

Performance Analysis of Two-Way AF Cooperative Networks with the Nth Relay Selection

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Abstract

In this letter, we propose a novel two-way amplify-and-forward (AF) relay selection scheme over independent flat Nakagami- m fading channels. In the proposed schemes, two users first sequentially broadcast their respective information to all the relays. Then, the Nth best-relay perform AF protocol on the received signals sent from the two users and forward them to both users. The Nth best-relay is selected to optimize the end-to-end performance of the system transmission. The performance of the proposed system is quantified by deriving the lower bound of the overall outage probability and average symbol error rate (SER). The effectiveness of our proposed selection scheme and analytical results is verified by simulations.

Keywords: Two-way relaying network, amplify-and-forward (AF), outage probability, symbol error rate.

1. Introduction

Two-way relaying system has attracted much research interest because of its potential in achieving higher spectral efficiency and throughput for wireless networks [1]. Based on different processing at the relay, such as decode-and-forward and amplify-and-forward, achievable rate regions for various two-way relaying systems have been studied in [2-3]. In [4], the authors investigated the performance of two-way amplify-and-forward relaying networks over independently but not necessarily identically distributed Nakagami- m fading channels in terms of outage probability, average SER, and average sum-rate. Adaptive two-way relaying, which the relay R adaptively selects AF or DF depending on the decodability of the two bi-directional data streams received at R, has been introduced in [5]. Because of synchronization and a lot of messages exchanging among different relays, a proper relay selection scheme will play a key role in the multiple relay system performance. In [6], the authors derived a simple two-way amplify-and-forward relay selection criterion for the general case over Nakagami- m fading channels. In [7], the authors proposed the max-min sum rate selection algorithm for AF bidirectional network based on the outage probability. Two-way relay selection, which is introduced for differential modulation systems to improve system performance, has been proposed in [8].

One of the key features in the two-way relaying networks is that the two-way protocol should not only consider the downlink (broadcasting) but also the uplink (multiple-access) data flows in the same time, which can improve the overall performance of the two directional transmission. However, most of the recent research

focuses on the “best” single relay selection methods. In practice, the best relay may not always be selected, thus the study of the N th best relay selection will be very necessary.

In [9], the outage performance of the proposed relay selection scheme is analyzed under the outdated channel state information (CSI), and a tight closed-form lower bound and asymptotic value of the outage probability over Rayleigh fading channels are obtained. Analysis of amplify-and-forward with the N th best-relay cooperative diversity over Rayleigh fading channels is presented in [10]. The authors consider the adaptive DF and AF cooperative diversity systems with the N th best-relay selection scheme in [11]. In [12], authors assume that the relay employs the amplify-and-forward protocol, propose the N th worst relay selection scheme and analyze the system performance of the system under two different fading models: lognormal shadowing channels and generalized-K fading channels. In [13], the asymptotic symbol error rate (SER) and diversity gain of AF cooperative communications with the N th best-relay scheme over independent and nonidentical Nakagami- m fading channels are derived. The authors investigate the performance of N th best relay selection networks with secrecy constraints where several eavesdroppers try to overhear the source message in [14].

Motivated by all of the above, this letter presents a novel two-way relay selection scheme and we investigate the performance of the proposed system over independent flat Nakagami- m fading channels. Firstly, the lower bound of the overall outage probability for the proposed system is investigated. And then, the average symbol error rate (SER) are determined. Finally, Simulation results verify our analysis.

2. System Model

Consider a two-way amplify-and-forward (AF) relaying system, where two sources S_1 and S_2 exchange information with the help of K cooperating AF relay nodes k , $k \in \{1, \dots, K\}$, over independent flat Nakagami- m fading channels. We assume that all terminals are single-antenna devices and operate in a half-duplex mode, and there is no direct path between S_1 and S_2 . We also assume here the additive white Gaussian noise (AWGN) at all nodes is independent and identically distributed (i.i.d.) $\mathcal{CN}(0, N_0)$.

In the first time slot, S_1 and S_2 transmit their signals x_1 and x_2 to all the relays simultaneously. So the received signal at relay k can be expressed as

$$y_k = \sqrt{P_1}h_{1k}x_1 + \sqrt{P_2}h_{2k}x_2 + \eta_k, \quad (1)$$

where P_1 and P_2 are the transmitting power of S_1 and S_2 . h_{1k} and h_{2k} stand for the channel gains of $S_1 \rightarrow k$ and $S_2 \rightarrow k$, respectively.

In the next two time slots, the N th relays R_k is selected for amplifying its received signal and forwarding it. Thus, in the second time slot, Considering the channel is block time variation, the received signal at relay k is amplified with the fixed-gain [17], $G = \sqrt{1/CN_0}$, whereby C is a constant and retransmitted to S_1 and S_2 , which can be expressed by

$$y_{s1} = G\sqrt{P_1}g_{1k}^\dagger h_{1k}x_1 + \sqrt{P_2}g_{2k}^\dagger h_{2k}x_2 + g_{1k}^\dagger \eta_k + \eta_1, \quad (2)$$

$$y_{s2} = G\sqrt{P_1}g_{2k}^\dagger h_{1k}x_1 + \sqrt{P_2}g_{2k}^\dagger h_{2k}x_2 + g_{2k}^\dagger \eta_k + \eta_2, \quad (3)$$

respectively. where g_{1k}^\dagger , g_{2k}^\dagger are the conjugates of g_{1k} and g_{2k} .

After the self-interference parts are subtracted [1], the received SNRs of two-links $S_2 \rightarrow k \rightarrow S_1$ and $S_1 \rightarrow k \rightarrow S_2$ are given by

$$\gamma_{1k} = \frac{P_2 \gamma_{g_{1k}} \gamma_{h_{2k}}}{\gamma_{g_{1k}} + C}, \quad (4)$$

$$\gamma_{2k} = \frac{P_1 \gamma_{g_{2k}} \gamma_{h_{1k}}}{\gamma_{g_{2k}} + C}, \quad (5)$$

respectively, where $\gamma_{g_{sk}} = |g_{sk}|^2 / N_0$, $\gamma_{h_{sk}} = |h_{sk}|^2 / N_0$, $s = 1, 2$ and g_{1k} and g_{2k} stand for the channel gains of $k \rightarrow S_1$ and $k \rightarrow S_2$, respectively.

Since the overall performance of the considered system is governed by the performance of the weakest source, let $\gamma_k = \min(\gamma_{1k}, \gamma_{2k})$. Thus, the relay selection criterion can be expressed as

$$(k^*) = \arg N_{th} \max_{k \in \{1, \dots, N\}} \{\gamma_k\}, \quad (6)$$

Where $N_{th} \max_{k \in C\{\cdot\}}$ denotes to select the N th maximum value from the set C .

3. Performance Analysis

In this section, performance of the above-mentioned relay selection schemes under Nakagami-m is analyzed. First of all, closed-form lower bound for the overall outage probability is derived. Then, useful insights into practical systems implementation are obtained by quantifying SER.

3.1. Outage Probability

With the above selection scheduling, the overall outage probability is given by

$$P_{out} = P_r [N_{th} \max_{k \in \{1, \dots, K\}} \{\gamma_k\} \leq \gamma_0]. \quad (7)$$

Since that is hard to obtain the exact result of (7), we derive the tight lower bound of the outage probability. First, from (2) and (3), we can obtain

$$\gamma_{1k} = \frac{G^2 |g_{1k}|^2 |h_{2k}|^2 \gamma_2}{G^2 |g_{1k}|^2 + 1} < \gamma_2 |h_{2k}|^2, \quad (8)$$

$$\gamma_{2k} = \frac{G^2 |g_{2k}|^2 |h_{1k}|^2 \gamma_1}{G^2 |g_{2k}|^2 + 1} < \gamma_1 |h_{1k}|^2. \quad (9)$$

Then, the overall outage probability is given by

$$\begin{aligned} P_{out} &> P_{out}^{lb} \\ &= F_{r_k^*}(\gamma_0) \\ &= \Pr [N_{th} \max_{k \in \{1, \dots, K\}} \min \{\gamma_{1k}, \gamma_{2k}\} \leq \gamma_0] \end{aligned} \quad (10)$$

where $F_{\gamma_k}(x) = \Pr[\min(\gamma_{1k}, \gamma_{2k}) \leq x]$ is the CDF of γ_k .

Under Nakagami-m fading, $|h_{1k}|^2$ and $|h_{2k}|^2$ for any $k \in \{1, \dots, N\}$ are independent but not necessarily identically distributed with shape parameter m_{1k} , m_{2k} and scale parameter $\Omega_{1k} = E[|h_{1k}|^2]$, $\Omega_{2k} = E[|h_{2k}|^2]$ denotes the expectation operator. Hence, the cumulative distribution function (CDF) of the fading amplitude $|h_{sk}|^2$, $s = 1, 2$, can be given by

$$F_{h_{sk}}(x_s) = 1 - \frac{\Gamma(m_{sk}, \frac{m_{sk}}{\Omega_{sk}} x_s)}{\Gamma(m_{sk})}, \quad (11)$$

where $\Gamma(\cdot)$ is the gamma function, $\Gamma(\cdot, \cdot)$ is the incomplete gamma function [19].

The CDF for γ_i can be expressed by

$$\begin{aligned}
 F_{\gamma_k}(x) &= \Pr[\min\{\gamma_{1k}, \gamma_{2k}\} \leq x] \\
 &= F_{h_{1k}}(x) + F_{h_{2k}}(x) - F_{h_{1k}}(x)F_{h_{2k}}(x) \\
 &= \frac{\Gamma(m_{1k}, \frac{m_{1k}}{\Omega_{1k}}x)}{\Gamma(m_{1k})} + \frac{\Gamma(m_{2k}, \frac{m_{2k}}{\Omega_{2k}}x)}{\Gamma(m_{2k})} - \frac{\Gamma(m_{1k}, \frac{m_{1k}}{\Omega_{1k}}x)}{\Gamma(m_{1k})} \frac{\Gamma(m_{2k}, \frac{m_{2k}}{\Omega_{2k}}x)}{\Gamma(m_{2k})}
 \end{aligned} \tag{12}$$

By substituting (12) into (10), the desired result of the lower bound of the overall outage probability P_{out}^{lb} can be derived as in (13).

$$\begin{aligned}
 P_{out}^{lb} &= \sum_{k=1}^K \binom{K}{k} \left[\frac{\Gamma\left(m_{1k}, \frac{m_{1k}}{\Omega_{1k}}\gamma_0\right)}{\Gamma(m_{1k})} + \frac{\Gamma\left(m_{2k}, \frac{m_{2k}}{\Omega_{2k}}\gamma_0\right)}{\Gamma(m_{2k})} - \frac{\Gamma\left(m_{1k}, \frac{m_{1k}}{\Omega_{1k}}\gamma_0\right)}{\Gamma(m_{1k})} \frac{\Gamma\left(m_{2k}, \frac{m_{2k}}{\Omega_{2k}}\gamma_0\right)}{\Gamma(m_{2k})} \right]^k \\
 &\quad \left(1 - \frac{\Gamma\left(m_{1k}, \frac{m_{1k}}{\Omega_{1k}}\gamma_0\right)}{\Gamma(m_{1k})} \right)^{K-k} \left(1 - \frac{\Gamma\left(m_{2k}, \frac{m_{2k}}{\Omega_{2k}}\gamma_0\right)}{\Gamma(m_{2k})} \right)^{K-k} \\
 &= \sum_{k=N}^K \binom{K}{k} \sum_{p=1}^k \binom{k}{p} (-1)^p \left(1 - \frac{\Gamma\left(m_{1k}, \frac{m_{1k}}{\Omega_{1k}}\gamma_0\right)}{\Gamma(m_{1k})} \right)^{K-k+p} \left(1 - \frac{\Gamma\left(m_{2k}, \frac{m_{2k}}{\Omega_{2k}}\gamma_0\right)}{\Gamma(m_{2k})} \right)^{K-k+p}
 \end{aligned} \tag{13}$$

3.2. SER analysis

We invoke the cumulative distribution function (CDF) approach described in [18] and derive the exact SER in terms of the CDF of γ_{k^*} , $F_{\gamma_{k^*}}(x)$, as

$$P_e = \frac{a}{2} \sqrt{\frac{b}{\pi}} \int_0^\infty \gamma^{-\frac{1}{2}} \exp(-b\gamma) F_{\gamma_{k^*}}(\gamma) d\gamma \tag{14}$$

This approach embraces all general modulation schemes that have an SER expression of the form $P_e = E\{Q(\sqrt{c\gamma})\}$, where a and b are modulation specific constants. The modulation formats to which (14) apply include binary phase-shift keying (BPSK) ($a = 1, b = 1$), quadrature phase-shift keying (QPSK) ($a = 1, b = 0.5$), and M-ary pulse amplitude modulation (M-PAM) ($a = 2(M-1)/M, b = 3/(M^2-1)$).

Then, Inserting (13) into (14), we evaluate the BER performance evaluate the BER of binary phase shift keying (BPSK), the SER can be expressed as

$$P_e = \frac{1}{2} \sqrt{\frac{1}{\pi}} \int_0^\infty \sum_{k=N}^K \binom{K}{k} \sum_{p=1}^k \binom{k}{p} (-1)^p \gamma^{-\frac{1}{2}} \exp(-b\gamma)$$

$$\left(1 - \frac{\Gamma(m_{1k}, \frac{m_{1k}}{\Omega_{1k}} \gamma_0)}{\Gamma(m_{1k})}\right)^{K-k+p} \left(1 - \frac{\Gamma(m_{2k}, \frac{m_{2k}}{\Omega_{2k}} \gamma_0)}{\Gamma(m_{2k})}\right)^{K-k+p} d\gamma \quad (15)$$

Unfortunately, there is no closed-form expression for the integral in (15).

Under the assumption of linear modulation and AWGN, the SER conditioned on the instantaneous received SNR is approximately $Q(\sqrt{c\gamma})$, where c is a constant determined by the modulation format (e.g. $c=2$ for phase shift keying (PSK)), and $Q(x) = \frac{1}{2\pi} \int_x^\infty \exp(-t^2/2) dt$. The average SER is therefore

$$P_e = E\{Q(\sqrt{c\gamma})\}, \quad (16)$$

where $E(\cdot)$ denotes the expectation operator. This integral can be computed by using the Gauss-Hermite integration as (17), where M denotes the number of points for the Gauss-Hermite integration, x_n and $\omega(x_n)$ stands for the abscissas and weights of Gauss-Hermite integration, respectively.

$$P_e = \frac{1}{2\sqrt{\pi}} \sum_{k=N}^K \binom{K}{k} \sum_{p=1}^k \binom{k}{p} (-1)^p \sum_n^M \omega(x_n) \left(1 - \frac{\Gamma\left(m_{1k}, \frac{m_{1k}}{\Omega_{1k}} \gamma_0\right)}{\Gamma(m_{1k})}\right)^{K-k+p} \left(1 - \frac{\Gamma\left(m_{2k}, \frac{m_{2k}}{\Omega_{2k}} \gamma_0\right)}{\Gamma(m_{2k})}\right)^{K-k+p} \quad (17)$$

4. Numerical Results

In this section, we present Monte-Carlo simulations to confirm the derived analytical results. We assume that the average SNR in links from two sources to relay is equal, i.e. $P_1 = P_2$. The distance between two sources S_1 and S_2 is normalized to 1, the normalized distance from S_1 to relay k ($k = 1, \dots, K$) is d_{1k} , and the path loss exponent is 3. Thus, $\Omega_{1k} = d_{1k}^{-3}$, $\Omega_{2k} = (1 - d_{1k})^{-3}$. Without loss of generality, we set $d_{1k} = 0.5$ for all the relays. Furthermore, we assume $m_{1k} = 0.5$, $m_{2k} = 0.5$, $k = 1, \dots, K$, and the targeted transmission rate $R = 1$ bit/s/Hz.

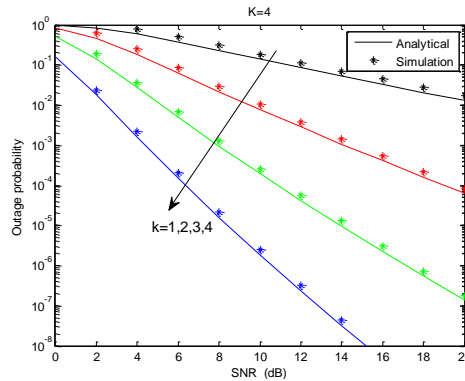


Figure 1. Outage Probability AGAINST SNR for $K = 4$

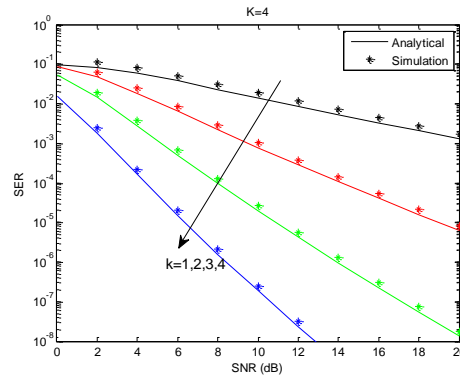


Figure 2. Average Symbol Error Rate against SNR for $K = 4$

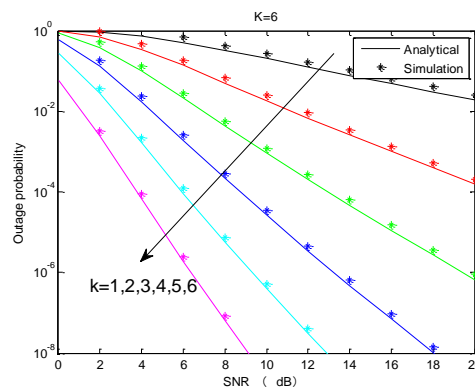


Figure 3. Outage Probability against SNR for $K = 6$

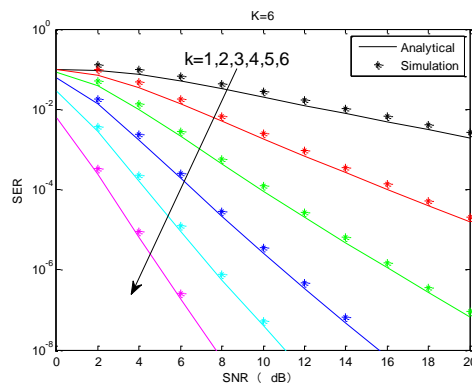


Figure 4. Average Symbol Error Rate against SNR for $K = 6$

Figure 1 and Figure 3 shows the theoretical results for the overall outage probability, and actual simulation of the proposed scheme for $K = 4$ and $K = 6$, respectively. From Figure 1 and Figure 3, it is observed that analytical results exactly coincide with Monte Carlo simulations and therefore verify the accuracy of our analysis. It can be seen that the theoretical analysis is validated in all SNR regimes. As expected, we can see that the number of relay k also affects the difference of the overall outage probability between the proposed scheme and the “best” single relay selection protocol, that is, as K gets bigger, the difference becomes more obvious. It can also be seen that the relay gain given by (10) performs a tight lower bound for (6) for the system outage performance.

Figure 2 and Figure 4 shows the theoretical results for the average symbol error rate against SNR, and actual simulation of the proposed scheme for $K = 4$ and $K = 6$,

respectively. From Figure 2 and Figure 4, it is observed that analytical results also exactly coincide with Monte Carlo simulations and therefore verify the accuracy of our analysis. As expected, we can see that the number of relay k also affects the difference of SER, that is, as K gets bigger, the difference becomes more obvious. The selection scheme can greatly improve the overall system SER and diversity performance.

5. Conclusion

In this letter, we present the amplify-and-forward relay system with the N th best-relay cooperative diversity over independent flat Nakagami- m fading channels. Closed-form expressions for the probability density functions of the output SNR is obtained. Furthermore, closed-form expressions for the outage probability and SER are determined. Results show that the diversity order increases linearly with the number of relays and decreases linearly with the order of the relay. And then, numerical results verify that the proposed relay selection scheme has better outage performance than that of the single relay selection protocol.

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