critical one.

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