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# Topology Modeling and Vulnerability Analysis of China Mine Power Grid Based on Complex Network Theory

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## *Abstract*

*A structural vulnerability analysis of China mine power grid, which considering the particular of topological structure and the increase gas power generation nodes, is proposed based on complex network theory. According to transmission capacity of weighted network model, this paper analyses the network characteristics, including degree distribution, clustering coefficient and average path length, to prove that the mine power grid is a scale -free network. The node importance, node betweenness, edge weight and edge betweenness are regarded as the index of identification and the remaining load capacity after attack is chosen as evaluation index to analyze the structural vulnerability of mine power grid to have a fault simulation of the mine power grid before and after adding gas power generation nodes. The results of fault simulation show that the model is in line with the actual situation of the China mine power grid, and can better identify and assess the vulnerability of the mine power grid.*

***Keywords****: complex network; mine grid; vulnerability; gas power generation; power supply capacity*

## 1. Introduction

In recent years, with the advancement of high productive, high effective and safe mining strategy, the increase of capacity of the China mine power grid, the popularization of high-power equipment, and the application of the low concentration gas power generation system, have brought severe challenges to the safe and stable operation of the mine power grid network [1-2].

The concepts of random network [3] and small-world [4] are proposed respectively in 1959 and 1998, and characteristics of scale-free networks are analyzed in 1999 [5], which lays foundation for the study on complex networks theory. At present, complex network is used to analyse the structural vulnerability of power grid. Due to the particular of the industry, the topological structure of mine power grid has its own characteristics. Since the complex network theory was applied to power systems, the domestic and foreign scholars have made deep researches on the large-scale regional power network and drawn some significant conclusions [6-11], but few researches on mine grid. The relationship between network topology and cascading failures is studied in [12]. Combining complex network theory with electrical characteristics of the power network, the research [13] established the weighted network of the standard test system. Thus, it is meaningful to strengthen the research on mine grid, to find the weak link of mine grid, and to study the influence of the gas generators on the mine grid.

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In this paper, the complex network theory is applied to the mine power grid. The different characteristics of the mine power grid and the large regional power network are analyzed. Based on the limitations of several vulnerability assessment indexes, the identification index applicable to the mine power grid is proposed and tested. Considering there are more and more gas generators in the mine grid, this paper analyzes the influence of adding gas generation nodes on the characteristics of mine power grid, and then compares the ability of two networks to resist attack. The simulation of random attacks and intentional attacks on the mine grid validate the effectiveness of the analytical method.

## 2. Topology Modeling for China Mine Power Grid

Figure 1, shows the topology model of the power supply system of a coal mine power system in our country. In this paper, the main line that the voltage class is 6kV and above is extracted and numbered, and the sites in the power network is replaced by the nodes with number.

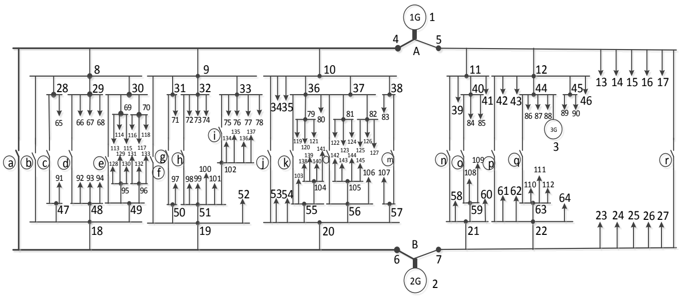


**Figure 1. Topology of the Mine Grid**

In the Figure 1, nodes 1, 2 are the power supply nodes, which means 110kV inlet line; node 3 is gas generation node; nodes 4, 6 are 35KV bus bar; nodes 5,7 are 6kV bus bar; nodes 10 ~ 18, 20~8 are the low voltage bus of the 35kV station; nodes 11, 12,21, 22 are the low voltage bus of the 6kV station; node A and node B are the virtual nodes of the 110kV three-winding main transformers, and all other nodes are load nodes.

In this paper, it also needs to analyze the network with gas power nodes added, so consulting the access method of original gas power node 3, the paper add nine gas power nodes where in the heavy load area. The nine nodes number are 30, 32, 33, 36, 37, 49, 51, 55, 56.

According to the topology of the nine grid in Figure 1, the 145 nodes weighted model programmed based on the Pajek platform is shown in Figure 2.

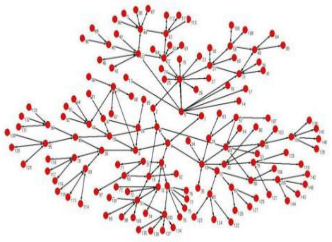


**Figure 2. Topology Model of the Mine Grid**

## 3. Topological Characteristics Analysis of Mine Power Grid

In complex network theory, there are four basic criteria - degree, average path length, clustering coefficient, and betweenness, which are used to analyze topological characteristics of network. In this section, the topological characteristics of the mine grid are analyzed according to the criteria.

### 3.1. The Weight of Edges and Degree of Nodes

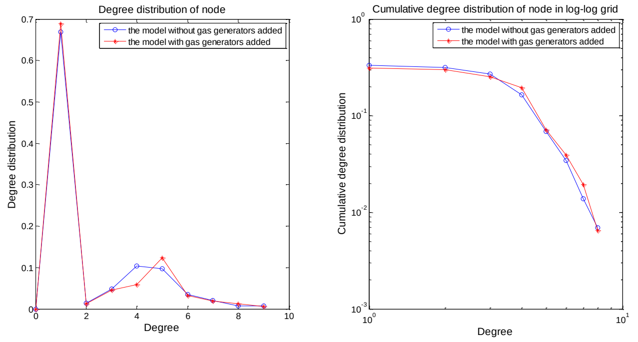
In complex networks, the weight of edges is used to describe the relation of the nodes. The importance of the edge is directly related to its transmission power in the coal mine power system. The weight of edge $w_{ij}$ between node $i$ and node $j$ is defined as the ratio of the absolute value of $S_{ij}$ and $S_N$, that is

$$w_{ij} = \frac{\left|S_{ij}\right|}{S_N} \tag{1}$$

where $S_{ij}$ is the actual transmission power and $S_N$ is the rated capacity of the power grid. Taking the loss and direction during the energy transmission into account, $S_{ij}$ is defined as the rated capacity between the node $i$ and node $j$ subtracts the loss of energy, and it is positive when power flow from the node $i$ to node $j$.

The degree of node $i$ is the number of nodes connected to it. The degree expresses the adjacency relationship between node $i$ and other nodes, which shows the significance of it in the network.

The degree distribution of nodes in the network which with and without gas generators added are shown in Figure 3(a). Figure 3(b), shows the curves of cumulative degree distribution of node in log-log grid.



(a) (b)

**Figure 3. Node Degree Distribution of Topology Model**

It can be seen from the Figure 3, that the addition of gas generation nodes has little influence on the degree distribution of the original model. And the mine power grid has same characteristics with scale-free network: the degree of most nodes are small and the curve of cumulative degree distribution of node in log-log grid presented here is linear.

The weight of each node in the mine grid is calculated and ranked in sequence. Table 1, lists the 18 nodes with higher value of weight and their degrees.

**Table 1. The Weight and Degree of Some Nodes**

| order | the model without gas generators added | | | the model with gas generators added | | |
|---|---|---|---|---|---|---|
| | number | degree | weight | number | degree | weight |
| 1 | 2 | 2 | 0.494429 | 1 | 2 | 0.33844 |
| 2 | 1 | 2 | 0.477716 | 2 | 2 | 0.271588 |
| 3 | 6 | 5 | 0.2785515 | 7 | 8 | 0.215877 |
| 4 | 4 | 5 | 0.2688022 | 5 | 9 | 0.208914 |
| 5 | 7 | 8 | 0.2158774 | 6 | 5 | 0.167131 |
| 6 | 5 | 9 | 0.2089136 | 4 | 5 | 0.129526 |
| 7 | 20 | 6 | 0.1149025 | 22 | 6 | 0.083565 |
| 8 | 10 | 7 | 0.1114206 | 19 | 5 | 0.066156 |
| 9 | 19 | 5 | 0.0940111 | 12 | 7 | 0.062674 |
| 10 | 9 | 5 | 0.0877437 | 11 | 5 | 0.059889 |
| 11 | 22 | 6 | 0.0835655 | 21 | 5 | 0.059889 |
| 12 | 8 | 5 | 0.0696379 | 20 | 6 | 0.059192 |
| 13 | 18 | 5 | 0.0696379 | 10 | 7 | 0.05571 |
| 14 | 12 | 7 | 0.0696379 | 8 | 5 | 0.041783 |
| 15 | 11 | 5 | 0.0598886 | 18 | 5 | 0.041783 |
| 16 | 21 | 5 | 0.0598886 | 104 | 5 | 0.038997 |
| 17 | 37 | 4 | 0.0529248 | 51 | 8 | 0.036212 |
| 18 | 36 | 4 | 0.0348189 | 105 | 5 | 0.033426 |

Table 1, shows that most of the nodes have both high weight and high degree, but some nodes with low degree are also important in the network, such as nodes 1, 2, *etc.*, Thus, it is more comprehensive to evaluate the importance of nodes by the two parameters. Gas power generator nodes share the load of a number of nodes, so there is some change in the important degree raking of most nodes, and the ranking of nodes with higher weight before is generally decreased.

### 3.2. The Weight of Nodes and Clustering Coefficient

For the coal mine power supply system, the importance of a node is reflected in its transmission power. Defining the weight $w_{ij}$ of node $i$ as the ratio of $S_i$ and $S_N$, that is

$$w_i = \frac{S_i}{S_N} \tag{2}$$

where $S_i$ is the transmission power of node $i$ ,which defined as the sum of $S_{ij}$ that is positive value.

Assuming that there are $k_i$ edges connecting node $i$ to other nodes, these $k_i$ nodes are named as the neighbors of node $i$ . Obviously, there are $k_i(k_i-1)/2$ edges at most between these $k_i$ nodes. Defining the number of the practical existing edges between the $k_i$ nodes as $E_i$ , the clustering coefficient of node $i$ is named as $C_i$ which the ratio of the couples number of the practical existing edges to the maximum number of the existing edges between these $k_i$ nodes, that is

$$C_i = \frac{2E_i}{k_i(k_i - 1)} \tag{3}$$

The clustering coefficient, the other parameters of mine grid with and without gas generation nodes added the average path length $L_{random}$ and the average clustering coefficient $C_{random}$ of the same scale random network are ca1culated, as shown in Table 2. The complex network parameters of some typical power grid in China were list in Table 2, as comparisons to analyze the specialty of coal mine power grid [14].

**Table 2. Grid Topology Parameter Statistics**

| network name | node number | edge number | average degree | clustering coefficient | average path length | $C_{random}$ | $L_{random}$ |
|---|---|---|---|---|---|---|---|
| Northern China Power Grid | 8092 | 9018 | 2.23 | 0.0017 | 32 | 0.00028 | 11.2 |
| Northeast China Power Grid | 1144 | 1309 | 2.29 | 0.00342 | 14 | 0.002 | 8.5 |
| North China Power Grid | 3706 | 4045 | 2.18 | 0.00123 | 20.7 | 0.0006 | 10.55 |
| Central China Power Grid | 2379 | 2756 | 2.32 | 0.0044 | 21.8 | 0.001 | 9.238 |
| Sichuan-Chongqing (China) Power Grid | 853 | 898 | 2.11 | 0.0017 | 19.63 | 0.0025 | 9.038 |
| Guangdong (China) Power Grid | 1871 | 2000 | 2.14 | 0.00084 | 15.1 | 0.0011 | 9.92 |
| the model without gas generators added | 145 | 161 | 2.21 | 0 | 6.02 | 0.0041 | 3.9 |
| the model with gas generators added | 154 | 170 | 2.19 | 0 | 5.93 | 0.0087 | 3.96 |

As shown in Table 2, although the scale of mine grid is far less than that of large area power grid, the connection status between nodes is similar; The reason why clustering coefficient is zero is that the mine grid is radial network, most of the load are exclusively for the line; Because of its low voltage level, short power distance and less collusion line series, the mine grid has smaller average path length and network diameter.

The mine power grid only satisfies the criterion of small-world network [15]: the average path length $L \geq L_{random}$, therefore just like Sichuan_Chongqing power grid and Guangdong power grid, it does not belong to the small-world network. And combining with the degree distribution of topological model, this paper determines that the mine power grid belongs to scale-free network. Therefore, from the point of its degree distribution, the mine power grid belongs to scale-free network.

For the mine grid with gas generation nodes added, the average degree $k$ and the average path length $L$ are slightly decreased, while the clustering coefficient is the same. It means the addition of gas generators nodes have little impact on the network topology and the new grid is still a scale-free network.

### 3.3. Parameters Redefinition and Betweenness

The shortest path is defined as the path that has the minimum reactance value between the two nodes. And the numbers of the edges of the shortest path is defined as $d_{ij}^{'}$. Thus, the average path of the network, which is labeled as $L^{'}$, is defined as the average value of the distances between all the node couples, that is

$$L^{'}=\frac{1}{N(N-1)/2}\sum_{i\geq j}d_{ij}^{'} \tag{4}$$

where $N$ is the number of nodes in the network.

Node betweenness and edge betweenness are two kinds of betweenness. The betweenness of node $i$, represented as $B_i$, means the sum of the relative values, which is the ratio of the number of the shortest paths passing by node $i$ to the number of all the shortest paths in network. That is

$$B_i=\sum_{j,k\in N,j\neq k}\frac{n_{jk}(i)}{n_{jk}} \tag{5}$$

where $n_{jk}$ is the number of the shortest paths between node $j$ and node $k$, $n_{jk}(i)$ is the number of the shortest paths between node $j$ and node $k$ that pass by node $i$.

The betweenness of edge has the same definition as that of node. Explicitly, betweenness shows the role that node and edge play in the connectivity of network.

By calculating the betweenness of node and edge, this paper found that the nodes and edges with high betweenness represent the transformer substations and the main line, which are important in the power system. As all the gas generation nodes are added in the heavy load area, the periphery of the whole topological network, which has few influence on central nodes and edges.

As networks of the control system are growing, vulnerability is also increasing [16], so this paper analyses the vulnerability of mine power grid. In the vulnerability analysis of mine power grid, the betweenness of nodes and edges can identify the weak link.

## 4. Vulnerability Analysis of Mine Power Grid

### 4.1. Evaluating Criterion

The mine power grid with small network scale, simple structure and low capacity of power supply, only has two power supplies in most cases. Due to its special structural characteristics, the average shortest path length,the maximum connectivity [17-23], and the transmission efficiency [24] are not suitable for the analysis of mine power grid.

In view of the relatively small scale of mine power grid, each node and edge should be considered in the identification of weak links [25-27]. The complex network parameters of nodes and edges in the topological model are used to identify their vulnerability, and the power supply capacity of the network is measured by the remaining load capacity after being attacked [28].

$$S=\sum_{i=1}^{N}\min\{g_i,p_i\} \tag{6}$$

where $g_i$ is the power supply capacity of the sub-network after separation, $p_i$ is the load capacity of the sub-network after separation.

The remaining load capacity can quantify the power supply capacity of the attacked mine grid, which is superior in the vulnerability analysis of small-scale power grid.

According to the features of double transformer and single-bus section of mine grid, cascading failures triggered[29] by power redistribution are neglected in the calculation of the remaining load.

### 4.2. Vulnerability Analysis of Mine Power Grid

The simulated fault attack of mine power grid is divided in two modes, which are node attacks and edge attacks.

Four kinds of node attacking modes are defined as below to study the importance of single node:

1) Random Attacking 16 nodes in the network, that is, they were removed randomly;
2) Degree Attacking 16 nodes in sequence according to the degree of each node;
3) Weight Attacking 16 nodes in sequence according to the weight of each node;
4) Betweenness Attacking 16 nodes in sequence according to the betweenness of each node.

Three kinds of edge attacking modes are defined as following:

1) Random Attacking 16 edges in the network, that is, there were removed randomly;
2) Betweenness Attacking 16 edges in sequence according to the betweenness of each edge;
3) Weight Attacking 16 edges in sequence according to the weight of each edge.

The remaining loads capacity of the mine power grid without gas generation nodes after each round of attacking on nodes and edges are shown in Figure 4, and Figure 5.

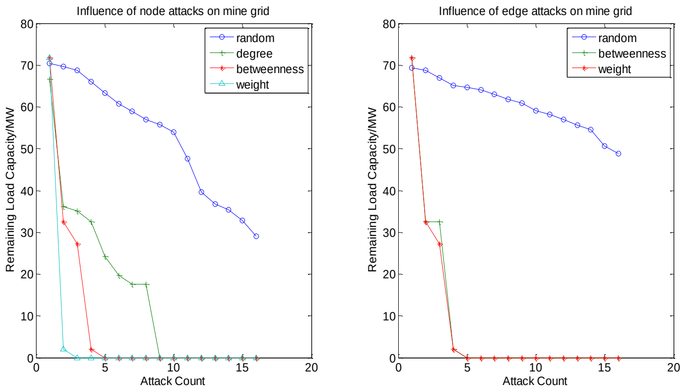


**Figure 4. and Figure 5. Influence of Attacks on Mine Power Grid Without Gas Generators Added**

The curves of node attacks Figure 4, show that the mine power grid is robust under the random attack, and the influence of the intentional attack on the power supply capacity is much greater. In terms of influence of power supply capacity, attacking in sequence according to the weight is the greatest, and attacking in sequence according to the degree is the smallest.

The curves of edge attacks Figure 5, show that the impact of the random removal of edges on the power supply capacity is very small. Due to the breakdown of mine power grid, the remaining load capacity variation curves of attacking in sequence according to weight and betweenness are almost coincident.

The remaining loads capacity of the mine power grid with gas generation nodes after attacking are shown in Figure 6, and Figure 7.

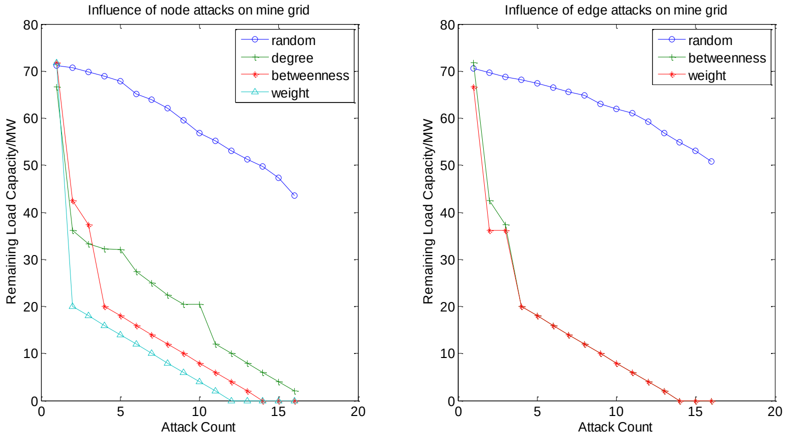


**Figure 6. and Figure 7. Influence of Attacks on Mine Power Grid with Gas Generators Added**

The results of random attacks show that the mine power grid with and without gas generations nodes added have same ability to resist line fault, but the former is more robust when nodes are attacked. And it can be seen from the curves of intentional attacks that the mine power grid is very vulnerable in response to the attacking of important nodes and lines, but has a strong ability to resist subsequent attacks when added gas generator nodes. Therefore, from the aspect of topological structure, the joining of the gas generator can enhance the ability to resist the attack, reduce the impact of the fault, and improve the power supply capacity of the mine power grid.

## 5. Conclusion

In this paper, the complex network theory is applied to the mine power grid, and the topological modeling and vulnerability analysis of the mine power grid are carried out. The results of research show that: by analyzing the network characteristics, this paper found that gas generation nodes added only has a certain impact on the degree of nodes, that is, the influence of the power distribution in mine power grid, has little effect on other indexes. Beside, the topological network belongs to scale-free network whether added gas scale-free network whether added gas generation nodes or not. With different attacking modes, the static vulnerability of the mine power grid is analyzed. The results show that the mine power grid is robust when attacked randomly, the attack according to the degree of nodes has the greatest influence on the power supply capacity, the attacks according to the betweenness and weight of edges present the same effect, and the addition of the gas generators can enhance the ability of the mine power grid to resist the attack and improve the power supply capacity.

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