International Journal of Grid Distribution Computing
Vol. 8, No.5, (2015), pp.73-80
http://dx.doi.org/10.14257/ijgdc.2015.8.5.06

OD-DE: An Improved Orthogonal Direction Algorithm Based on Direction Error

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Abstract

In the orthogonal direction (OD) algorithm, the iteration direction is inconsistent with the direction that leads to the iteration error, thereby, degrading the convergence performance of adaptive filtering. In this paper, we propose a new self-adaptive orthogonal direction algorithm (called OD-DE) by introducing iteration direction error. In the OD-DE algorithm, the iteration direction is changed to be consistent with the direction that causes the iteration error, both equal to the direction vector established for input signal. Besides, we also estimate the optimal step-size for the OD-DE algorithm, to further improve the convergence performance of adaptive filtering. Last, the simulation experiments demonstrate the effectiveness of our proposed algorithm.

Keyword: *affine projection, iteration direction, adaptive filtering, system identification*

1. Introduction

The adaptive filtering techniques have obtained wide applications in many domains such as noise and echo cancellation, equalization and beam forming. The normalized least mean square (NLMS) algorithm [1] is a widely-used adaptive filtering technique due to its simple computation. However, for the input signal of highly self-correlation, the NLMS algorithm has the shortage of slow convergence speed. In order to solve the problem, researchers proposed many self-adaptive filtering algorithms that are not only computationally efficient, but also rapidly convergent. For example, in [2], from the geometric viewpoint of affine subspace projection, the affine projection (AP) algorithm was proposed for the first time. In [3], the orthogonal direction (OD) algorithm was proposed by introducing the concept of the direction vector of input signal. In [4], the NLMS (NLMS-OCF) algorithm with orthogonal correction factors was proposed based on the thought that the convergence speed of adaptive filtering can be improved by making successive input signals orthogonal to each other. However, for these algorithms, the iteration direction is inconsistent with the direction that leads to the iteration error, thereby, reducing the convergence speed of adaptive filtering.

In order to improve the convergence performance of adaptive filtering, in [5], a new AP algorithm was presented based on the consideration that the direction vector of input signal can be reused. In [6], an AP algorithm with exponential smoothing factor was proposed based on the consideration that a variable step-size can improve the adaptive filtering performance. In [7], an AP algorithm with the variable regularization factor was proposed. In [8], the authors used the evolutionary method to automatically determine the projection order of the AP algorithm, consequently improving the self-adaptive filtering performance. In [9, 10], a new affine projection algorithm based on regressive estimated error was introduced by redefining the iteration error of the AP algorithm, and the convergence behavior was also analyzed.

ISSN: 2005-4262 IJGDC

For the OD algorithm, its iteration error is caused by the input vector that is constructed based on input signal, while its iteration direction is the direction vector of input signal, i.e., the iteration direction is inconsistent with the direction that leads to the iteration error. In this paper, to solve the above problem, a self-adaptive orthogonal direction algorithm based on direction error (called OD-DE for short) is proposed by analyzing the iteration error in the iteration direction of adaptive filtering. In the OD-DE algorithm, under a condition free from measurement noise, the iteration error is directly caused by the direction vector that is also the iteration direction of adaptive filtering. Besides, the optimal step-size is obtained by setting the weight error to be 0 in the direction vector. Finally, simulation experiments demonstrate that the OD-DE algorithm can effectively improve the convergence performance of adaptive filtering.

2. Problem Statement

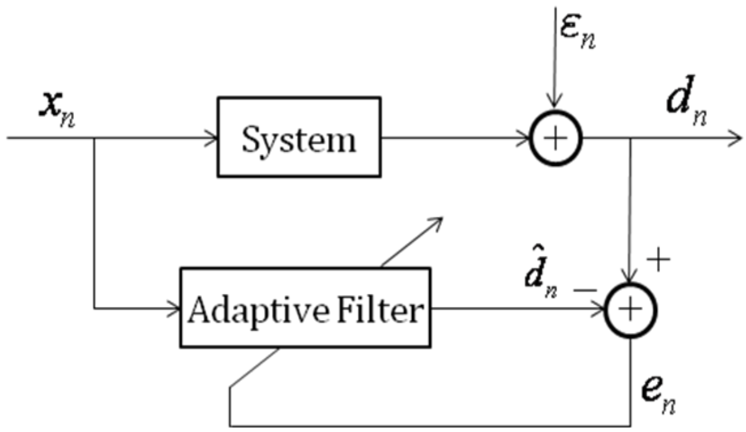

Figure 1. The System Identification Model of an Adaptive Filter

Figure 1 presents an adaptive filter used in the system identification model, where:

(1) The system input is x_n, which the wide sense stationary input signal is;

(2) The system output signal is d_n corresponding to the input signal;

(3) The estimate output signal of the adaptive filter is $\hat{d}_n$;

(4) The system measurement noise is ε_n, which zero-mean Gaussian white noise is.

First of all, using a tapped delay line (TDL) [4], the input signal x_n can be converted into an input vector x_n as follows,

$$\mathbf{x}_n = \begin{bmatrix} x_n & x_{n-1} & \cdots & x_{n-N+1} \end{bmatrix}^T \tag{1}$$

Now, the objective of the adaptive filter can be described as follows: estimating an N-dimensional weight vector $\mathbf{w}_n$ by using the most recent input vector $\mathbf{x}_n$ available at the nth instant for the adaptive filter, so as to minimize the mean-square error (MSE) between the actual output signal d_n and the estimate output signal $\hat{d}_n$ (that is obtained based on the weight vector $\mathbf{w}_n$).

The error e_n (called iteration error) between the actual output signal d_n and the estimate output signal $\hat{d}_n$ can be defined as

$$e_n = d_n - \hat{d}_n \tag{2}$$

, where the output signal $\hat{d}_n$ is estimated as

$$\hat{d}_n = \mathbf{w}_n^T \mathbf{x}_n \tag{3}$$

In [3], the OD algorithm presents an iterative method for updating the weight vector $\mathbf{w}_n$, which can be described by the following equation

$$\mathbf{w}_{n+1} = \mathbf{w}_n + \mu \frac{\boldsymbol{\varphi}_n}{\boldsymbol{\varphi}_n^T \boldsymbol{\varphi}_n} e_n \tag{4}$$

, where μ denotes a step-size, which is used to control the performance of adaptive filtering; and $\boldsymbol{\varphi}_n$ denotes the direction vector of input signal, which is defined as

$$\boldsymbol{\varphi}_n = \mathbf{x}_n - \hat{\mathbf{Z}}_{n-1} \hat{\mathbf{b}}_n \tag{5}$$

In Equation (5), the vector $\hat{\mathbf{b}}_n$ is given as

$$\hat{\mathbf{b}}_n = \left[\hat{\mathbf{Z}}_{n-1}^T \hat{\mathbf{Z}}_{n-1} \right]^{-1} \hat{\mathbf{Z}}_{n-1}^T \mathbf{x}_n \tag{6}$$

In Equations (5) and (6), $\hat{\mathbf{Z}}_{n\text{-}1}$ is an input matrix, which consists of the direction vectors of the most recent m input signals (orthogonal to each other), and thus it can be defined as follows

$$\hat{\mathbf{Z}}_{n-1} = \left[\boldsymbol{\varphi}_{n-1} \quad \boldsymbol{\varphi}_{n-2} \quad \cdots \quad \boldsymbol{\varphi}_{n-m} \right] \tag{7}$$

, where the initial matrix is given as

$$\hat{\mathbf{Z}}_0 = \left[\mathbf{0} \quad \mathbf{0} \quad \cdots \quad \mathbf{0} \right].$$

For ease of the analysis of the OD algorithm, we suppose that there truly exists an optimal N-dimensional weight vector $\mathbf{w}^0$ for the adaptive filter. Then, the actual system output d_n can be rewritten as

$$d_n = \mathbf{w}^{0T} \mathbf{x}_n + \varepsilon_n \tag{8}$$

Now, based on Equations (2), (3) and (8), the corresponding iteration error e_n can be rewritten as

$$e_n = \tilde{\mathbf{w}}_n^T \mathbf{x}_n + \varepsilon_n \tag{9}$$

, where $\tilde{\mathbf{w}}_n$ is a vector, which denotes the weight error of the adaptive filter at the nth instant, and thus it can be defined as

$$\tilde{\mathbf{w}}_n = \mathbf{w}^0 - \mathbf{w}_n \tag{10}$$

On the one hand, from Equation (9), it can be concluded that at the nth instant, the iteration error is caused by the input vector $\mathbf{x}_n$.

On the other hand, from Equation (4), it can be seen that the iteration direction is equal to the direction vector $\boldsymbol{\varphi}_n$ of input signal. Therefore, for the OD algorithm, the iteration direction is inconsistent with the direction that leads to the iteration error, resulting in some degree of deviation for the iteration error estimation, thereby, reducing the convergence performance of adaptive filtering.

To improve the convergence performance of adaptive filtering, a new self-adaptive orthogonal direction algorithm (called OD-DE) is proposed in the next section. In the OD-DE algorithm, under a condition free from measurement noise, the iteration direction would be changed to be consistent with the direction that causes the iteration error, both equal to the direction vector.

3. OD-DE Algorithm

In the section, we present the OD-DE algorithm. First, we describe how the weight vector is estimated. In the OD-DE algorithm, based on a new definition of direction error, the iteration direction would be consistent with the direction of iteration error. Second, we describe how to estimate the optimal step-size for the OD-DE algorithm, to improve the convergence speed of adaptive filtering.

3.1. Estimating the Weight Vector

Combining Equations (4) and (10), the weight vector $\mathbf{w}_n$ in the OD algorithm can be given by the following iterative equation

$$\tilde{\mathbf{w}}_{n+1} = \tilde{\mathbf{w}}_n - \mu \frac{\boldsymbol{\varphi}_n}{\boldsymbol{\varphi}_n^T \boldsymbol{\varphi}_n} e_n \tag{11}$$

In [3], it has been pointed out that the direction vector $\boldsymbol{\varphi}_n$ of input signal is orthogonal to the input matrix $\hat{\mathbf{Z}}_{n\text{-}1}$, i.e., $\boldsymbol{\varphi}_n^T \hat{\mathbf{Z}}_{n-1} = 0$. Now, after multiplying Equation (11) by $\boldsymbol{\varphi}_n^T$ (i.e., the inverse of the direction vector) and $\hat{\mathbf{Z}}_{n-1}^T$ (i.e., the inverse of the input matrix), respectively, we have

$$\boldsymbol{\varphi}_n^T \tilde{\mathbf{w}}_{n+1} = \boldsymbol{\varphi}_n^T \tilde{\mathbf{w}}_n - \mu e_n \tag{12a}$$

$$\hat{\mathbf{Z}}_{n-1}^T \tilde{\mathbf{w}}_{n+1} = \hat{\mathbf{Z}}_{n-1}^T \tilde{\mathbf{w}}_n \tag{12b}$$

From Equations (7) and (12b), we have

$$\begin{cases} \boldsymbol{\varphi}_{n-\delta}^T \tilde{\mathbf{w}}_{n+1} = \boldsymbol{\varphi}_{n-\delta}^T \tilde{\mathbf{w}}_n;\ 1 \le \delta \le m \\ \boldsymbol{\varphi}_{n-1-\tau}^T \tilde{\mathbf{w}}_n = \boldsymbol{\varphi}_{n-1-\tau}^T \tilde{\mathbf{w}}_{n-1};\ 1 \le \tau \le m-1 \\ \boldsymbol{\varphi}_{n-2-\kappa}^T \tilde{\mathbf{w}}_{n-1} = \boldsymbol{\varphi}_{n-2-\kappa}^T \tilde{\mathbf{w}}_{n-2};\ 1 \le \kappa \le m-2 \\ \vdots \\ \boldsymbol{\varphi}_{n-m-\gamma+1}^T \tilde{\mathbf{w}}_{n-m+2} = \boldsymbol{\varphi}_{n-m-\gamma+1}^T \tilde{\mathbf{w}}_{n-m+1};\ \gamma = 1 \end{cases} \tag{13}$$

Based on Equation (13), we obtain

$$\begin{bmatrix} \boldsymbol{\varphi}_{n-1}^T & \boldsymbol{\varphi}_{n-2}^T & \cdots & \boldsymbol{\varphi}_{n-m}^T \end{bmatrix} \tilde{\mathbf{w}}_n = \begin{bmatrix} \boldsymbol{\varphi}_{n-1}^T \tilde{\mathbf{w}}_n & \boldsymbol{\varphi}_{n-2}^T \tilde{\mathbf{w}}_{n-1} & \cdots & \boldsymbol{\varphi}_{n-m}^T \tilde{\mathbf{w}}_{n-m+1} \end{bmatrix} \tag{14}$$

After plugging Equation (12a) into Equation (14), we have

$$\begin{bmatrix}\boldsymbol{\varphi}_{n-1}^T & \boldsymbol{\varphi}_{n-2}^T & \cdots & \boldsymbol{\varphi}_{n-m}^T\end{bmatrix}\tilde{\mathbf{w}}_n = \begin{bmatrix}\boldsymbol{\varphi}_{n-1}^T\tilde{\mathbf{w}}_{n-1}-\mu e_{n-1} & \boldsymbol{\varphi}_{n-2}^T\tilde{\mathbf{w}}_{n-2}-\mu e_{n-2} & \cdots & \boldsymbol{\varphi}_{n-m}^T\tilde{\mathbf{w}}_{n-m}-\mu e_{n-m}\end{bmatrix} \quad (15)$$

In order to improve the filtering performance, the iteration error of the adaptive filter should be generated in the iteration direction of the adaptive filter. Based on such a consideration, we redefine the iteration error (we call it direction error to distinguish it from the iteration error defined in Equation (9)) as follows

$$\hat{e}_n = \tilde{\mathbf{w}}_n^T \boldsymbol{\varphi}_n \quad (16)$$

Now, based on Equation (16), Equation (15) can be rewritten as

$$\begin{bmatrix}\boldsymbol{\varphi}_{n-1}^T & \boldsymbol{\varphi}_{n-2}^T & \cdots & \boldsymbol{\varphi}_{n-m}^T\end{bmatrix}\tilde{\mathbf{w}}_n = \begin{bmatrix}\hat{e}_{n-1}-\mu e_{n-1} & \hat{e}_{n-2}-\mu e_{n-2} & \cdots & \hat{e}_{n-m}-\mu e_{n-m}\end{bmatrix} \quad (17)$$

From Equations (5), (7) and (16), we have

$$\hat{e}_n = \tilde{\mathbf{w}}_n^T\mathbf{x}_n - \begin{bmatrix}\tilde{\mathbf{w}}_n^T\boldsymbol{\varphi}_{n-1} & \tilde{\mathbf{w}}_n^T\boldsymbol{\varphi}_{n-2} & \cdots & \tilde{\mathbf{w}}_n^T\boldsymbol{\varphi}_{n-m}\end{bmatrix}\hat{\mathbf{b}}_n \quad (18)$$

After putting Equation (17) into Equation (18), the direction error $\hat{e}_n$ is represented as follows

$$\hat{e}_n = \tilde{\mathbf{w}}_n^T\mathbf{x}_n - \begin{bmatrix}\hat{e}_{n-1}-\mu e_{n-1} & \hat{e}_{n-2}-\mu e_{n-2} & \cdots & \hat{e}_{n-m}-\mu e_{n-m}\end{bmatrix}\hat{\mathbf{b}}_n \quad (19)$$

However, in the actual system, it is necessary to consider the measurement noise. Thus, based on Equation (9), the direction error $\hat{e}_n$ of the adaptive filter is approximately estimated as follows

$$\hat{e}_n = e_n - \begin{bmatrix}\hat{e}_{n-1}-\mu e_{n-1} & \hat{e}_{n-2}-\mu e_{n-2} & \cdots & \hat{e}_{n-m}-\mu e_{n-m}\end{bmatrix}\hat{\mathbf{b}}_n \quad (20)$$

As a result, the weight vector $\mathbf{w}_n$ can be redefined in the OD-DE algorithm by the following new iterative equation

$$\mathbf{w}_{n+1} = \mathbf{w}_n + \mu\frac{\boldsymbol{\varphi}_n}{\boldsymbol{\varphi}_n^T\boldsymbol{\varphi}_n}\hat{e}_n \quad (21)$$

Finally, Equations (2), (5), (6), (7), (20) and (21) in a certain order constitute the OD-DE algorithm.

From Equations (16) and (21), it can be seen that (1) for the OD-DE algorithm, the iteration direction is equal to the direction vector $\boldsymbol{\varphi}_n$ of input signal; and (2) under a condition free from measurement noise, the iteration direction error $\hat{e}_n$ is also equal to the direction vector $\boldsymbol{\varphi}_n$. Therefore, our proposed OD-DE algorithm, by redefining the iteration error, can well improve the convergence performance of adaptive filtering.

In addition, since the direction vectors of the most recent m input signals are orthogonal to each other, the inverse of the matrix $\hat{\mathbf{Z}}_{n-1}^T\hat{\mathbf{Z}}_{n-1}$ in Equation (6) can be rewritten as

$$\left[\hat{\mathbf{Z}}_{n-1}^T\hat{\mathbf{Z}}_{n-1}\right]^{-1} = diag\begin{bmatrix}\frac{1}{\boldsymbol{\varphi}_{n-1}^T\boldsymbol{\varphi}_{n-1}} & \frac{1}{\boldsymbol{\varphi}_{n-2}^T\boldsymbol{\varphi}_{n-2}} & \cdots & \frac{1}{\boldsymbol{\varphi}_{n-m}^T\boldsymbol{\varphi}_{n-m}}\end{bmatrix} \quad (22)$$

Based on Equation (22), the OD-DE algorithm can further improve the computation accuracy by avoiding calculating the inverse of the matrix.

3.2. Estimating the Iterative Step

Equation (21) uses a fixed step-size μ (which is used to control the performance of adaptive filtering). In order to further improve the convergence performance of adaptive filtering, in the OD-DE algorithm, we use the variable step-size μ_n to replace the fixed step-size m, thus we rewrite Equation (21) as follows

$$\mathbf{w}_{n+1} = \mathbf{w}_n + \mu_n \frac{\boldsymbol{\varphi}_n}{\boldsymbol{\varphi}_n^T \boldsymbol{\varphi}_n} \hat{e}_n \tag{23}$$

Below, we discuss how to estimate the optimal value $\mu_{n,opt}$ for the variable step-size μ_n. Suppose that at the （n+1）th instant, the optimal step-size $\mu_{n,opt}$ would make the weight error equal to zero on the direction vector. Thus, from Equation (12a), the optimal step-size $\mu_{n,opt}$ is obtained as

$$\mu_{n,opt} = \frac{\boldsymbol{\varphi}_n^T \tilde{\mathbf{w}}_n}{e_n} \tag{24}$$

Based on an inference process similar to that from Equation (12) to Equation (20), we have

$$\hat{e}_n = \boldsymbol{\varphi}_n^T \tilde{\mathbf{w}}_n = e_n - \left[\hat{e}_{n-1} - \mu_{n-1,opt} e_{n-1} \quad \hat{e}_{n-2} - \mu_{n-2,opt} e_{n-2} \quad \cdots \quad \hat{e}_{n-m} - \mu_{n-m,opt} e_{n-m}\right] \hat{\mathbf{b}}_n \tag{25}$$

From Equations (24) and (25), the optimal step-size $\mu_{n,opt}$ can be obtained as

$$\mu_{n,opt} = 1 - \frac{\left[\hat{e}_{n-1} - \mu_{n-1,opt} e_{n-1} \quad \hat{e}_{n-2} - \mu_{n-2,opt} e_{n-2} \quad \cdots \quad \hat{e}_{n-m} - \mu_{n-m,opt} e_{n-m}\right] \hat{\mathbf{b}}_n}{e_n} \tag{26}$$

Finally, Equations (2), (5), (6), (7), (23), (25) and (26) in a certain order constitute the OD-DE algorithm with the optimal step-size.

4. Simulation Experiment

In this section, by simulation experiments, we show the mean-square error learning (MSE) curves of the OD-DE algorithm with the optimal step-size.

In the simulation experiments, the initial weight vector of the adaptive filter is estimated as $\mathbf{w}_0 = 0$, which is a 32-dimensional vector. The system is set to have a 32-point long impulse with a signal-to-noise ratio 60 dB. The initial value $\mu_{0,opt}$ for the optimal step-size is set to 0.2. The optimal weight vector $\mathbf{w}^0$ for the adaptive filter is thought to be the vector with the maximum entropy, i.e., each direction (each dimension) of the vector has the same value. Here, we assume that the amplitude of each direction is equal to 1, thus we have

$$\mathbf{w}^0 = \mathbf{1} \tag{27}$$

, where $\mathbf{1}$ denotes a 32-dmensional vector where each element is equal to 1.

Besides, it should be pointed out that each learning curve of the experimental result is obtained by averaging 100 learning curves independent to each other (which are obtained by 100 independent experiments).

Case 1: Consider an ARMA (1, 2) input signal given by $y_n = z_n + 0.5z_{n-1} - 0.75y_{n-1}$ where z_n is zero-mean white Gaussian noise. We set the parameter $m = 2$. Figure 2 presents the MSE learning curves for the OD-DE algorithm with the optimal step-size. It can be observed that the proposed algorithm can improve the convergence speed of adaptive filtering.

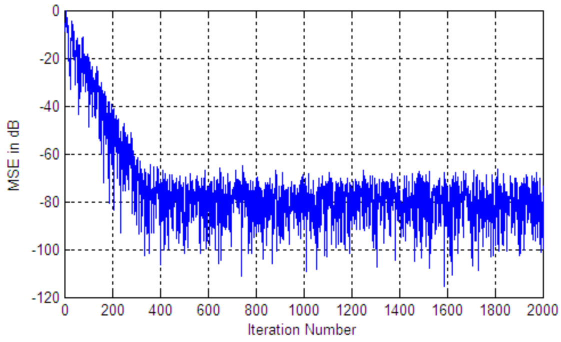

Figure 2. Learning Curves for the OD-DE Algorithm with ARMA (1, 2) Input

Case 2: Let us consider another ARMA (1, 3) input signal, which is given by $y_n = 0.95z_n - 0.72z_{n-1} + 0.5z_{n-3} - 0.95y_{n-1}$. We set the parameter $m = 3$. The MSE learning curves predicted by the OD-DE algorithm with the optimal step-size is shown in Figure 3. It can be conclude that the proposed algorithm can effectively improve the adaptive filtering performance.

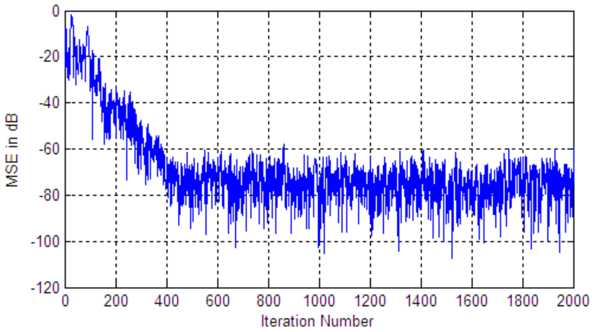

Figure 3. Learning Curves for the OD-DE Algorithm with ARMA (1, 3) Input

5. Conclusion

In the classical OD algorithm, the iteration direction is inconsistent with the direction that leads to the iteration error, thereby, degrading the convergence performance of adaptive filtering. In this paper, we proposed an improved OD algorithm (called OD-DE) to solve this problem. The contributions of the OD-DE algorithm are twofold.

First, in the OD-DE algorithm, we used the direction vector to define the direction error to replace the original iteration error in the OD algorithm, consequently, making that under a condition free from measurement noise, the iteration direction and the direction that leads to the iteration error are both consistent with the direction vector. Second, in the OD-DE algorithm, by setting the weight error to be zero in the direction vector, we presented the optimal step-size for the OD-DE algorithm, thereby further improving the convergence performance of adaptive filtering.

Finally, the simulation experiments demonstrated the effectiveness of our proposed OD-DE algorithm.

Acknowledgement

The author gratefully acknowledges the funding of Zhejiang Provincial Natural Science Foundation of China under Grant No.LY12F01016, No.LY13F020023, No.LY14E080018.

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