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Normal Estimation for Mass Point Clouds of Irregular Model in the 3D Reconstruction based on Fuzzy Inference

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Abstract

This paper presents a fuzzy normal estimate for mass point clouds of irregular models in reconstruction. The irregular model is complex object that some part is smooth and some parts are irregular including sharp features. Therefore, we put kNN and curvature of mass point clouds to fuzzy inference system to divide the kind of point clouds and the output of FIS can determine which part of tooth point clouds belong to. For different kinds point clouds, corresponding algorithm is given. Point clouds in the smooth area are estimated normal by PCA directly and ones in other regions of thin or sharp area are estimated by checker and attach points. This method is simpler than those complex methods used on the whole point clouds directly. The experiment results show that much time is saved and surface reconstruction is very fine than PCA and WLOP.

Keywords: *mass point clouds, irregular model, surface reconstruction, normal estimate, fuzzy inference, implicit function*

1. Introduction

A variety of technologies are involved during the three-dimensional reconstruction of irregular models. Normal and its orientation is one of the essential properties of point in the mass point clouds. It decides the inside/outside of irregular model playing an important role. In general, normal of point can be gotten from the preparatory appliance in plaster models or real objects while the position of point is scanning by scanner. However, some unavoidable noise and physical errors will be included during the data acquisition. In addition, irregular model is blocked by itself and some parts of models contain sharp feature. Therefore normal of point from scanner is incomplete and inaccurate. It is necessary to find a model to correct normal of point in order to obtain an excellent 3D irregular model.

There are two classes of normal estimation methods that one is based on local estimation named Principal Component Analysis (PCA) and the other is based on global estimation named Voronoi diagram. Classical PCA method [1-3] relies on Euclidean distances between points to infer orientation of normal. And it requests that the point should be uniform and thick enough. Otherwise, the normal computed is unreliable and this kind of method only suits to the uniform data. Huang, *et al.*, [4] propose Weighted

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Locally Optima Projection operator (WLOP) that refines a set of particles without denoised, out-free and distributed over the original point clouds. It is more reliable for estimation normal than PCA. In this way, the normal could be more reliable. But the process of reorganization is complex. But the efficiency of program is low due to the complex refining. The character of irregular model is complex that some part is uniform and smooth and other is contrary. It is the target for researchers to design an appropriate method. Chen, *et al.*, [5] propose an orientation inference algorithm as a graph optimization problem. An energy function is defined to penalize inconsistent orientation changes by checking the sign consistency between neighboring local surface. The two kinds of algorithm mentioned above require separate steps for redirection of normal vector. Wang, *et al.*, [6] integrate the un oriented normal estimation and the consistent normal orientation into one variation framework, which minimizing a combination of the Dirichlet energy and the coupled-orthogonality deviation can guarantee normal field with consistent orientation and continuously vary on the underlying shape. Voronoi Pole [7] is regarded as the normal of point in the Voronoi diagram. It is very simple to obtain normal. So building a Voronoi diagram is needed using point clouds. However, the default of Voronoi is more sensitive to noise than PCA. Alliez, *et al.*, [8] use advantages of the local character of PCA and the global partition quality of Voronoi and obtain more stable normal.

As mentioned above, these methods consider point clouds with noise and outliers. But it is difficult to deal with sharp features. A robust normal estimation method [9] detects the best local tangent plane for each point using robust statistics that it is suitable for point clouds located in high curvature regions or near/on sharp feature. An adaptive method [10] can handle models with non uniform sampling. It uses adaptive spherical cover to generate the triangular mesh and computes normal orientation of each point based on its nearest triangle. All kinds of method often can resolve a certain type of problem and ignore other type of one. Liu, *et al.*, [11] propose a method based on fuzzy inference that it gives fuzzy rules and corresponding algorithms according to different problems. But it does not give unified expression to solve problem.

In this paper, we propose a fuzzy normal estimation method. According to the character of mass point clouds of irregular model, we give the fuzzy rules from human experts and put them into the fuzzy inference system (FIS). The output of FIS can divide the degree of point complexity and adopt correspond algorithms to estimate. The method is simple and practical and solves the problem that sophisticated algorithm is inefficient.

The remainder of the paper is organized as follows. We firstly analysis Fuzzy Inference System (FIS) applied in normal estimation in Section 2. Then, we present fuzzy normal estimation method in Section 3. In Section 4, experimental results are demonstrated by mass point clouds of toy duck and pig. Finally, we conclude the paper with some future work.

2. FIS Applied in Normal Estimation of Mass Point Clouds

FIS [14] includes four main components, fuzzifier, fuzzy rule base, fuzzy inference engine and defuzzifier in Figure 1. The fuzzifier maps non-fuzzy input space to fuzzy sets and the defuzzifier performs an opposite task to map fuzzy sets to non-fuzzy output space. Fuzzy sets are characterized by their membership functions $u(.)$'s. The fuzzy inference engine performs a mapping from fuzzy sets in input space $U \in R^n$ to fuzzy sets in output space $V \in R^m$ based on fuzzy rules. Finally, it is assumed that:

$U = U_1 \times U_2 \times ... \times U_n$, $U_i \in R$ (i=1,2,…,n) and $V = V_1 \times V_2 \times ... \times V_n, V_i \in R$, (k=1,2,…,m). In this paper, the following FIS is introduced in detail.

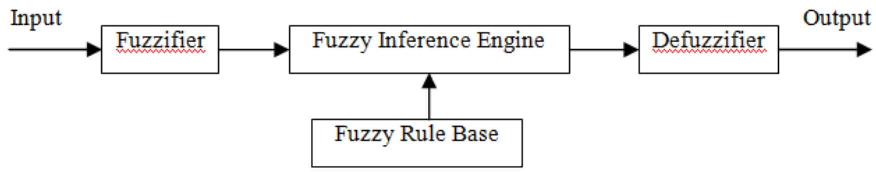


Figure 1. Components of FIS

The fuzzy rule base consists of a collection of fuzzy IF-THEN rules:

R_j: if x_{knn} is A_1^j and x_{cur} is A_2^j, then y is B_m

where x=$[x_{knn}, x_{cur}]^T$ and y are input vectors and output value of FIS, respectively. A_i^j (i=1,2,..,n) and B_k (k=1,2,…,m) are linguistic variables of fuzzy sets in subspace U_i and V_k described by their membership functions $\mu_{A_i^j}(x_i)$ and $\mu_{B_k^j}(y_k)$, j=1,2,…,M. M is total number of the fuzzy rules. By using product inference, singleton-fuzzifier, and center-average defuzzifier strategies, output of FIS can be expressed as

$$y(x) = \hat{\Omega}(x \mid \hat{\Theta}) = \frac{\sum_{j=1}^{M} \bar{y}^j (\prod_{i=1}^{n} \mu_{A_i^j}(x_i))}{\sum_{j=1}^{M} (\prod_{i=1}^{n} \mu_{A_i^j}(x_i))} = \hat{\Theta}\xi(x) \tag{1}$$

where $\mu_{A_i^j}(x_i)$ is membership function value of fuzzy variable x_i , $\bar{y}^j$ is the point at which $\mu_{B_k^j}(y_k)$ achieves its maximum value, and it is assumed that $\mu_{B_k^j}(\bar{y}^j) = 1$.

$$\hat{\Theta} = \begin{pmatrix} \bar{y}_1^1 & \bar{y}_1^2 & \cdots & \bar{y}_1^M \\ \bar{y}_2^1 & \bar{y}_2^2 & \cdots & \bar{y}_2^M \\ \vdots & \vdots & \ddots & \vdots \\ \bar{y}_m^1 & \bar{y}_m^2 & \cdots & \bar{y}_m^M \end{pmatrix} \tag{2}$$

Eq. (2) is adjustable parameter matrix and $\xi(x) = [\xi^1(x), \xi^2(x),..., \xi^M(x)]^T$ is fuzzy basis function vector, in which $\xi^j(x)$ is defined as

$$\xi^j(x) = \frac{\prod_{i=1}^{n} \mu_{A_i^j}(x_i)}{\sum_{j=1}^{M} \prod_{i=1}^{n} \mu_{A_i^j}(x_i)} (j = 1,2,..., M) \tag{3}$$

From analysis of the FIS, we must give fuzzy rules of mass point clouds, and then normal of point clouds can be estimated for different kind.

3. Fuzzy Normal Estimate Method for Point Clouds of Irregular Model

The model of irregular object is complex. In this section, we will analysis the character of irregular object. Because irregular object is not symmetric and changes of surface is not pattern. It is difficult to design a perfect method for point clouds. The main idea of fuzzy estimate normal method is that k-nearest neighbors (*k*NN) of each point data is used to measure the density of point clouds and its curvature is used to determine the model thickness or contain sharp feature. According to value of the two variables, we can determine the corresponding estimation method in each unit by fuzzy inference. And then fuzzy rules are given corresponding to different characters. At last a fuzzy model is given to solve all the conditions of dental point clouds.

In all the following sections, we assume that given a collection of *n* points $P=\{p_1,p_2,\ldots,p_n\}$ that are scattered in 3D domain, where $p_i = [x_i, y_i, z_i]^T \in R^3$.

3.1. The Character of Irregular Model

As can be seen in Figure 2, we divide three classes for point clouds according to the character of irregular model. The character of part one is uniform and smooth. The character of part two is uniform but curvature changes greatly. And the character of part three is non uniform and curvature changes irregular. It is difficult to design a perfect method for mass point clouds.

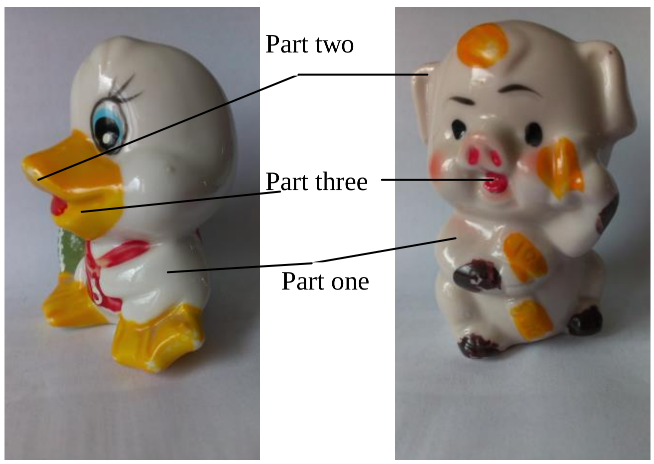


Figure 2. Classes of Irregular Model

In mass point clouds, these differences mainly reflect in *k*-nearest neighbors (*k*NN) of each point and the degree of curvature change. Once a point is selected, we compute the distances x^i_{knn} (i=1,2,…,n) from this point to its each neighbor and sort values of $|x^i_{knn}|$ into an ascending order and add $|x^i_{knn}|$ into a priority queue. Quadric is fitted using the points to spherical equation in Eq.(4). Curvature of the surface is reciprocal radius of sphere.

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = R^2 \tag{4}$$

where (x_0, y_0, z_0) is a point selected and (x,y,z) is its neighbor point. *R* is radius of sphere. Curvature is estimated as following:

$$x_{cur} = 1/\sqrt{R^2} \tag{5}$$

3.2. Description of Fuzzy Rules

The value of $[x_{knn}, x_{cur}]$ can express the density and sharp distribution of point clouds. The combinations of the two input variables with different values are inputted into fuzzy system and compared with the fuzzy rules. Which part of dental point clouds belong to can be judged by fuzzy inference engine. The fuzzy rules are expressed as follows:

R_1: if x_{knn} is small and $|x_{cur}|$ is small, then *y* belongs to part one;

R_2: if x_{knn} is small and $|x_{cur}|$ is large, then *y* belongs to part two;

R_3: if x_{knn} is large and $|x_{cur}|$ is large, then *y* belongs to part three ;

where "small" and "large" in the rules are linguistic values of fuzzy sets in subspace U_i described by their membership functions $\mu_{A_i^j}(x_i)$, *j*=1,2,…,*M*. *M* is total number of the fuzzy rules. For convenience, triangular-type membership functions for x_{knn} and x_{cur} are selected in this system in Figure 3.

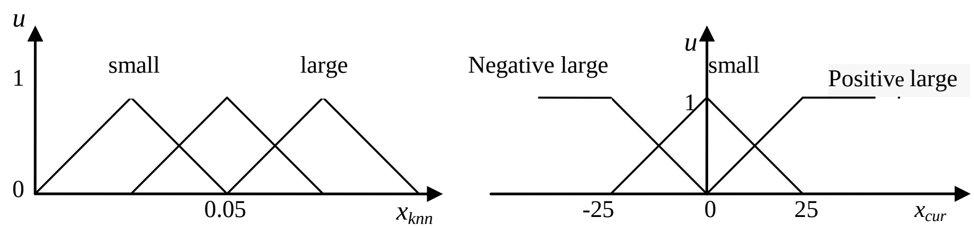


Figure 3. Triangular-type Membership Functions for x_{knn} and x_{cur}

3.3. Normal Estimation Framework

The main idea of fuzzy normal estimation is that normal is computed by PCA firstly and the checker and attach points are determined to use or not according to the value of *y(x)*. The framework of normal estimation is defined in detail.

$$N = N_{PCA} + \alpha N_{check} + \beta N_{attach} \tag{6}$$

where $[1 \quad \alpha \quad \beta]$ is the output *y(x)* of FIS. N_{PCA} is the algorithm of PCA in [1]. N_{check} is used to infer the orientation of part two and N_{attach} is suitable to part three. N_{PCA} can solve the part one enough. But part two and three need to deal with N_{check} and N_{attach} additionally.

The checker N_{check} [12] is used to check whether the position of point is on the edge of model or not. It works along the local tangential direction of two close-by points. The *k*NNs of point *pi* are projected onto its tangent plane, which determine the current unsigned normal at p_i. If the projection of p_i lies outside the convex hull of its *k*NN projections, then p_i is deemed to be at a thin feature. If the point is at thin surface, the normal of point can be oriented, but its orientation is not allowed to propagate other points.

In this way, we execute N_{check} during the normal propagation to detect and reverse orientation between close-by surface sheets. In particular, let p_i and p_j be a propagated point pair. If both $\left|\cos(\angle(n_i, p_i p_j))\right|$ and $\left|\cos(\angle(n_j, p_i p_j))\right|$ exceed a threshold ε , signifying a potential propagation along normal direction, we flip the normal orientation at point p_j. And then, the priority-driven propagation continues.

For part three, we firstly adopt Delaunay tetrahedralization to construct a surface of molar using the mass point clouds. It is difficult for this irregular surface to infer its inside/outside. Attach points N_{attach} [12] which are positioned slightly inside of the expected surface can help to recognize the orientation.

Attach points are generated by projecting from the given points P along normal N_{PCA}, positioned at β away from the given points. The set of attach points $\hat{P} = \{p_{n+1}, p_{n+2}, ..., p_{2n}\}$ are generated as follows:

$$p_{n+i} = p_i - \lambda n_i \tag{7}$$

where $N = \{n_1, n_2, ..., n_n\}$ is unoriented normal of each point computed by N_{PCA}. The value of λ is a small positive number. The set $\hat{P}$ are positioned slightly inside of the expected surface by setting appropriate value of λ. Because we contact the tetrahedrals both P and $\hat{P}$, in which each tetrahedral must include P and $\hat{P}$. We can obtain an appropriate orientation by extracting triangles with tetrahedral consist of both P and $\hat{P}$.

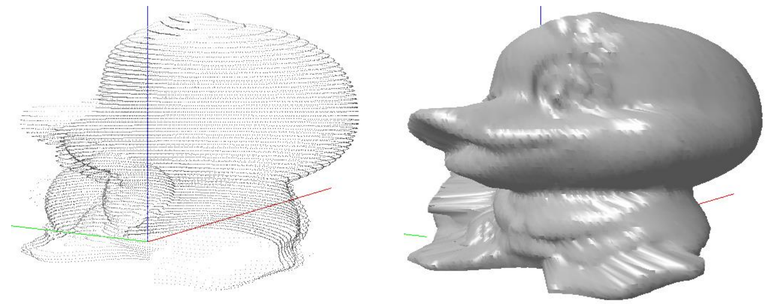
4. Experimental Results

The purpose of the experiments is to evaluate the performance of this proposed method by normal of irregular models. Experimental results are implemented on a PC equipped with an Intel Core 2 processor at 2.93GHz and 2GB main memory. And point clouds are from 3D laser scanner, PICZA LPX-250. In the experiment, the selected models of toy duck and pig do not include normal vector of each point.

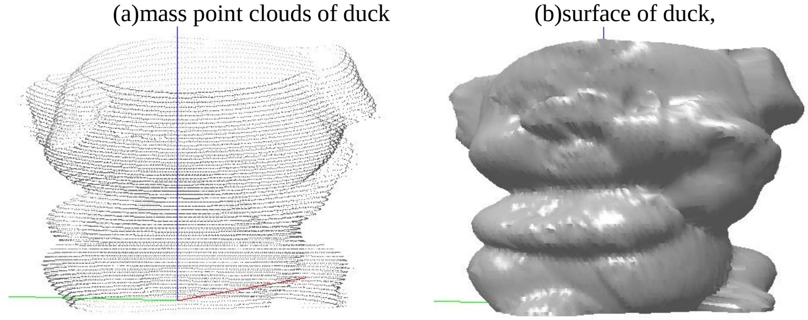
We can estimate normal by the given algorithm and then fit the surface by implicit function. The point clouds are divided into sub-domains by octree structure firstly. Then the *k*NN of each point is computed and connected the points in sub-domain, and therefore we can calculate the curvature. Here *k*=5 and the value of curvature is regarded as small if $x_{cur} \in [-25, 25]$ and as large if x_{cur} beyond the scope through this experiment. Next, input x_{knn}, x_{cur} into FIS and infer to the value of *y*. The corresponding normal estimation method is determined according to the value of *y*. At last, the models of denture are reconstructed using implicit function in Figure 4. At the same time, we compare with PCA and WLOP algorithm in Table 1. WLOP spent double times than fuzzy normal estimation computing the pig and triple times than this computing duck. Normal estimated by PCA is inaccurate while normal computed by WLOP and this algorithm is satisfied with 3D reconstruction. Because each of mass point clouds must be computed by WLOP, it takes much time. And this algorithm can divide mass point clouds to different classes and give corresponding algorithm to computer normal. In this way, much normal of point clouds in smooth area is estimated by PCA and few normal of point clouds in sharp area is done by the checker N_{check} or attach points N_{attach}. And the precision is protected and much time is saved.

Table 1. Comparison of Running Time (*s*) between the WLOP and this Algorithm on the two kinds of Mass Point Clouds

Model	PCA	WLOP	This algorithm
Toy duck	15.87	29.41	10.34
Toy pig	13.62	25.68	9.03



(a)mass point clouds of duck (b)surface of duck,



(c)mass point clouds of pig (d)surface of pig

Figure 4. The Models of Toy Duck and Pig

5. Conclusions

The irregular model is complex object that some part is smooth and some parts are irregular including sharp features. In order to simplify the algorithm, we present a fuzzy normal estimation method. The *k*NN of mass point clouds and curvature in the sub-domain are input into FIS, and the output of FIS can determine which part of tooth point clouds belong to. In this way, the point clouds in the smooth area are estimated normal by PCA directly and ones in other regions of thin or sharp area are estimated by checker and attach points. This method is simpler than those complex methods used on the whole point clouds directly. Further work concerns a universal fuzzy normal estimation algorithm applying to other complex model.

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