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Rademacher Functions and Their Applications to 3GPP Mobile Communication Systems

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Abstract

In this paper we investigate the applications of Rademacher functions in 3GPP mobile communication systems which are currently most widely being deployed in the world. Although the equations are simple enough to be expressed as signs of sine functions, it is very hard to find the limit of the usefulness in modern communication systems such as 3GPP and 3GPP2 society. In this paper we narrow the scope of the applications to 3GPP mobile communication systems for the better understanding, where the functions are used for OVSF code, TFCI and CQI codings.

Keywords*: Rademacher functions, OVSF code, TFCI coding, CQI coding*

1. Introduction

In 3GPP W-CDMA (Wideband Code Division Multiple Access) system, DPCCH (Dedicated Physical Control Channel) uses the TFCI (Transport Format Combinaion Indicator) coding to tramsmit TFI (Transport Format Indicator) information with error protection capability [1]-[3]. The transport channel is accompanied by TFI and the physical layer combines the TFI information from different transport channels to TFCI which is transmitted on DPCCH. For the downlink if one of DCH (Dedicated Channel) is associated with a DSCH (Downlink Shared Channel), (16,5) TFCI coding is used in split mode [3]-[4]. Considering the simplity of encoding and decoding of the TFCI coding with the optimum BER performance, 3GPP mobile communication systems decided to adopt the 1st order and 2nd order RM codes as the coding scheme of TFCI. The basis vectors of the RM codes are specially desinged to have maximized minimum Hamming distance. The basis sequences of the TFCI coding is based on the Rademacher functions.

The uplink feedback information corresponding to the HS-DSCH such as HARQ feedback and CQI (Channel Quality Indicator) is carried on the HS-DPCCH (High Speed Dedicated Physical Control Channel). HARQ feedback is ACK/NACK for the received packet of HS-DSCH and the CQI informs the base station scheduler of the date rate that the terminal is able to receive at a given time. The HS-DSCH is transport channel which is mapped on HS-PDSCH (High Speed Physical Downlink Shared Channel) and carries the user data with HSDPA (High Speed Downlink Packet Access) [5]. The CQI coding of HS-DPCCH (High Speed Dedicated Physical Control Channel) are effectively equivalent to those of (16,5) TFCI (Transport Format Combinaion Indicator) coding in W-CDMA system.

The OVSF (Orthogonal Variable Spreading Factor) codes of W-CDMA system which are used for channel allocation of traffic and control channels can be generated from Rademacher functions. It preserves orthogonality between these control and data channels and used to seperate data and control channels from the same UE (User Equipment) [4-5]. Thus, using the OVSF codes we can adjust the spreading facotr to be variable and maintain the orthogonality between different spreading codes of different lengths [6-7].

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Investing the Rademacher function, we can easily find the similarity and commonality between OVSF and TFCI codings of W-CDMA system and CQI codings of HSDPA system in 3GPP FDD mode and TDD mode specifications.

2. Rademacher Functions

The Rademacher functions which have many applications in 3GPP mobile communication shystem are expressed as [8]:

$$\mathrm{R}_n(t) = \mathrm{sgn}(\sin(2^n \pi t)) \tag{1}$$

, where $t \in (0,1)$, $n = 1,2,\dots,\log_2 N = K$, and

$$\mathrm{sgn}(x) = \begin{cases} +1, x > 0 \\ 0, x = 0 \\ -1, x < 0 \end{cases} \tag{2}$$

and $\mathrm{R}_0(t) = +1$. Table 1 and 2 show the Rademacher functions expressed as zeros or ones after mapping $0 \leftrightarrow +1$, $1 \leftrightarrow -1$ with $N = 2^K$, where $K = 1,2,3,4,5,6$.

Table 1. Rademacher Functions with K=1,2,3,4,5

N	K	$\mathrm{R}_n(t)$
2	1	$\mathrm{R}_1(t) = (01)$
4	2	$\mathrm{R}_1(t) = (0011)$ $\mathrm{R}_2(t) = (0101)$
8	3	$\mathrm{R}_1(t) = (00001111)$ $\mathrm{R}_2(t) = (00110011)$ $\mathrm{R}_3(t) = (01010101)$
16	4	$\mathrm{R}_1(t) = (0000000011111111)$ $\mathrm{R}_2(t) = (0000111100001111)$ $\mathrm{R}_3(t) = (0011001100110011)$ $\mathrm{R}_4(t) = (0101010101010101)$
32	5	$\mathrm{R}_1(t) = (00000000000000001111111111111111)$ $\mathrm{R}_2(t) = (00000000111111110000000011111111)$ $\mathrm{R}_3(t) = (00001111000011110000111100001111)$ $\mathrm{R}_4(t) = (00110011001100110011001100110011)$ $\mathrm{R}_5(t) = (01010101010101010101010101010101)$

Table 2. Rademacher functions with K=6

$N = 64, K = 6$
$R_1(t) = (0000000000000000000000000000000011111111111111111111111111111111)$
$R_2(t) = (0000000000000000111111111111111100000000000000001111111111111111)$
$R_3(t) = (0000000011111111000000001111111100000000111111110000000011111111)$
$R_4(t) = (0000111100001111000011110000111100001111000011110000111100001111)$
$R_5(t) = (0011001100110011001100110011001100110011001100110011001100110011)$
$R_6(t) = (0101010101010101010101010101010101010101010101010101010101010101)$

3. OVSF and TFCI Codes using Rademacher Functions

3.1. OVSF Coding

Figure 1 shows the code tree to generate OVSF codes in W-CDMA system [6-7]. Figure 2 shows the OVFS code generator using Rademacher functions as the basis sequences. This implies that the OVSF codes are linear combination of Rademacher functions. The OVSF code C_i^N is generated by setting $K = \log_2 N$ binary bits such as $i = (d_0, d_1, \dots, d_{K-1})_2$ which is the unsigned integer such that $i = \sum_{z=0}^{K-1} d_z 2^z$.

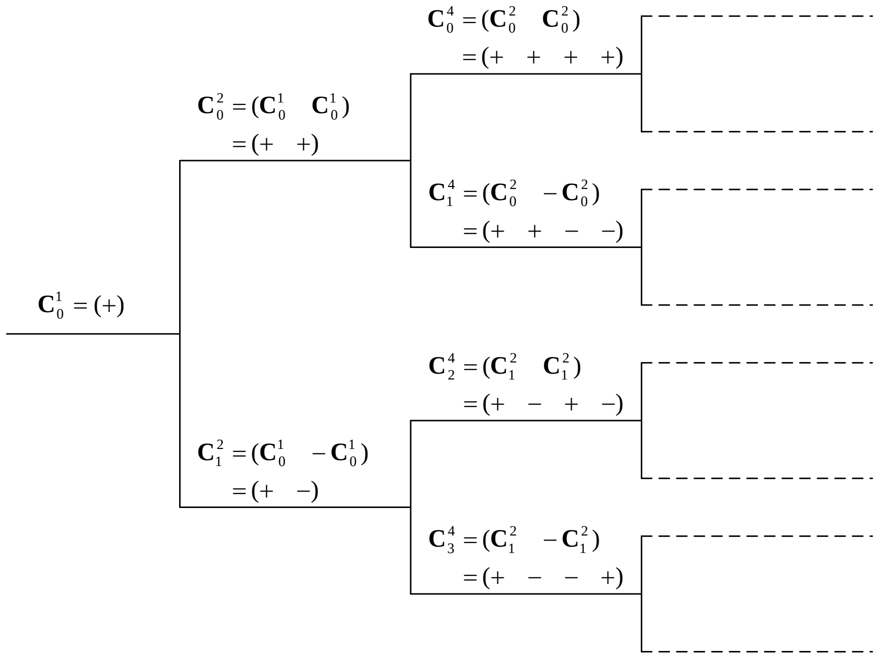

Figure 1. Code Tree to Generate OVSF Codes

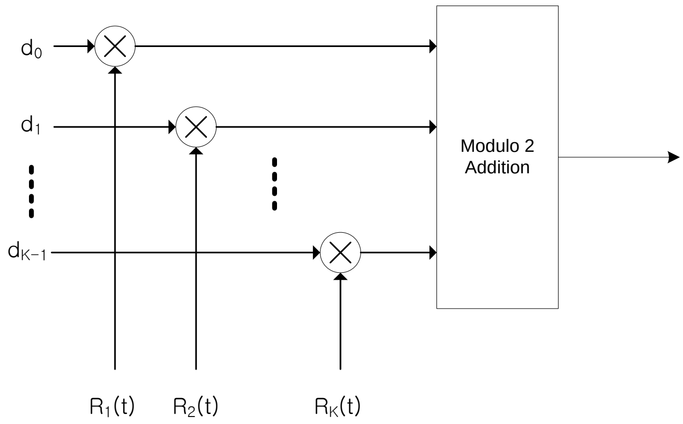

Figure 2. Generation of OVSF Codes using Rademacher Functions

3.2. TFCI Coding

3.2.1 (32, 10) TFCI Coding

In 3GPP FDD mode also called W-CDMA system, DPCCH uses the TFCI coding to tramsmit TFI information. Figure3 shows (32, 10) TFCI encoder to encode TFCI bits expressed by $a_0, a_1, \dots, a_{n-1}$, $1 \le n \le 10$, which are linearly combined with the basis sequences of Table 3 [1]-[3]. The number of TFCI bits is determined by upper layer. If the number of input TFCI bits is 1 to 4, the last symbol of TFCI code word of length 16 is always all "0". Thus in this case, puncturing the last symbol does not decrease the minimum Hamming distance. When the number of input TFCI bits is 5, puncturing the last 2 symbols decreases the minimum Hamming distance by 1 [9]-[12]. Figure4 illustrates how the basis sequences of (32, 10) TFCI code are made from the Rademacher functions. First the 0 and 16th bits of Rademacher functions are moved to the last two bits position. Second the Rademacher functions $\{R_1(t), R_2(t), R_3(t), R_4(t), R_5(t)\}$ are mapped to $\{M_4(t), M_3(t), M_2(t), M_1(t), M_0(t)\}$using barrel shifting [9]-[10]. The basis sequence $M_5(t)$ is all "1" pattern for bi-orthogonal coding. After linear combination of $\{M_0(t), M_1(t), M_2(t), M_3(t), M_4(t)\}$, we obtain the basis sequences $\{M_6(t), M_7(t), M_8(t), M_9(t)\}$ for 2nd order Reed Muller code. The (32,10) TFCI coding is also used for 3GPP TDD mode which is called TD-CDMA system.

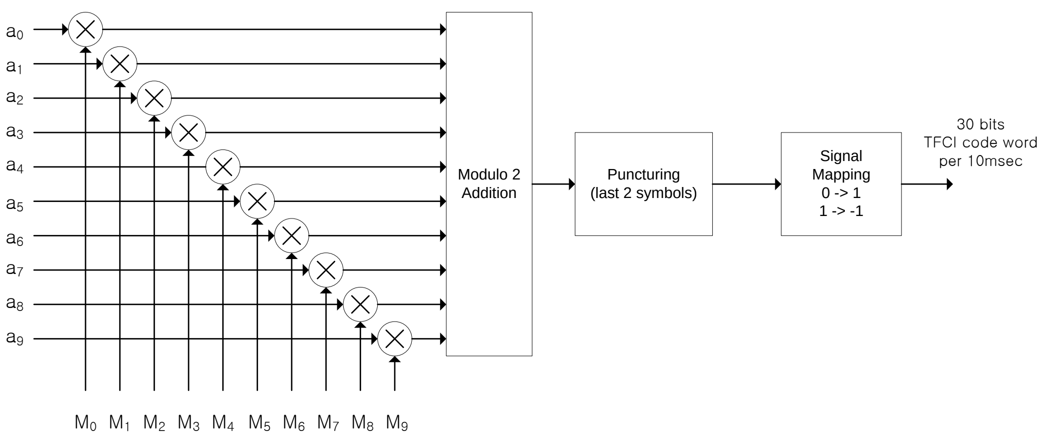

Figure 3. (32, 1 0) TFCI Encoder

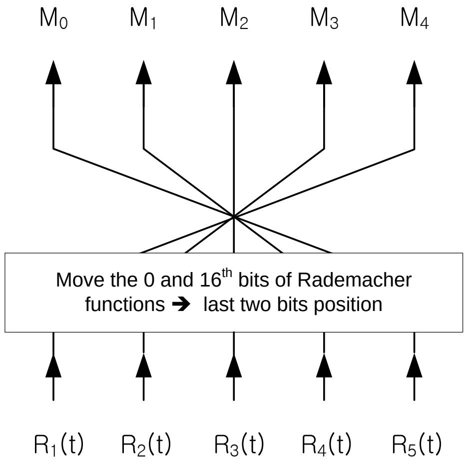

Figure 4. Barrel shift of Rademacher Functions for (32, 10) TFCI Code

Table 3. Basis Sequences of (32, 10) TFCI code

M_0	M_1	M_2	M_3	M_4	M_5	M_6	M_7	M_8	M_9
1	0	0	0	0	1	0	0	0	0
0	1	0	0	0	1	1	0	0	0
1	1	0	0	0	1	0	0	0	1
0	0	1	0	0	1	1	0	1	1
1	0	1	0	0	1	0	0	0	1
0	1	1	0	0	1	0	0	1	0
1	1	1	0	0	1	0	1	0	0
0	0	0	1	0	1	0	1	1	0
1	0	0	1	0	1	1	1	1	0
0	1	0	1	0	1	1	0	1	1
1	1	0	1	0	1	0	0	1	1
0	0	1	1	0	1	0	1	1	0
1	0	1	1	0	1	0	1	0	1
0	1	1	1	0	1	1	0	0	1
1	1	1	1	0	1	1	1	1	1
1	0	0	0	1	1	1	1	0	0
0	1	0	0	1	1	1	1	0	1
1	1	0	0	1	1	1	0	1	0
0	0	1	0	1	1	0	1	1	1
1	0	1	0	1	1	0	1	0	1
0	1	1	0	1	1	0	0	1	1
1	1	1	0	1	1	0	1	1	1
0	0	0	1	1	1	0	1	0	0
1	0	0	1	1	1	1	1	0	1
0	1	0	1	1	1	1	0	1	0
1	1	0	1	1	1	1	0	0	1
0	0	1	1	1	1	0	0	1	0
1	0	1	1	1	1	1	1	0	0
0	1	1	1	1	1	1	1	1	0
1	1	1	1	1	1	1	1	1	1
0	0	0	0	0	1	0	0	0	0
0	0	0	0	1	1	1	0	0	0

3.2.2 (16,5) TFCI coding

Figure 5 shows (16, 5) TFCI encoder to encode TFCI bits expressed by $a_0, a_1, \ldots, a_{n-1}$, $1 \leq n \leq 5$, which are linearly combined with the basis sequences of Table 4. The number of TFCI bits is determined by upper layer. If the number of input TFCI bits is 1 to 4, the last symbol of TFCI code word of length 16 is always all "0". Thus in this case, puncturing the last symbol does not decrease the minimum Hamming distance [9]-[12]. Figure 6 illustrates how the basis sequences of (16,5) TFCI code are constructed from the Rademacher functions with $K = 4$ of Table 1. First the 0th bits of Rademacher functions are moved to the last positions of the functions. Second the functions $\{R_1(t), R_2(t), R_3(t), R_4(t)\}$ are mapped to sequences $\{M_3(t), M_2(t), M_1(t), M_0(t)\}$, respectively [9]-[10]. The basis sequence $M_4(t)$ is all "1" for bi-orthogonal coding.

When we encode the short TFCI in 3GPP TDD, if the number of TFCI bits is 1 or 2, then repetition will be used for TFCI encoding and if the unber of TFCI bits is in the range of 3 to 5, the TFCI bits are encoded using (16,5) TFCI coding of Figure 5 with basis sequences of Table 4 [2].

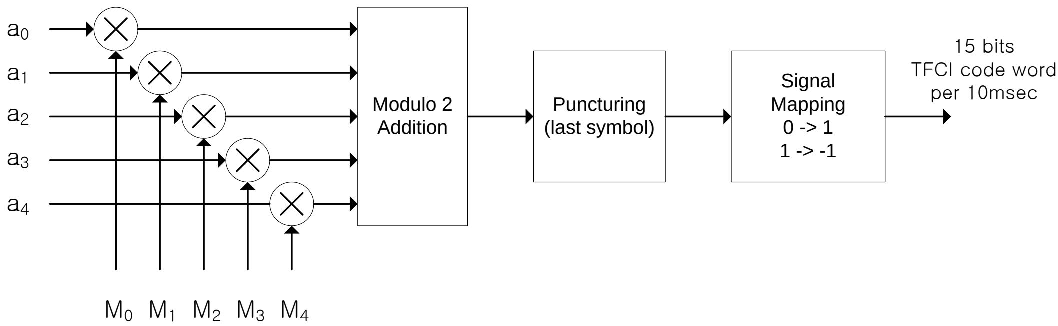

Figure 5. (16, 5) TFCI encoder

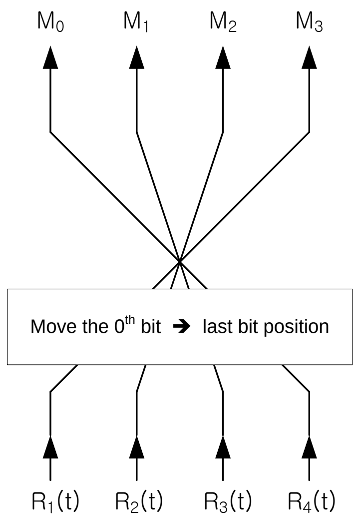

Figure 6. Generation of Basis Sequences for (16, 5) TFCI Code

Table 4. Basis Sequences of (16, 5) TFCI Code

M_0	M_1	M_2	M_3	M_4
1	0	0	0	1
0	1	0	0	1
1	1	0	0	1
0	0	1	0	1
1	0	1	0	1
0	1	1	0	1
1	1	1	0	1
0	0	0	1	1
1	0	0	1	1
0	1	0	1	1
1	1	0	1	1
0	0	1	1	1
1	0	1	1	1
0	1	1	1	1
1	1	1	1	1
0	0	0	0	1

3.2.3 (48, 10) TFCI Coding

TFCI encoding depends on the number of TFCI bits and the modulation in use [1-3]. When 2Mcps service is transmitted, 8PSK mdoulation is applied in 1.28Mcps TDD option. The (48,10) TFCI coding is used when the nuber of TFCI bits is in the rage of 6 to 10.

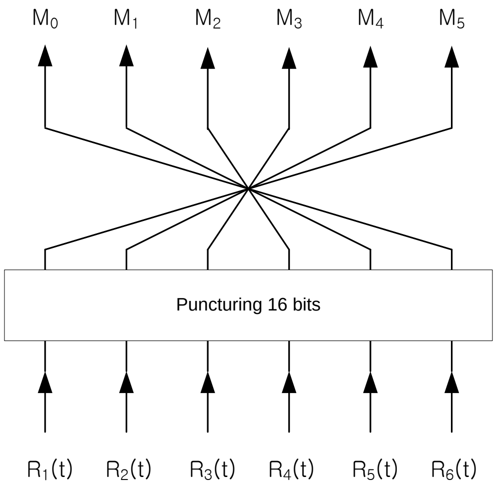

Figure 7. Generation of Basis Sequences for (48, 10) TFCI Code

Figure 7 shows how the basis sequences of (48,10) TFCI code in TDD mode are derived from the Rademacher functions with $K = 6$ of Table 2. First puncture the 16 bits in the positions of {0, 4, 8, 13, 16, 20, 27, 31, 34, 38, 41, 44, 50, 54, 57, 61st} bits of Rademacher functions. Second the Rademacher functions $\{R_1(t), R_2(t), R_3(t), R_4(t), R_5(t), R_6(t)\}$ are mapped to $\{M_5(t), M_4(t), M_3(t), M_2(t), M_1(t), M_0(t)\}$, respectively. The basis sequence $M_6(t)$ is all "1" pattern for bi-orthogonal coding. Basis sequences $\{M_7(t), M_8(t), M_9(t)\}$ are obtained by linear combination of basis sequences $\{M_0(t), M_1(t), M_2(t), M_3(t), M_4(t), M_5(t)\}$. Figure 8 is the (48,10) TFCI encoder whose basis sequences are shown in Table 5.

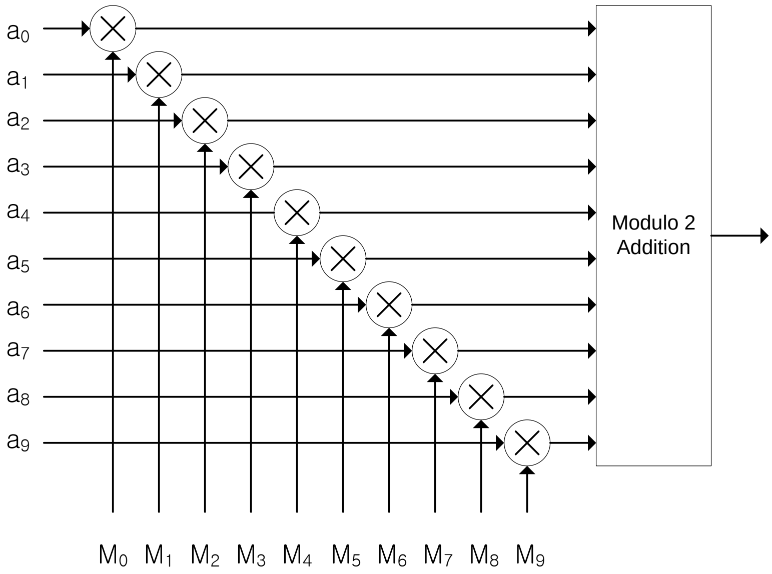

Figure 8. (48, 10) TFCI Encoder

Table 5. Basis Sequences of (48, 10) TFCI Code

M_0	M_1	M_2	M_3	M_4	M_5	M_6	M_7	M_8	M_9
1	0	0	0	0	0	1	0	1	0
0	1	0	0	0	0	1	1	0	0
1	1	0	0	0	0	1	1	0	1
1	0	1	0	0	0	1	1	1	0
0	1	1	0	0	0	1	0	1	0
1	1	1	0	0	0	1	1	1	0
1	0	0	1	0	0	1	1	1	1
0	1	0	1	0	0	1	1	0	1
1	1	0	1	0	0	1	0	1	0
0	0	1	1	0	0	1	1	0	0
0	1	1	1	0	0	1	1	0	1
1	1	1	1	0	0	1	1	1	1
1	0	0	0	1	0	1	0	1	1
0	1	0	0	1	0	1	1	1	0
1	1	0	0	1	0	1	0	0	1
1	0	1	0	1	0	1	0	1	1
0	1	1	0	1	0	1	1	0	0
1	1	1	0	1	0	1	1	1	0
0	0	0	1	1	0	1	0	0	1
1	0	0	1	1	0	1	0	1	1
0	1	0	1	1	0	1	0	1	0
0	0	1	1	1	0	1	0	1	0
1	0	1	1	1	0	1	1	0	1
0	1	1	1	1	0	1	1	1	0
0	0	0	0	0	1	1	1	0	1
1	0	0	0	0	1	1	1	1	0
1	1	0	0	0	1	1	1	1	1
0	0	1	0	0	1	1	0	1	1
1	0	1	0	0	1	1	1	0	1
1	1	1	0	0	1	1	0	1	1
0	0	0	1	0	1	1	0	0	1
0	1	0	1	0	1	1	0	0	1
1	1	0	1	0	1	1	1	1	1
1	0	1	1	0	1	1	0	0	1
0	1	1	1	0	1	1	1	1	0
1	1	1	1	0	1	1	1	0	1
0	0	0	0	1	1	1	1	1	0
1	0	0	0	1	1	1	0	1	1
1	1	0	0	1	1	1	1	1	1
0	0	1	0	1	1	1	1	0	0
1	0	1	0	1	1	1	1	0	0
1	1	1	0	1	1	1	1	1	1
0	0	0	1	1	1	1	1	1	1
0	1	0	1	1	1	1	0	1	0
1	1	0	1	1	1	1	0	1	0
0	0	1	1	1	1	1	0	1	1
0	1	1	1	1	1	1	0	0	1
1	1	1	1	1	1	1	1	0	0

3.2.4. (24, 5) TFCI Coding

When 2Mcps service is transmitted, 8PSK mdoulation is applied in 1.28Mcps TDD option [2]. If the number of TFCI bits in the range of 3 to 5, the (24,5) TFCI coding is used. The basis sequences of (24,5) TFCI code in TDD mode can be obtained from the Rademacher functions with $K = 5$ of Table 1, whose functions

$\{R_1(t), R_2(t), R_3(t), R_4(t), R_5(t)\}$ are mapped to $\{M_4(t), M_3(t), M_2(t), M_1(t), M_0(t)\}$, respectively after puncturing the first 8 bits of them. (24,5) TFCI encoder is depicted in Figure 9 whose basis sequences are shown in Table 6.

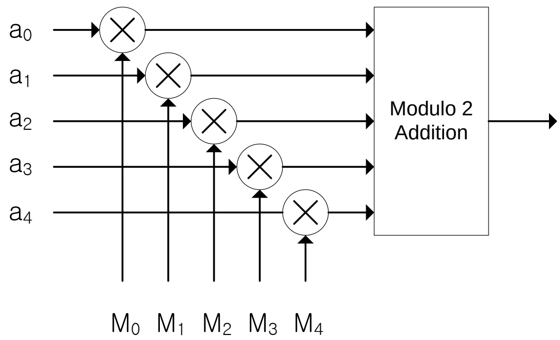

Figure 9. (24, 5) TFCI Encoder

Table 6. Basis sequences of (24, 5) TFCI code

M_0	M_1	M_2	M_3	M_4
0	0	0	1	0
1	0	0	1	0
0	1	0	1	0
1	1	0	1	0
0	0	1	1	0
1	0	1	1	0
0	1	1	1	0
1	1	1	1	0
0	0	0	0	1
1	0	0	0	1
0	1	0	0	1
1	1	0	0	1
0	0	1	0	1
1	0	1	0	1
0	1	1	0	1
1	1	1	0	1
0	0	0	1	1
1	0	0	1	1
0	1	0	1	1
1	1	0	1	1
0	0	1	1	1
1	0	1	1	1
0	1	1	1	1
1	1	1	1	1

4. CQI Coding and RTBS Coding

In this section we discuss the relationship between CQI, RTBS, and TFCI codings.

4.1. (20,5) CQI Coding

The CQI is carried on the HS-DPCCH (High Speed Dedicated Physical Control Channel) to inform the base station scheduler of the data rate that the terminal is able to receive at a given time. We use Figure 10 as the encoder for (20,5) CQI code whose basis sequences Q_0, Q_1, Q_2, Q_3 are exactly same as the M_0, M_1, M_2, M_3 of (16,5) TFCI code just simply adding 4 consecutive zeros to them and $Q_4 = M_4 = \mathbf{1}$ [13]. The basis vector set $\{Q_0, Q_1, Q_2, Q_3, Q_4\}$ is as follows:

$$Q_0 = [10101010101010100000]$$
$$Q_1 = [01100110011001100000]$$
$$Q_2 = [00011110000111100000]$$
$$Q_3 = [00000001111111100000]$$
$$Q_4 = [11111111111111111111]$$

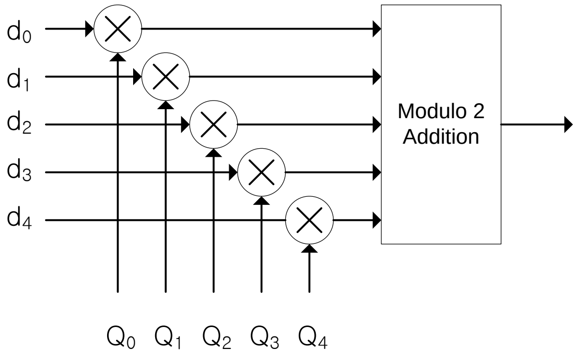

Figure 10. (20, 5) CQI Encoder

4.2. (32, 6) RTBS Coding

(32, 10) TFCI coding is also used for encoding of RTBS (Recommended Transport Block Size) bits of CQI (Channel Quality Information) for HS-SICH (Shared Information Channel for HS-DSCH) [2]. Figure 11 is the (32,6) TRBS encoder which is the simplified version of (32,10) TFCI encoder with 6 basis sequences $\{M_0(t), M_1(t), M_2(t), M_3(t), M_4(t), M_5(t)\}$ of Table 3.

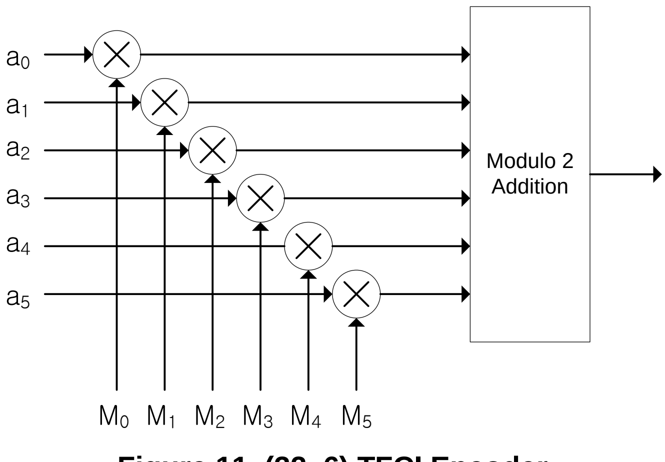

Figure 11. (32, 6) TFCI Encoder

4. Conclusion

This paper has demonstrated that the Rademacher functions are widely applied for OVSF code and (32, 10) TFCI, (16,5) TFCI, (24,5) TFCI, (48,10) TFCI, (20,5) CQI, (32,6) RTBS codes in 3GPP FDD mode and TDD mode mobile communication systems. The OVSF codes are obtained by linearly combining the Rademacher functions. And the basis sequences of TFCI codes are constructed by using barrel shifting of Rademacher functions. Furthermore, we also see that (20,5) CQI and (32,6) RTBS codings are equivalent to (16,5) TFCI coding and (32,10) TFCI coding, respectively.

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