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A Complete Mathematical Modeling and Simulation of Ergodic Channel Capacity of Wireless Random MIMO System

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Abstract

In this article we described the channel capacity of random MIMO channel when CSI is not available at transmitter side. The channel correlation is directly related to capacity of MIMO channel. Also we consider the capacity of MIMO channel when the channel gain between transmitter and receiver is correlated. We compute mathematical equation for calculation of ergodic channel capacity of random MIMO system. MIMO system have very large impact on channel capacity, so it is very necessary to analyze all parameter which are responsible for very high speed data transmission. Because day by day there is very high need of technologies which provide high data transmission. All the simulation in this article are done with the help of MATLAB/Simulink software.

Keywords*: MIMO, Channel, Random, Capacity, Antenna, CSI, Communication.*

1. Introduction

As compared to a conventional single antenna system, the channel capacity of a multiple antenna system with N number of transmitting and receiving antennas can be increased without using additional transmit power or spectral bandwidth[1-3]. As we know that in future the three is very high demand of very high speed data transmission, So MIMO system can be used to meet these demands.MIMO system can be deployed for different broadband wireless network techniques, such as Cellular Communication, WiMax *etc*.[4-7].

Sometimes we have wireless channel with great channel capacity but still we need best methods to achieve high speed transmission with high reliability[8].There are two types of technique for multiple antenna. These are spatial multiplexing and diversity technique [9]. In first technique there are multiple data are transmitted simultaneously with the help of multiple transmitting antennas [10]. Therefore to achieve very high rate of data transmission. In second technique is deals with reliability of data transmission. In this technique same data is received with the help of multiple receiving antennas. Therefore this technique is improve reliability of data transmission [11].

The performance of wireless communication systems is mainly governed by the wireless channel environment. As opposed to the typically static and predictable characteristics of a wired channel, the wireless channel is rather dynamic and unpredictable, which makes an exact analysis of the wireless communication system often difficult. In recent years, optimization of the wireless communication system has become critical with the rapid growth of mobile communication services and emerging broadband mobile Internet access services.

In fact, the understanding of wireless channels will lay the foundation for the development of high performance and bandwidth-efficient wireless transmission technology. In wireless communication, radio propagation refers to the behaviour of radio waves when they are propagated from transmitter to receiver. In the course of propagation, radio waves are mainly affected by three different modes of physical phenomena: reflection, diffraction, and scattering [12]. Reflection is the physical phenomenon that occurs when a propagating electromagnetic wave impinges upon an object with very large dimensions compared to the wavelength, for example, surface of the earth and building. It forces the transmit signal power to be reflected back to its origin rather than being passed all the way along the path to the receiver. Diffraction refers to various phenomena that occur when the radio path between the transmitter and receiver is obstructed by a surface with sharp irregularities or small openings. It appears as a bending of waves around the small obstacles and spreading out of waves past small openings.

The secondary waves generated by diffraction are useful for establishing a path between the transmitter and receiver, even when a line-of-sight path is not present. Scattering is the physical phenomenon that forces the radiation of an electromagnetic wave to deviate from a straight path by one or more local obstacles, with small dimensions compared to the wavelength. Those obstacles that induce scattering, such as foliage, street signs, and lamp posts, are referred to as the scatters. In other words, the propagation of a radio wave is a complicated and less predictable process that is governed by reflection, diffraction, and scattering, whose intensity varies with different environments at different instances[13].

A unique characteristic in a wireless channel is a phenomenon called ‘fading,’ the variation of the signal amplitude over time and frequency. In contrast with the additive noise as the most common source of signal degradation, fading is another source of signal degradation that is characterized as a non-additive signal disturbance in the wireless channel. Fading may either be due to multipath propagation, referred to as multi-path (induced) fading, or to shadowing from obstacles that affect the propagation of a radio wave, referred to as shadow fading.

The fading phenomenon in the wireless communication channel was initially modeled for HF (High Frequency, 3_30MHz), UHF (Ultra HF, 300_3000 GHz), and SHF (Super HF, 3_30 GHz) bands in the 1950s and 1960s. Currently, the most popular wireless channel models have been established for 800MHz to 2.5 GHz by extensive channel measurements in the field. These include the ITU-R standard channel models specialized for a single-antenna communication system, typically referred to as a SISO (Single Input Single Output) communication, over some frequency bands. Meanwhile, spatial channel models for a multi-antenna communication system, referred to as the MIMO (Multiple Input Multiple Output) system, have been recently developed by the various research and standardization activities such as IEEE 802, METRA Project, 3GPP/3GPP2, and WINNER Projects, aiming at high-speed wireless transmission and diversity gain[14].

We organize this article in following manner. Second section deals with basic theory of MIMO system.

Third section describe mathematical formulation and simulation of ergodic channel capacity of MIMO channel. Fourth section describe the effect of fading on wireless communication. Last section describe overall work done in this article, which contain discussion about the result analysis and applicability of our analysis in future research.

2. Theory of MIMO System

MIMO also known multi input multi output is very powerful technique for wireless communication system. The structure of MIMO consist of multiple antennas at both ends of transmitter and receiver. The main purpose of using multiple antennas to improve the performance. It provide higher capacity as well as improved quality of service without affecting the power of antennas.

There are various advantage of using MIMO in wireless communication it provide increase in array gain, multiplexing gain and spatial diversity. The array gain is the gain in

SNR due to use of multiple antennas which leads to increase in range as well as coverage [15].MIMO offers multiplexing gain by transmitting multiple data streams which leads to increase in throughput.

MIMO propagation model consist of transmitted data modulator, transmitting and receiving antennas, demodulator and wireless channel as shown in Figure (1).

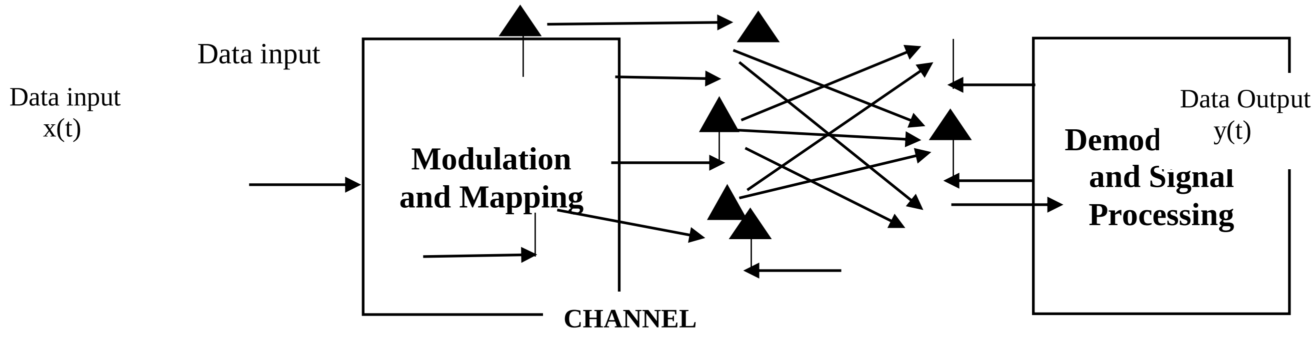

Figure 1. Simple Model of MIMO System

Channel capacity of MIMO channel is measured by maximum amount of information it can transmitted over a channel and received with minimum probability of error [16]. The capacity of MIMO system when transmitter and receiver are aware of channel matrix is

$$C = max\ log|I^N + HQH^\rho| \tag{1}$$

3. Analysis of Channel Capacity of Random MIMO System

Here we have assumed that MIMO channel are deterministic. But in general MIMO channel change randomly. H is random matrix which means that its capacity of channel is also varying with time [17]. The channel capacity of MIMO channel is given by this equation.

$$\overline{C} = E\{C(H)\} = E\left\{\max_{T_r(R_{xx})=N_T} \log_2 \det(I_{N_R} + \frac{E_X}{N_T N_0} H R_{xx} H^H)\right\} \tag{2}$$

Equation (2) represents the ergodic channel capacity for open loop system without using CSI at transmitter side.

$$\overline{C_{OL}} = E\left\{\sum_{i=1}^{r} \log_2(1 + \frac{E_X}{N_T N_0}\lambda_i\right\} \tag{3}$$

$$\overline{C_{CL}} = E\left\{\max_{\sum_{i=1}^{r}\gamma_i = N_T} \sum_{i=1}^{r} \log_2(1 + \frac{E_X}{N_T N_0}\gamma_i\lambda_i\right\}$$

$$= E\left\{\sum_{i=1}^{r} \log_2(1 + \frac{E_X}{N_T N_0}\gamma_i^{opt}\lambda_i\right\} \tag{4}$$

Similarly ergodic channel capacity for closed loop system without using CSI at transmitter side.
And we can define Outage Probability as

$$P_{Out}(R) = \Pr(C(H) < R) \tag{5}$$

In other words the system is said to be in outage if the decoding error of probability cannot be made arbitrarily small with the transmission rate of R bps/Hz. Then the ε-outage channel capacity is defined as the largest possible date rate such that the outage probability in equation (5) is less than ε. In other words it is corresponding to C_{ε} such that P(C (H)≤ C_{ε})=ε

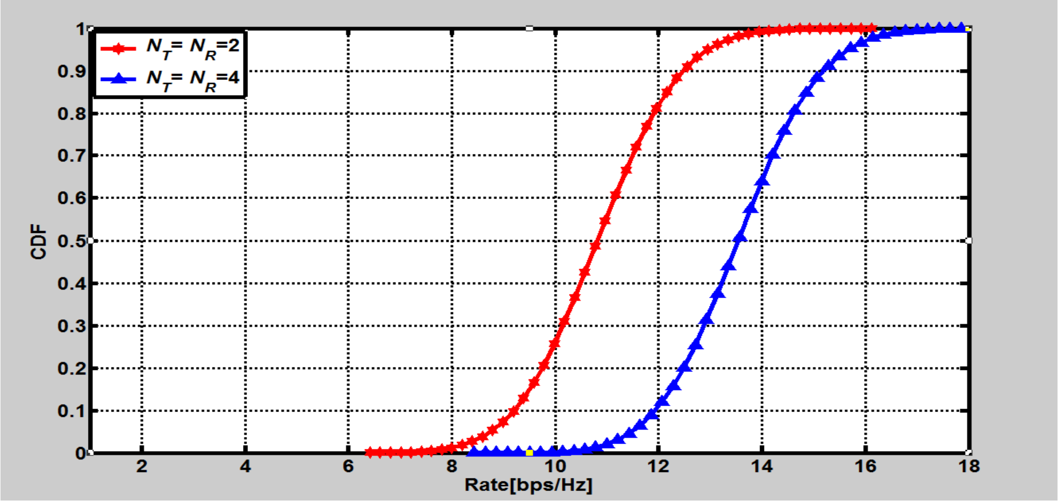

Figure 2. MIMO Channel Capacity (SNR=10dB) When CSI not Available at Transmitter

The Figure 2 shows the cumulative distribution function of the random 2×2 and 4×4 MIMO channel capacities when signal to noise ratio is 10dB.It is clear from fig 2 that if we increase the number of antenna in MIMO system its channel capacity is improving in drastic manner. So MIMO system play very important role to communication where we need high channel capacity [18-19].

In Figure 3 we compute the ergodic capacity of MIMO channel as signal to noise ratio is varied, when CSI is not known at the transmitter side. Figure 3 shows the ergodic channel capacity as varying the number of antennas.

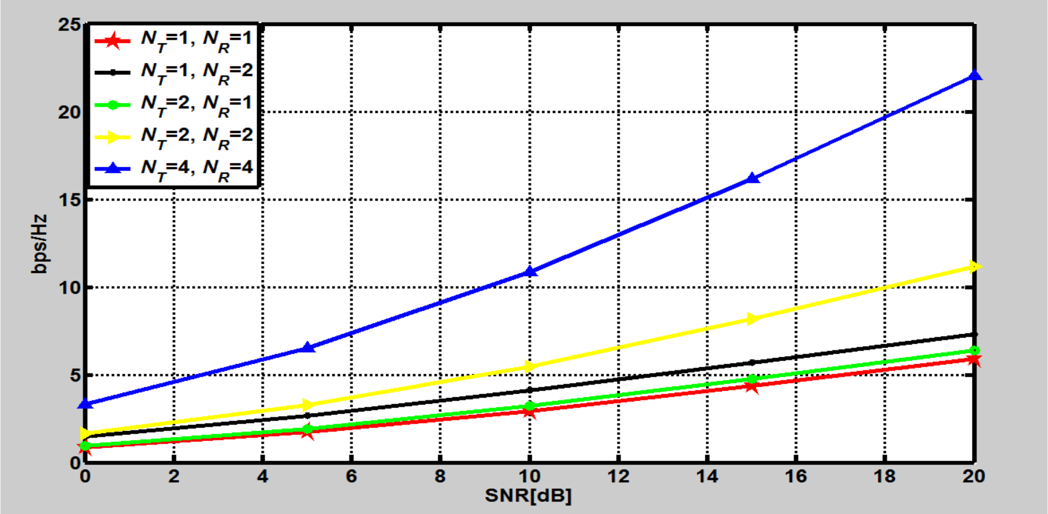

Figure 3. Ergodic Channel Capacity When CSI not Present at Transmitter

In general the MIMO channel gain are not independent and identically disturbed. The channel correlation is directly related to capacity of MIMO channel. Also we consider the capacity of MIMO channel when the channel gain between transmitter and receiver is correlated [20-21]. When the SNR is high the deterministic channel capacity can be written as

$$C = \max_{Tr(R_{xx})=N} \det(R_{xx}) + \log_2 \det(\frac{E_x}{NN_o} H_W H_W^H) \tag{6}$$

From above equation (6) we can see that the second term is constant and while first term is involving $\det(R_{xx})$ is maximum when$R_{xx} = I_N$. Consider the following correlated channel model.

$$H = R_r^{1/2} H_W R_t^{1/2} \tag{7}$$

Where R_t is the correlation matrix, reflecting the correlation between the transmitting, R_r is the correlation matrix reflecting the correlation between the receiving antennas and H_w denotes the Rayleigh fading channel gain matrix [22]. The diagonal entries of R_t and R_r are considered as a unity. The MIMO channel is given as

$$C = \log_2 \det(I_{N_R} + \frac{E_x}{NN_o} H_W H_W^H R_t R_t^{1/2} R_r^{H/2}) \tag{8}$$

If $N_T = N_R = N$, R_t and R_r are full rank and SNR is high and above equation (8) can be approximated as

$$\mathrm{C} \approx \log_2 \det\left(\frac{E_x}{NN_o} H_W H_W^H\right) + \log_2 \det(R_r) + \log_2 \det(R_t) \tag{9}$$

From above equation (9) it is found that the MIMO channel capacity has been reduced and the amount of capacity reduction due to correlation between the transmit and receive antennas is

$$\log_2 \det(R_r) + \log_2 \det(R_t) \tag{10}$$

It is shown that the value of above equation (10) is always negative by the fact that $\log_2 \det(R) \leq 0$ for any coreelation matrix R. since R is symmetric matrix [23]. Since the determinant of unitary matrix is unity the determinant of a correlation matrix can be expressed as

$$\det(\mathrm{R}) = \prod_{i=1}^{N} \lambda_i \tag{11}$$

Note that geometric mean is bounded by the arithmetic mean that is

$$\left(\prod_{i=1}^{N} \lambda_i\right)^{\frac{1}{N}} \leq \frac{1}{N} \sum_{i=1}^{N} \lambda_i = 1 \tag{12}$$

From above equation (11) and (12) it is clear that

$$\log_2 \det(R) \leq 0 \tag{13}$$

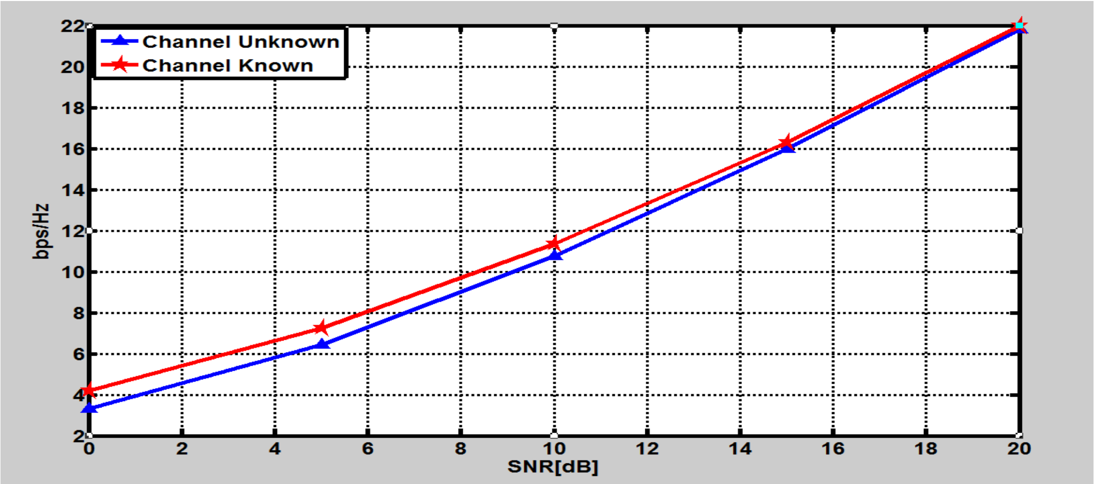

Figure 4. Ergodic Channel Capacity $N_T = N_R = 4$

The equality in above equation (13) holds when the correlation matrix is the identity matrix. Therefore the quantities in equation are all negative.the corelation matrix for ergodic channel capacity when there exists a correlation between the transmit and receive antenna with $R_r = I_4$

$$R_t = \begin{bmatrix} 1 & 0.76e^{j0.17\pi} & 0.43e^{j0.35\pi} & 0.25e^{j0.53\pi} \\ 0.76e^{-j0.17\pi} & 1 & 0.76e^{j0.17\pi} & 0.43e^{j0.35\pi} \\ 0.43e^{-j0.35\pi} & 0.76e^{-j0.17\pi} & 1 & 0.76e^{j0.17\pi} \\ 0.25e^{-j0.53\pi} & 0.43e^{-j0.35\pi} & 0.76e^{-j0.17\pi} & 1 \end{bmatrix} \quad (14)$$

$R_r = I_4$ states that no correlation exists between the receive antenna. Figure 4 shows the result for ergodic channel capacity.The Figure 4 shows the ergodic capacities of 4×4 MIMO channel with and without using transmitter at the transmitter side. It shows that the closed loop system provide more capacity than open loop system [24] [25].

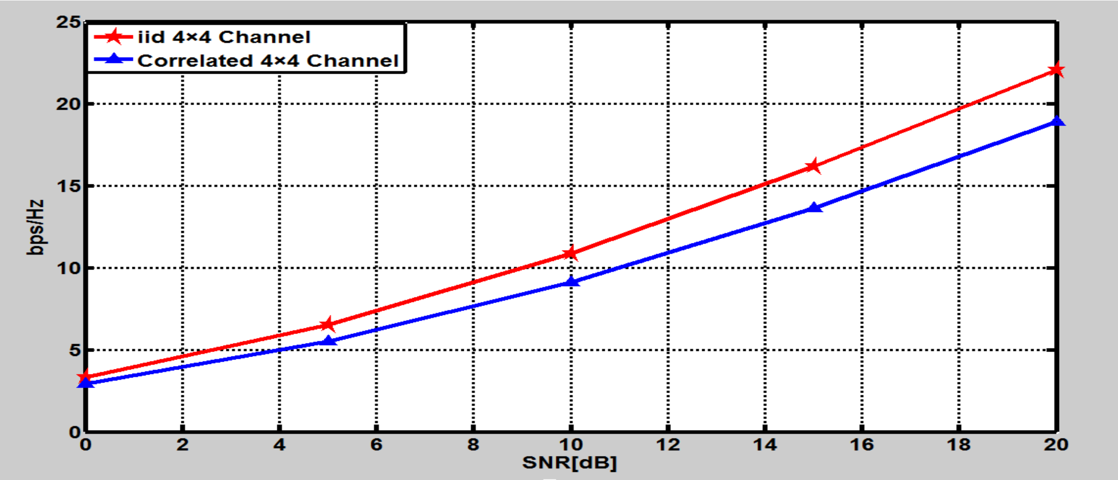

Figure 5. Capacity Reduction Due to Channel Correction

However, we can see that CSI availability does not help to improve the channel capacity when average SNR is extremely high. It implies that even the lowest SNR is good enough to *get al*most the same transmit power allocated as the highest SNR mode when the average SNR is extremely high.The Figure 5 shows that a capacity of 3.3 bps/Hz is lost due to channel correlation when SNR is 18dB.

4. Analysis of Fading Effect on Wireless MIMO Channel

The free space model is used to predict the strength of power at receiver end, when there is no obstacle present in communication channel.it is general used for satellite communication. Let d is the distance between transmitter and receiver [18][19]. Gain of transmitting antenna is G_t and gain of receiving antenna is G_r. Received power at distance d is expressed by well-known Friss equation [20-24].

$$P_r(d) = \frac{G_t G_r P_t \lambda^2}{(4\pi)^2 d^2 L} \tag{15}$$

Any received signal in the propagation environment for wireless channel can be considered as sum of the received signal from an infinite number of scatters. By the central limit theorem the received signal can be represented by the Gaussian random channel. In other words a wireless channel can be represented by a complex Gaussian random variable.

Let X denote the amplitude of complex Gaussian random variable$W_1 + jW_2$, such that

$$X=\sqrt{W_1^2 + W_2^2}$$

Then X is a Rayleigh random variable with the following density function

$$f_x(x) = \frac{x}{\sigma^2} e^{\frac{x^2}{2\sigma^2}} \tag{16}$$

Where $2\sigma^2 = E\{X^2\}$

Now we will discuss how to generate the Rayleigh random variable X first of all we generate two i.i.d Gaussian random variable with zero mean and unit variance [22].

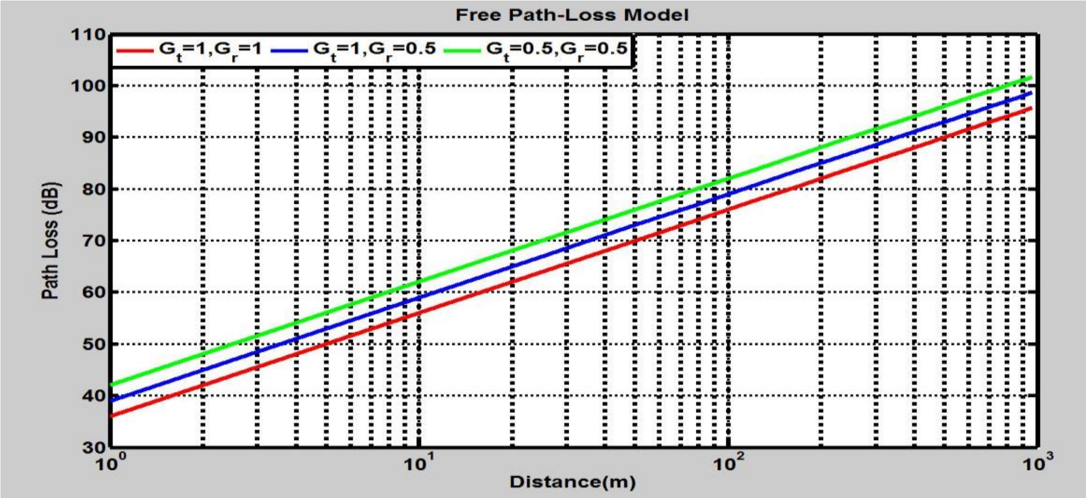

Figure 6. Free Space Communication Path Loss Model

The log-distance path loss model in which the path loss at distance d is given as

$$PL_{LD}(d)[dB] = PL_F(d_0) + 10n\,log\frac{d}{d_0} \tag{17}$$

Where d_0 is a reference distance at which or closer to path loss. The path loss exponent can vary 2 to 6 depending upon propagation of environment. The Figure 6 shows the free space path loss at carrier frequency of 1.5GHz for different antenna gains as the distance varies. It is obvious that the path loss increased by reducing the antenna gain [25-28].

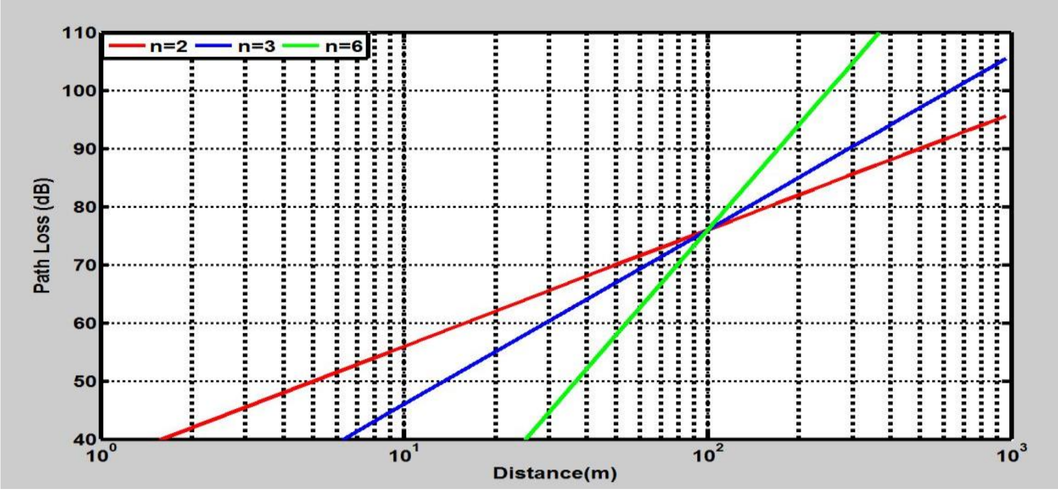

Figure 7. Log-Distance Path Loss Model

Figure 7 shows the log distance path loss at the carrier frequency of 1.5GHz. it is clear that the path loss increase with the path loss exponent n. even if the distance between transmitter and receiver is equal to each other every path have different path loss[29][30][31][32][33][34][35]. Let X_σ denote a Gaussian random variable with zero mean and standard deviation of σ then the log normal shadowing model can be written as

$$PL(d)[dB]=\overline{PL}(d)+X_\sigma=PL_F(d_0)+10n\log\frac{d}{d_0}+X_\sigma \tag{18}$$

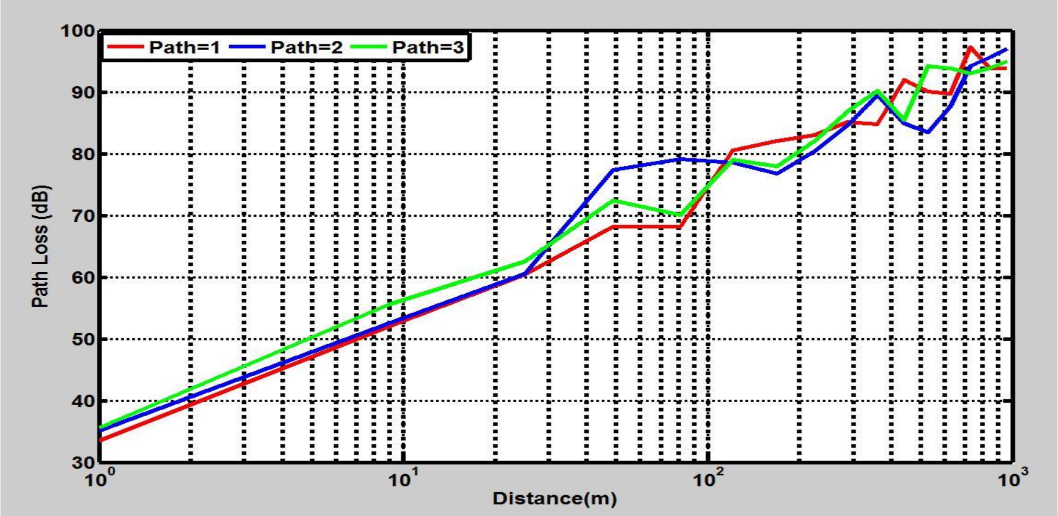

Figure 8. Log Normal Shadowing Path Loss Model

The Figure 8 shows the path loss that follows the log normal shadowing model for carrier frequency 1.5GHz and σ=3dB and n=2. It is clearly shows the random effect of shadowing that is imposed on the deterministic nature of log distance path loss model.

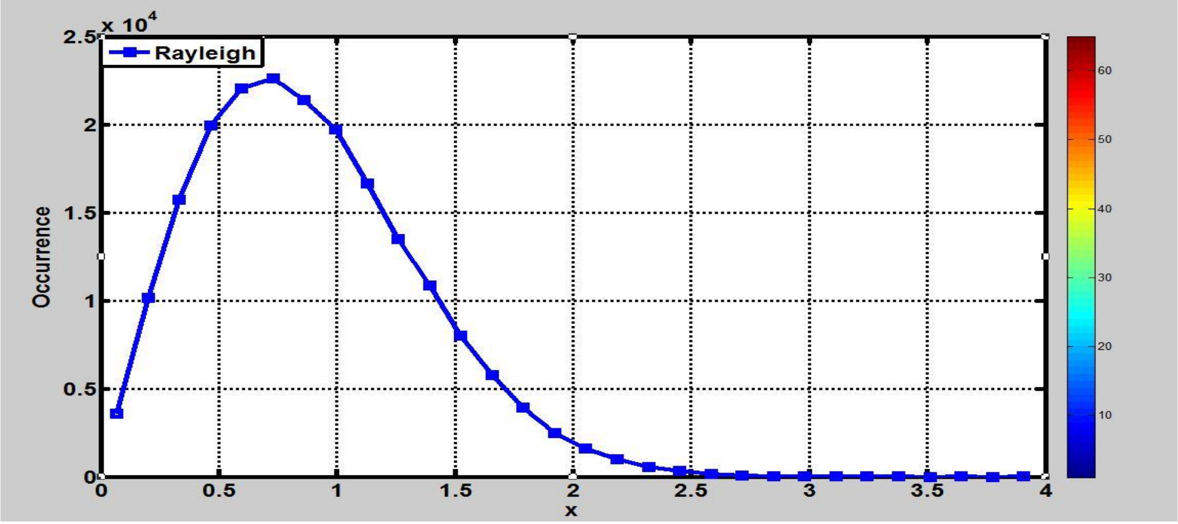

Figure 9. Rayleigh Fading Channel Distribution

Figure9 shows the Rayleigh distribution and Gaussian distribution when k=-40dB and k=15dB.

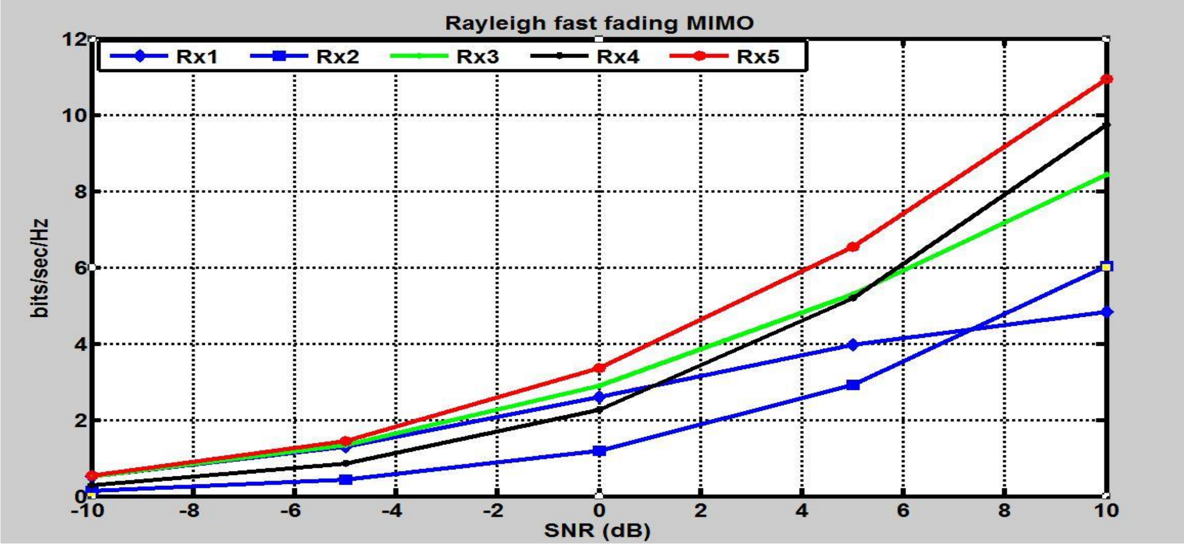

Figure 10. Rayleigh Fast Fading MIMO System using Different Receiver

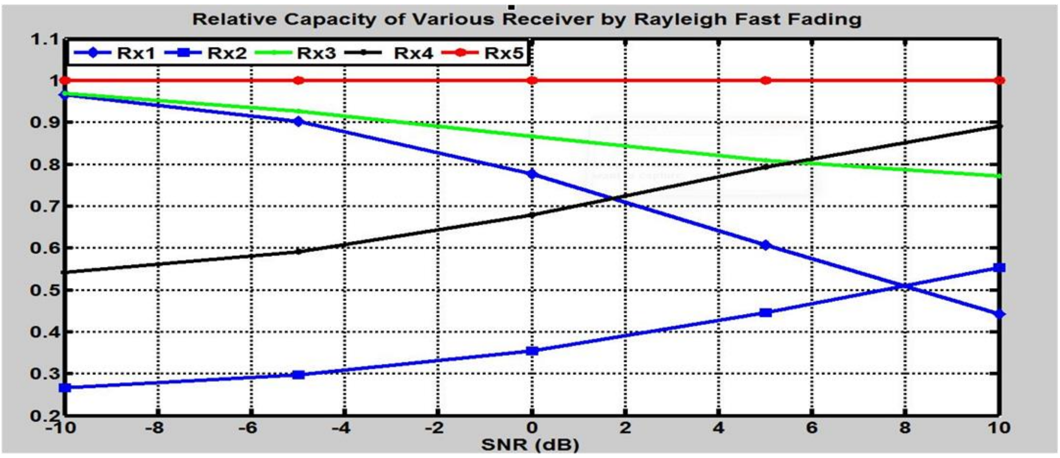

Figure 11. Relative Capacity of Different Receiver using Rayleigh Fast Fading

Figure 10 shows the simulation result for Rayleigh fast fading for multi input multi output system for different receiver. Figure 11 shows Relative Capacity of Different Receiver using Rayleigh Fast Fading using different receiver at receiving point.

5. Conclusion

In this paper, we describe the ergodic channel capacity of random MIMO channel. Here we formulate mathematical equation in this article and also simulate all the equation with the help of simulation tool. MIMO channel in wireless communication provide increase in array gain, multiplexing gain and spatial diversity. Here we have taken four antennas in transmitting side and also four antenna at reception side. Also observe that there is capacity reduction due to channel correction. Apart from this we also discuss the effect of fading on wireless communication. For this analysis, we done simulation to prove our results using MATLAB.

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