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The Delay-dependent Condition for Uncertain T-S Fuzzy Lurie Control Systems with Time-delay

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Abstract

The problem of delay-dependent condition for a new class of Takagi-Sugeno (T-S) fuzzy Lurie control systems with time-delay and time-variant uncertainties is investigated in this paper, which is different from existing ones. We use T-S fuzzy models to describe Lurie system in the form of a weighted sum of some simple linear subsystems, and a new delay-dependent absolute stability criterion for such systems is derived by utilizing Lyapunov-Krasovskii functional (LKF), linear matrix inequality (LMI) approach and novel techniques. Finally, a numerical example and its simulation results have shown that the proposed result is feasible and effective.

Keywords*: Takagi-Sugeno (T-S) fuzzy Lurie systems, delay-dependent, uncertain systems, absolute stability, Lyapunov-Krasovskii functional (LKF), linear matrix inequality (LMI)*

1. Introduction

Lurie control system is an important nonlinear system. Like many other nonlinear physical systems, it can be expressed as a feedback connection of a linear dynamical system and a nonlinear element, where the nonlinearities always satisfy a sector condition. The notion of absolute stability was introduced by Lur'e in [1] based on these class of nonlinear systems. As the time-delay phenomenon is frequently encountered in several of engineering systems such as chemical process, biological systems, medical systems, mechanical systems, economic systems, long transmission lines and so on, the systems with time-delay are concerned extensively. Since the existence of time-delay is often the main source of instability and poor performance, some stability criteria of Lurie systems with time-delay have also been derived over the past years [2]. But the stability conditions mentioned above are all delay-independent, which are often conservative when time-delay is small. Based on this, a considerable number of delay-dependent absolute stability conditions have been proposed [3]. Moreover, since Lurie direct type control systems include a class of plants without any practicality in engineering practice, some delay-dependent stability conditions have also been proposed in[4,5] for uncertain Lurie indirect systems.

On the other hand, the Takagi-Sugeno (T-S) fuzzy models described in [6] for the first time are powerful tools, which can provide an effective representation for complex nonlinear systems Therefore, the stability analysis and control synthesis of T-S fuzzy systems have attracted great attention from numerous researchers. In recent years, by using the LMI-based approach, several stability conditions for uncertain T-S fuzzy systems were derived in [7, 8].

However, to the authors' knowledge, the absolute stability of T-S fuzzy Lurie control systems with time delay and time-variant uncertainties has not been addressed up to now,

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which motivates the present study. In this paper, a new class of T-S fuzzy Lurie control systems with time-delay and time-variant uncertainties in the state and the nonlinearities are investigated. By utilizing Lyapunov-Krasovskii functional (LKF) together with the free weighting matrix technique, a novel delay-dependent absolute stability criterion for such new uncertain T-S fuzzy Lurie control systems with time-delay is derived in the form of LMIs. Finally, a numerical example will be provided to demonstrate feasibility and effectiveness of the proposed result.

2. Problem Formulation

In this section, we consider a class of uncertain T-S fuzzy Lurie control systems with time-delay, which is described by a Takagi-Sugeno (T-S) fuzzy model composed of a set of fuzzy implication. Each implication is expressed by a nonlinear time-delay Lurie control system and the i th rule of the T-S fuzzy model for each $i=1,2,\cdots r$ is represented as follows:

Plant Rule i: If $s_1(t)$ is μ_{i1} and $s_2(t)$ is μ_{i2} $\cdots$ and $s_g(t)$ is μ_{ig} THEN

$$\begin{cases} \dot{x}(t)=\left(A_i+\Delta A_i(t)\right)x(t)+\left(B_i+\Delta B_i(t)\right)x(t-\tau)+\left(C_i+\Delta C_i(t)\right)u(t) \\ \qquad +\left(D_i+\Delta D_i(t)\right)f\left(\sigma(t)\right), t\geq 0 \\ \dot{\sigma}(t)=c^T x(t)-\rho f\left(\sigma(t)\right), \\ x(\theta)=\varphi(\theta), \quad \theta\in[-\tau,0] \end{cases} \tag{1}$$

where μ_{ij} is the fuzzy set and r is the number of IF-THEN rules; $x(t)\in R^n$ denotes the sate vector; $u(t)\in R^q$ is the control input; $c\in R^n, \rho\in R$; $\tau>0$ is the time-delay; A_i, B_i, C_i and D_i are known real constant matrices; $\Delta A_i(t), \Delta B_i(t)$, $\Delta C_i(t)$ and $\Delta D_i(t)$ are real-valued unknown matrices representing time-varying parameter uncertainties, and are assumed to be of the form:

$$\left[\Delta A_i(t)\ \Delta B_i(t)\ \ \Delta C_i(t)\ \ \Delta D_i(t)\right]=M_i F_i(t)\left[E_{1i}\ E_{2i}\ E_{3i}\ E_{4i}\right] \qquad i=1,2\cdots r \tag{2}$$

Where $M_i, E_{1i}, E_{2i}, E_{3i}, E_{4i}$ are known real constant matrices of appropriate dimensions and $F_i(\cdot):\mathbb{R}\to\mathbb{R}^{l_1\times l_2}$ is an unknown time-varying matrix function satisfying

$$F_i^T(t)F_i(t)\leq I, \quad i=1,2\cdots r. \tag{3}$$

And $\varphi(\cdot)\in C\left([-\tau,0],R^n\right)$ is a continuous vector valued initial function; the nonlinearity function $f(\cdot)$ satisfy the following sector condition:

$$f(\cdot)\in K[0,\infty]=\left\{f(\cdot)\,|\,f(0)=0, 0<\sigma f\left(\sigma(t)\right)<\infty, \sigma\neq 0\right\}.$$

By using a center-average defuzzifier, product fuzzy interference, and singleton fuzzifier, the dynamic fuzzy model can be represented in following form:

$$\begin{cases} \dot{x}(t)=\sum_{i=1}^{r}h_i\left(s(t)\right)\left\{\Delta A_i x(t)+\Delta B_i x(t-\tau)+\Delta C_i u(t)+\Delta D_i f\left(\sigma(t)\right)\right\}, \quad t\geq 0 \\ \dot{\sigma}(t)=c^T x(t)-\rho f\left(\sigma(t)\right), \\ x(\theta)=\varphi(\theta), \quad \theta\in[-\tau,0] \end{cases} \tag{4}$$

Here we define:

$$\Delta A_i = A_i + \Delta A_i(t), \Delta B_i = B_i + \Delta B_i(t), \Delta C_i = C_i + \Delta C_i(t), \Delta D_i = D_i + \Delta D_i(t),$$

With $A_i, B_i, C_i, D_i, \Delta A_i(t), \Delta B_i(t), \Delta C_i(t)$ and $\Delta D_i(t)$ are the same as the corresponding items in (1). The fuzzy basis functions are described by:

$$h_i(s(t)) = \frac{\omega_i(s(t))}{\sum_{j=1}^{r} \omega_j(s(t))}, \tag{5}$$

Where $\omega_i(s(t)) = \prod_{j=1}^{g} \mu_{ij}(s_j(t)), \quad s(t) = \left[s_1(t), s_2(t), \cdots, s_g(t)\right]^T, i = 1, 2, \cdots, r,$

And $\mu_{ij}(s_j(t))$ is the grade of membership of $s_j(t)$ in μ_{ij}. Then, it can be seen that

$$\omega_i(s(t)) \geq 0, \sum_{j=1}^{r} \omega_j(s(t)) > 0, i = 1, 2, \cdots, r, \forall t \geq 0$$

$$h_i(s(t)) \geq 0 \quad i = 1, 2 \cdots r, \sum_{i=1}^{r} h_i(s(t)) = 1. \forall t \geq 0$$

In this paper, a state feedback T-S fuzzy-model-based controller will be designed for the stabilization of the T-S fuzzy system (4). The i th controller rule is

Control rule i: If $s_1(t)$ is μ_{i1} and $s_2(t)$ is μ_{i2} $\cdots$ and $s_g(t)$ is μ_{ig} THEN

$$u(t) = K_i x(t) \quad i = 1, 2, \cdots r,$$

Where K_i ($i = 1, 2, \cdots r$) are the local control gains. Then, the overall fuzzy sate feedback controller is given by $u(t) = \sum_{i=1}^{r} h_i(s(t)) K_i x(t) \quad i = 1, 2, \cdots r$.

In order to verify the main results of this paper, we shall use the following lemmas:

Lemma1 (see [9]) given $A, D, S, W,$ and F be real matrices with appropriate dimensions such that $W > 0$ and $F^T F \leq I$. Then we have the following:

(1) For any scalar $\varepsilon > 0$ and vector x and y of appropriate dimensions.

$$2x^T DFSy \leq \varepsilon^{-1} x^T DD^T x + \varepsilon y^T S^T Sy.$$

(2) For any scalar $\varepsilon > 0$ such that $W - \varepsilon DD^T > 0$ and

$$(A + DFS)^T W^{-1} (A + DFS) \leq A^T \left(W - \varepsilon DD^T\right)^{-1} A + \varepsilon^{-1} S^T S.$$

Lemma2 (see [10]) For any constant symmetric matrix $M \in R^{n \times n}, M > 0,$ scalar $h > 0$. Vector function $\dot{x}(\cdot) \in \left(\left[-h, 0\right], R^n\right)$ such that the integrations in the following are well defined, then $h \int_0^h \dot{x}^T(s) M \dot{x}(s) ds \geq \left(\int_0^h \dot{x}(s) ds\right)^T M \left(\int_0^h \dot{x}(s) ds\right).$

Lemma3 (see [11]) suppose that matrices $\{M_i\}_{i=1}^{r} \in R^{N \times M}$ and a semi-positive-definite matrix $P \in R^{N \times N}$ are given, then $\left(\sum_{i=1}^{r} h_i M_i\right)^T P \left(\sum_{i=1}^{r} h_i M_i\right) \leq \sum_{i=1}^{r} h_i M_i^T P M_i$, where $h_i\left(i = 1, 2 \cdots r\right)$ are fuzzy basis function defined by (5).

3. Main Results

Theorem1. The system described by (4) is absolutely stable if there exist symmetric positive definite matrices $P, Z, Q,$ and matrices K_i, scalars $\varepsilon_{1ij} > 0, \varepsilon_{2ij} > 0,$ such that the following LMIS hold for all $i = 1, 2 \cdots r, j = 1, 2, \cdots r$

$$\Psi_{ii} = \begin{bmatrix} \Pi_{1ii} & \Pi_{2ii} & \Pi_{3ii} & \tau A_i^T Z + \tau K_i^T C_i^T Z & 0 & PM_i \\ * & \Pi_{4ii} & \Pi_{5ii} & \tau B_i^T Z & 0 & 0 \\ * & * & \Pi_{6ii} & \tau D_i^T Z & 0 & 0 \\ * & * & * & -Z & ZM_i & 0 \\ * & * & * & * & -\varepsilon_{2ii} I & 0 \\ * & * & * & * & * & -\varepsilon_{1ii} I \end{bmatrix} < 0, \quad i = j, \quad (6)$$

$$\Psi_{ij} = \begin{bmatrix} \Psi_{1ij} & \Psi_{2ij} \\ * & \Psi_{3ij} \end{bmatrix}, \quad 1 \le i < j \le r, \quad (7)$$

$$\Psi_{1ij} = \begin{bmatrix} \Pi_{1ij} + \Pi_{1ji} & \Pi_{2ij} + \Pi_{2ji} & \Pi_{3ij} + \Pi_{3ji} \\ * & \Pi_{4ij} + \Pi_{4ji} & \Pi_{5ij} + \Pi_{5ji} \\ * & * & \Pi_{6ij} + \Pi_{6ji} \end{bmatrix},$$

$$\Psi_{2ij} = \begin{bmatrix} \tau A_i^T Z + \tau K_j^T C_i^T Z & 0 & \tau A_j^T Z + \tau K_i^T C_j^T Z & 0 & PM_i & PM_j \\ \tau B_i^T Z & 0 & \tau B_j^T Z & 0 & 0 & 0 \\ \tau D_i^T Z & 0 & \tau D_j^T Z & 0 & 0 & 0 \end{bmatrix},$$

$$\Psi_{3ij} = \begin{bmatrix} -Z & ZM_i & 0 & 0 & 0 & 0 \\ * & -\varepsilon_{2ij} I & 0 & 0 & 0 & 0 \\ * & * & -Z & ZM_j & 0 & 0 \\ * & * & * & -\varepsilon_{2ji} I & 0 & 0 \\ * & * & * & * & -\varepsilon_{1ij} I & 0 \\ * & * & * & * & * & -\varepsilon_{1ji} I \end{bmatrix},$$

Where $*$ denotes the elements below the main diagonal of a symmetric block matrix,

$\Pi_{1ij} = P\left(A_i + C_i K_j\right) + \left(A_i + C_i K_j\right)^T P - Z + Q + \varepsilon_{ij}\left(E_{1i} + E_{3i} K_j\right)^T \left(E_{1i} + E_{3i} K_j\right),$

$\Pi_{2ij} = PB_i + Z + \varepsilon_{ij}\left(E_{1i} + E_{3i} K_j\right)^T E_{2i},\ \Pi_{3ij} = PD_i + c + \varepsilon_{ij}\left(E_{1i} + E_{3i} K_j\right)^T E_{4i},$

$\Pi_{4ij} = -Z - Q + \varepsilon_{ij} E_{2i}^T E_{2i},\ \Pi_{5ij} = \varepsilon_{ij} E_{2i}^T E_{4i},\ \Pi_{6ij} = \varepsilon_{ij} E_{4i}^T E_{4i} - 2\rho,$

$\varepsilon_{ij} = \varepsilon_{1ij} + \tau^2 \varepsilon_{2ij}.\left(1 \le i < j \le r\right)$

Proof According to the Lyapunov stable theory, we define the Lyapunov functional candidate:

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t),$$

$$V_1(t) = x^T(t) P x(t),\ V_2(t) = \tau \int_{-\tau}^{0} \int_{t+\beta}^{t} \dot{x}^T(\alpha) Z \dot{x}(\alpha) d\alpha d\beta,$$

$$V_3(t) = \int_{t-\tau}^{t} x^T(\alpha) Q x(\alpha) d\alpha,\ V_4(t) = 2\int_0^{\sigma(t)} f(\sigma) d\sigma.$$

Then, the time derivative of V (t) along the trajectory of system (4) is given by

$$\dot{V}(t) = \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t) + \dot{V}_4(t), \quad (8)$$

Where

$$\dot{V}_1(t) = 2x^T(t)P\dot{x}(t) \tag{9}$$

Use Lemma 1(1), there holds

$$\dot{V}_1(t) \le 2\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(s(t))h_j(s(t))x^T(t)P\bar{A}_{ij}\eta(t) + \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(s(t))h_j(s(t)) \left[\varepsilon_{1ij}{}^{-1}x^T PM_iM_i{}^T Px + \varepsilon_{1ij}\eta^T(t)\bar{E}_{ij}^T\bar{E}_{ij}\eta(t)\right], \tag{10}$$

Where

$$\bar{A}_{ij} = \left[A_i + \mathrm{C}_i\mathrm{K}_j \;\; B_i \;\; D_i\right], \quad \eta(t) = \left[x^T(t) \quad x^T(t-\tau) \quad f^T(\sigma(t))\right]^T,$$
$$\bar{E}_{ij} = \left[E_{1i} + E_{3i}k_j \;\; E_{2i} \;\; E_{4i}\right]$$

And

$$\dot{V}_2(t) = \tau^2\dot{x}^T(t)Z\dot{x}(t) - \tau\int_{t-\tau}^{t}\dot{x}^T(\alpha)Z\dot{x}(\alpha)d\alpha, \tag{11}$$

$$\dot{V}_3(t) = x^T(t)Qx(t) - x^T(t-\tau)Qx(t-\tau), \tag{12}$$

$$\dot{V}_4(t) = 2\dot{\sigma}(t)f(\sigma(t)) = 2\left[c^T x(t)f(\sigma(t)) - \rho f^2(\sigma(t))\right]. \tag{13}$$

Now, using lemma 2, it can be shown that

$$\begin{aligned}\dot{V}_2(t) &\le \tau^2\dot{x}^T(t)Z\dot{x}(t) - \left[\int_{t-\tau}^{t}\dot{x}(\alpha)d\alpha\right]^T Z\left[\int_{t-\tau}^{t}\dot{x}(\alpha)d\alpha\right] \\ &= \tau^2\dot{x}^T(t)Z\dot{x}(t) - \left[x(t)-x(t-\tau)\right]^T Z\left[x(t)-x(t-\tau)\right].\end{aligned} \tag{14}$$

By lemma3 and lemma 1(2), it can be also verified that

$$\begin{aligned}\dot{x}^T(t)Z\dot{x}(t) &= \left\{\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(s(t))h_j(s(t))\left[\bar{A}_{ij} + M_iF_i(t)\bar{E}_{ij}\right]\eta(t)\right\}^T Z \\ &\quad \left\{\sum_{k=1}^{r}\sum_{l=1}^{r}h_k(s(t))h_l(s(t))\left[\bar{A}_{kl} + M_kF_k(t)\bar{E}_{kl}\right]\eta(t)\right\} \\ &\le \sum_{i=1}^{r}\sum_{j=1}^{r}h_i(s(t))h_j(s(t))\eta^T(t)\left\{\left[\bar{A}_{ij} + M_iF_i(t)\bar{E}_{ij}\right]^T Z\left[\bar{A}_{ij} + M_iF_i(t)\bar{E}_{ij}\right]\right\}\eta(t) \\ &\le \sum_{i=1}^{r}\sum_{j=1}^{r}h_i(s(t))h_j(s(t))\eta^T(t)\left\{\bar{A}_{ij}^T\left(Z^{-1} - \varepsilon_{2ij}^{-1}M_iM_i^T\right)^{-1}\bar{A}_{ij} + \varepsilon_{2ij}\bar{E}_{ij}^T\bar{E}_{ij}\right\}\eta(t),\end{aligned} \tag{15}$$

Then

$$\begin{aligned}\dot{V}_2(t) &\le \tau^2\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(s(t))h_j(s(t))\eta^T(t)\left\{\bar{A}_{ij}^T\left(Z^{-1} - \varepsilon_{2ij}^{-1}M_iM_i^T\right)^{-1}\bar{A}_{ij} + \varepsilon_{2ij}\bar{E}_{ij}^T\bar{E}_{ij}\right\}\eta(t) \\ &\quad - \left[x(t)-x(t-\tau)\right]^T Z\left[x(t)-x(t-\tau)\right].\end{aligned} \tag{16}$$

It then follows from (10), (12), (13) and (16) that

$$\begin{aligned}\dot{V}(t) &\le \sum_{i=1}^{r}\sum_{j=1}^{r}h_i(s(t))h_j(s(t))\eta^T(t)\left\{\Omega_{ij} + \tau^2\left[\bar{A}_{ij}^T\left(Z^{-1} - \varepsilon_{2ij}^{-1}M_iM_i^T\right)^{-1}\bar{A}_{ij} + \varepsilon_{2ij}\bar{E}_{ij}^T\bar{E}_{ij}\right]\right\}\eta(t) \\ &= \sum_{i=1}^{r}\sum_{j=1}^{r}h_i(s(t))h_j(s(t))\eta^T(t)W_{ij}\eta(t),\end{aligned}$$

Where

$$W_{ij} = \Omega_{ij} + \tau^2\bar{A}_{ij}{}^T\left(Z^{-1} - \varepsilon_{2ij}{}^{-1}M_iM_i{}^T\right)^{-1}\bar{A}_{ij} + \varepsilon_{ij}\bar{E}_{ij}{}^T\bar{E}_{ij}.$$

If $W_{ij} < 0$, by the schur complement formular, it provides that

$$\begin{bmatrix} \Omega_{ij} + \varepsilon_{ij}\bar{E}_{ij}^{T}\bar{E}_{ij} & \tau\bar{A}_{ij}^{T}Z \\ * & -Z + \varepsilon_{2ij}^{-1}ZM_{i}M_{i}^{T}Z \end{bmatrix} < 0,$$

so, according to the schur complement formula again, we have

$$\begin{bmatrix} \Omega_{ij} + \varepsilon_{ij}\bar{E}_{ij}^{T}\bar{E}_{ij} & \tau\bar{A}_{ij}^{T}Z & 0 \\ * & -Z & ZM_{i} \\ * & * & -\varepsilon_{2ij}I \end{bmatrix} < 0,$$

That is

$$\begin{bmatrix} \Psi_{i} & PB_{i} + Z + \varepsilon_{ij}\left(E_{1i} + E_{3i}K_{j}\right)^{T}E_{2i} & PD_{i} + c + \varepsilon_{ij}\left(E_{1i} + E_{3i}K_{j}\right)^{T}E_{4i} & \tau\left(A_{i} + C_{i}K_{j}\right)^{T}Z & 0 \\ * & -Q - Z + \varepsilon_{ij}E_{2i}^{T}E_{2i} & \varepsilon_{ij}E_{2i}^{T}E_{4i} & \tau B_{i}^{T}Z & 0 \\ * & * & \varepsilon_{ij}E_{4i}^{T}E_{4i} - 2\rho & \tau D_{i}^{T}Z & 0 \\ * & * & * & -Z & ZM_{i} \\ * & * & * & * & -\varepsilon_{2ij}I \end{bmatrix} < 0,$$

Where

$$\Psi_{i} = P\left(A_{i} + C_{i}K_{j}\right) + \left(A_{i} + C_{i}K_{j}\right)^{T}P - Z + Q + \varepsilon_{ij}\left(E_{1i} + E_{3i}K_{j}\right)^{T}\left(E_{1i} + E_{3i}K_{j}\right) + PM_{i}\varepsilon_{1ij}^{-1}M_{i}^{T}P.$$

By using schur complement formula again, the LMI (6) and (7) in Theorem1 can be verified. From (6) and (7), we have that $\dot{V}(t) \le -\varepsilon\left\|x(t)\right\|^{2}$ for $x(t) \ne 0$, which shows that the uncertain T-S fuzzy Lurie system with time-delay described by (4) is absolutely stable. This completes the proof.

4. Numerical Example

In this section, a simulation example will be provided to illustrate the theoretical result developed in this paper. The uncertain T-S fuzzy Lurie system with time-delay considered in this example is with two rules for $i = 2,$ $i = j, u(t) \equiv 0$ and $f\left(\sigma(t)\right) = \sigma(t)$ $(i = 1,2)$, $\rho = 0.5,$ $\tau = 0.7$. The fuzzy basis functions for Rule 1 and Rule 2 are $h_{1}\left(s_{1}(t)\right) = \sin^{2}\left(\pi s_{1}(t)\right)$ and $h_{2}\left(s_{1}(t)\right) = \cos^{2}\left(\pi s_{1}(t)\right)$.

We let $A_{1} = \begin{bmatrix} -0.19 & 0 \\ -0.2 & -0.1 \end{bmatrix}, A_{2} = \begin{bmatrix} -0.1 & 0.16 \\ -0.1 & -0.2 \end{bmatrix}, B_{1} = \begin{bmatrix} -0.2 & 0.3 \\ -0.1 & 0 \end{bmatrix}, B_{2} = \begin{bmatrix} -0.25 & -0.2 \\ -0.2 & 0 \end{bmatrix}$

$$D_{1} = \begin{bmatrix} -0.3 \\ 0.2 \end{bmatrix}, D_{2} = \begin{bmatrix} -0.2 \\ 0.1 \end{bmatrix}, E_{11} = \begin{bmatrix} 0.2 & 0.2 \\ -0.2 & 0.2 \end{bmatrix}, E_{21} = \begin{bmatrix} 0.2 & 0.15 \\ 0.2 & 0.1 \end{bmatrix}$$

$$E_{41} = \begin{bmatrix} 0.3 \\ 0.2 \end{bmatrix}, E_{42} = \begin{bmatrix} 0.2 \\ 0.3 \end{bmatrix}, E_{12} = \begin{bmatrix} 0.2 & 0.25 \\ 0.1 & 0.2 \end{bmatrix}, E_{22} = \begin{bmatrix} 0.2 & 0.3 \\ 0.1 & 0.2 \end{bmatrix}$$

$$M_{1} = \begin{bmatrix} 0.3 & 0.2 \\ 0.1 & 0.3 \end{bmatrix}, M_{2} = \begin{bmatrix} 0.3 & 0.1 \\ 0.3 & 0.2 \end{bmatrix}, c = \begin{bmatrix} 0.1 \\ -0.2 \end{bmatrix}, F_{i}(t) = \begin{bmatrix} \sin(t) & 0 \\ 0 & \cos(t) \end{bmatrix}, i = 1,2.$$

Using the MATLAB LMI Toolbox to solve the LMI, we obtained a set of feasible solutions as follows:

$$P=\begin{bmatrix}0.5697 & -0.3787\\ -0.3787 & 0.9896\end{bmatrix}, Q=\begin{bmatrix}0.0256 & 0.0192\\ 0.0192 & 0.0791\end{bmatrix}, Z=\begin{bmatrix}1.1681 & -0.7916\\ -0.7916 & 0.7045\end{bmatrix},$$

$$\varepsilon_{11}=0.3103,\quad \varepsilon_{21}=0.1581,\quad \varepsilon_{12}=0.3130,\quad \varepsilon_{22}=0.0485.$$

And the maximum value of τ is 0.77 in this numerical experiment.

By using the MATLAB Simulink Toolbox, the state response of the system (4) is shown in Fig.1.The numerical and simulated results have shown that all the conditions of Theorem 1 are satisfied.

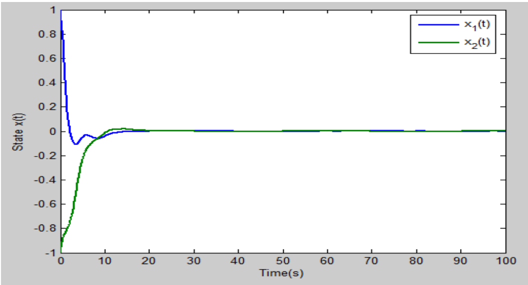

Figure 1. Response of the State x (t) With Uncertainties

5. Conclusion

In this paper, the problem of absolute stability for a class of uncertain T-S fuzzy Lurie control systems with time-delay is considered. We use T-S fuzzy models to describe lurie system in the form of a weighted sum of some simple linear subsystems, and we use LMI method, which has many advantages such as the use of simple, easy to computer implementation. Based on these point, a new system model is created, which is different from existing ones. Appropriate Lyapunov functional candidate have been defined, and a new delay-dependent condition for such system is obtained and described in the form of LMIs by using Lyapunov-Krasovskii functional (LKF) together with linear matrix inequality (LMI) approach. Mean-while, the results obtained by the free weight matrix method are less conservative than the fixed weight matrix method. Finally, the results of a numerical example have shown that the proposed result is feasible and effective.

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