

# A Complete Analysis of Channel Estimation and Peak to Average Power Ratio in Wireless Communication Using Discrete Fourier Transform

Sandeep Thakur

*Dept. of Electronics & Communication, AIGIET Markapur, India*  
*Sandeepatism@gmail.com*

## **Abstract**

*In wireless communication peak to average power and channel estimation are important parameters which should be considered for efficient communication. At the receiver end, the signal is normally distorted due to channel interference and noise. In this paper we focus on complete analysis of PAPR and channel estimation in wireless communication. For this purpose we use discrete Fourier transform which has many application. In this paper we analyze the effect of increase in number of subcarrier on the performance of PAPR using discrete Fourier transform technique. The paper also describes how discrete Fourier transform can be used to remove the distortion from received signal. Our main motive is to improve the peak to average power ratio with the help of discrete Fourier transform approach. This will improve the power efficiency. All the mathematical equations and simulation for our analysis are done with the help of MATLAB software.*

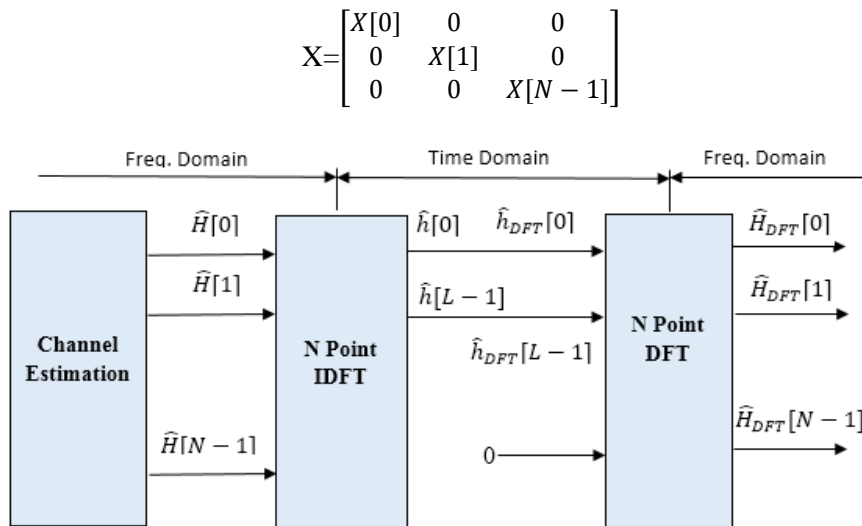
**Keywords:** *Communication, DFT, LFDMA, Fourier transform, Distortion, PAPR, Estimation, Channel.*

## **1. Introduction**

As the advancement of the technology we need very high rate and efficient communication. The Important parameter in communication are high transmission rate with low power. This means that the energy requirement for each transmitted bit should be reduced. To solve this problem there are many methods are implemented which can be used for all types of communication such as wired and wireless communication. For high spectral efficiency, rate and modulation purpose less expensive and efficient method is required. In this paper we describe the technique with mathematical formulation as well as simulation. Now a days Orthogonal Frequency division multiplexing technique is used by all types of communication. Because it is multi carrier scheme for modulation. The basic principle behind OFDM is to divide high rate stream into low rate sub streams and then modulated using subcarrier which are orthogonal in nature to each other [1, 2]. One of the major problem in wireless communication is limited frequency spectrum. The best method to overcome this problem orthogonal frequency division multiple access technique. The important thing in transmission of data is to estimate the wireless channel before demodulation. Because after demodulation there are many types of channel interference and noise get added into the picture. [3].

## **2. Channel Estimation Theory**

Least Square channel estimation is used to estimate the channel is such a way so that it can reduced the cost [4, 5]. The mathematical formulation for this technique is given below. We are assuming all the subcarrier are orthogonal in nature and we form matrix using N subcarrier.



**Figure 1. Block Diagram of Channel Estimation Technique**

And given channel gain is  $H[k]$  for every subcarrier then received signal  $Y(k)$  is given as below.

$$Y \triangleq \begin{bmatrix} Y[0] \\ Y[1] \\ \vdots \\ Y[N-1] \end{bmatrix} = \begin{bmatrix} X[0] & 0 & 0 \\ 0 & X[1] & 0 \\ 0 & 0 & X[N-1] \end{bmatrix} \begin{bmatrix} H[0] \\ H[1] \\ \vdots \\ H[N-1] \end{bmatrix} + \begin{bmatrix} Z[0] \\ Z[1] \\ \vdots \\ Z[N-1] \end{bmatrix}$$

Now

$$J \hat{H} = \|Y - X\hat{H}\|^2 = (Y - X\hat{H})^T (Y - X\hat{H}) \quad (1)$$

Now by differentiation it w.r.t  $\hat{H}$  we get

$$\frac{\partial J \hat{H}}{\partial \hat{H}} = -2(X^H Y) + 2(X^H X \hat{H}) = 0 \quad (2)$$

Now we have  $(X^H X \hat{H}) = (X^H Y)$

Which provide us solution for channel estimation

$$\hat{H}_{LS} = (X^H X)^{-1} (X^H Y) = (X^{-1} Y) \quad (3)$$

For each subcarrier we can write  $\hat{H}_{LS}$  as given below

$$\hat{H}_{LS}(k) = \frac{Y[k]}{X[k]} \quad (4)$$

Here  $k=0,1,2,3,\dots,(N-1)$

### 3. Working Principle

As we know that the discrete Fourier transform has wide range of application in mathematics, electronics as well as in communication. In this paper we describe how we can use discrete Fourier transform to estimate the channel so that we can remove the unwanted distortion at the receiver end [6][7][8]. The block diagram for this technique is

shown in Figure.1. Let's us suppose  $\hat{H}[k]$  is gain of estimated channel of  $k$ th subcarrier. Now by taking inverse discrete Fourier transform.

$$\text{IDFT} \{ \hat{H}[k] \} = h[n] + z[n] \triangleq \hat{h}[n] \quad (5)$$

Here  $n=0,1,2,\dots,N-1$

$z[n]$  is taken as noise component in time domain and we removing  $\hat{h}[n]$  because it contain noise only.

$$\hat{h}_{DFT}[n] = \begin{cases} h[n] + z[n] & , n = 0,1,2, \dots, L-1 \\ 0 & \text{Otherwise} \end{cases}$$

Now transforming the L elements into frequency component.

$$\hat{H}_{DFT}[k] = \text{DFT} \{ \hat{h}_{DFT}[n] \} \quad (6)$$

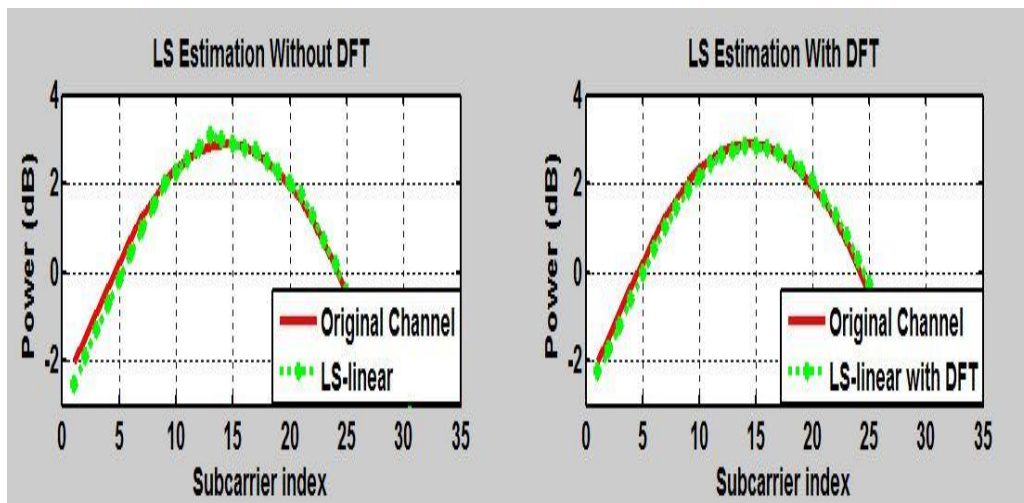
In Figure. 1 we chose the LS estimation for this purpose maximum delay should be known.

#### 4. Results and Observation for Channel Estimation

In this paper we estimate the channel using least square method and we shows the result before the estimation of channel using discrete Fourier transform and after Fourier transform are shown below [9].

**Table 1. Parameter Used**

Parameter	Specification
Number of symbol	16
Guard interval	4
Symbol length	36
Symbol duration	100
Pilot spacing	4
Number of pilot	8
Data per symbol	24
Number of bits per symbol	4



**Figure 2. (A) LS Channel Estimation without DFT B) LS Channel Estimation after DFT**

Table 1 contain all the parameter used during simulation. The simulation results are shown in Figure. 2 and in Figure. 3. All the simulation in this paper are done with the help of MATLAB software [15].As shown in Figure. 2(a) it is clearly shows that the original channel and LS estimation difference. But after applying discrete Fourier transform the LS estimation is almost close to original channel. So that we can get efficient signal at the receiver side by applying this technique. In Figure 2 (b) LS estimated channel is closely follow the original channel. Here in this case we use LS linear estimation method. Similarly we use least square spline estimation and we show the results how LS estimation using discrete Fourier transform affect the received signal. As shown in Figure 3 (a) Least square spline estimation does not follow the original channel. The green dotted line shows the LS Spline estimation. In Figure 3(b), after applying the discrete Fourier transform the original channel is closely followed. So by observing these results we can say that after applying the discrete Fourier transform we can estimate the channel for efficient communication.

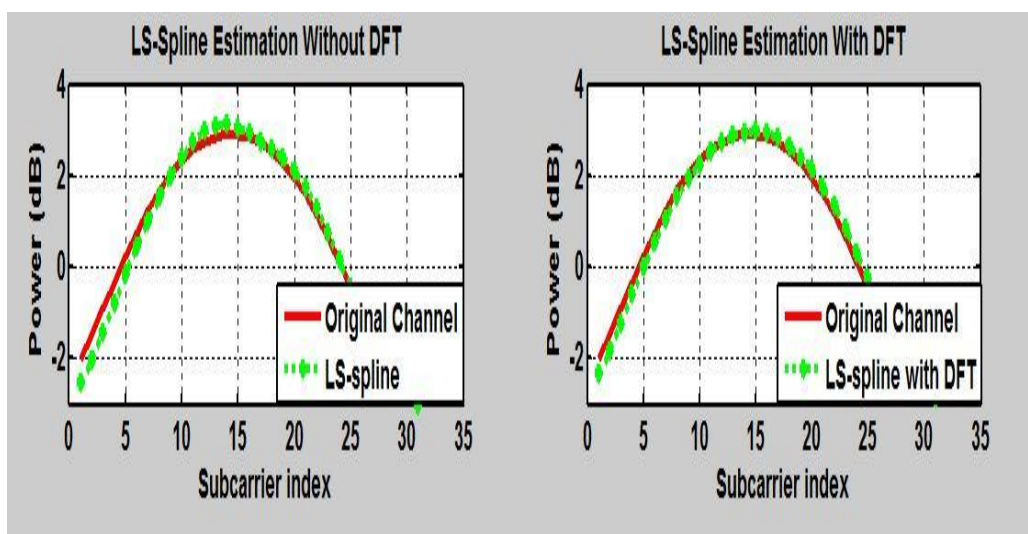


Figure 3. (A) LS Spline Before Transform (B) LS Spline After Transform

### 5. PAPR Performance Analysis Using DFT Technique

For the analysis of PAPR performance here we consider orthogonal frequency division multiple access system. Figure 4. Shows the block diagram of model for single carrier we used for analysis of PAPR [10, 11].

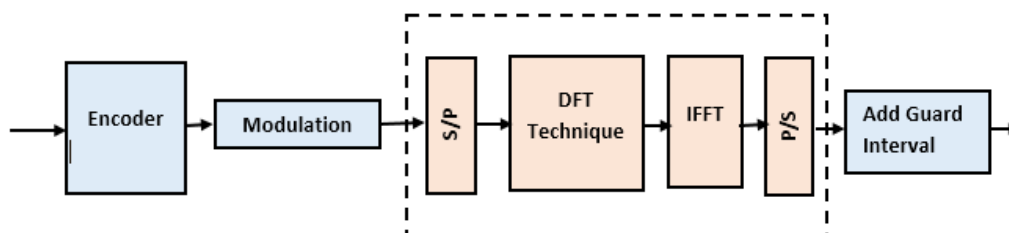


Figure 4. Block Diagram for Analysis PAPR Performance Using DFT Technique for Single Carrier

In discrete Fourier transform technique we divide subcarrier using two types of method. First is distributed frequency division multiple access and second is localized

frequency division multiple access. In this case input data is  $X[m]$  and after DFT technique it generates  $X[i]$ .

$$\hat{X}[k]=\begin{cases} X[K/S] & m_1 = 0,1,2,3 \dots (M-1) \\ 0 & \text{Otherwise} \end{cases}$$

$$K=S \cdot m_1$$

Then IFFT sequence is given as

$$\tilde{X}[n] = \frac{1}{N} \sum_{k=0}^{N-1} \hat{X}[k] e^{j2\pi \frac{n}{N} k} \quad (7)$$

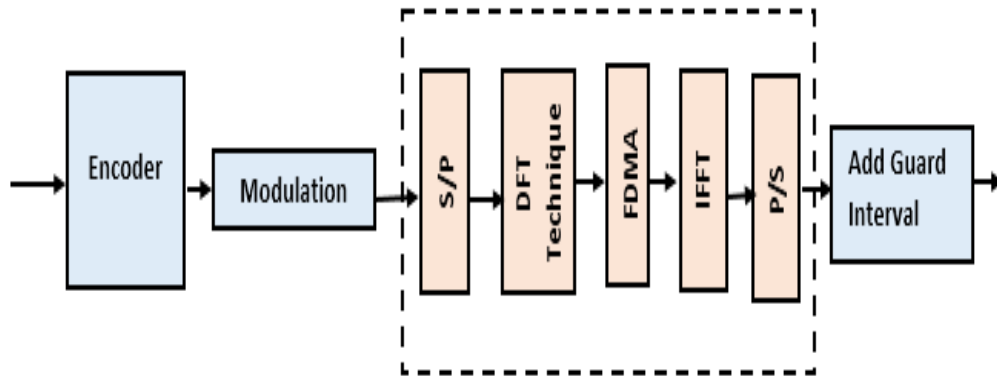
$$= \frac{1}{S} \frac{1}{M} \sum_{m_1=0}^{M-1} X[m_1] e^{j2\pi \frac{n}{M} m_1}$$

Here  $n = M \cdot s + m$   $S=0, 1, 2, 3 \dots (S-1)$

$$\begin{aligned} &= \frac{1}{S} \frac{1}{M} \sum_{m_1=0}^{M-1} X[m_1] e^{j2\pi \frac{M \cdot s + m}{M} m_1} \\ &= \frac{1}{S} \left( \frac{1}{M} \sum_{m_1=0}^{M-1} X[m_1] e^{j2\pi \frac{m}{M} m_1} \right) \end{aligned} \quad (8)$$

$$= \frac{1}{S} \cdot x[m] \quad (9)$$

This shows that the original signal is scaled by  $1/S$  factor.



**Figure 5. Block Diagram for Analysis PAPR Performance with DFT Technique of IFDMA**

In IFDMA we taken here  $r$ th subcarrier. So here  $r=0, 1, 2, 3 \dots (S-1)$

$$\hat{X}[k]=\begin{cases} X[(K-r)/S] & m_1 = 0,1,2,3 \dots (M-1) \\ 0 & \text{Otherwise} \end{cases}$$

$$K=S \cdot m_1 + r$$

Output sequence is

$$\tilde{X}[n] = \tilde{X}[Ms + m] \quad (10)$$

$$= \frac{1}{N} \sum_{k=0}^{N-1} \hat{X}[k] e^{j2\pi \frac{n}{N} k} \quad (11)$$

$$= \frac{1}{S} \frac{1}{M} \sum_{k=0}^{N-1} \hat{X}[m_1] e^{j2\pi \left( \frac{n}{M} m_1 + \frac{n}{N} r \right)} \quad (12)$$

$$= \frac{1}{S} \frac{1}{M} \sum_{k=0}^{N-1} \hat{X}[m_1] e^{j2\pi \left( \frac{Ms+m}{M} m_1 \right)} e^{j2\pi \frac{n}{N} r} \quad (13)$$

$$= \frac{1}{S} \cdot x[m] \cdot e^{j2\pi \frac{n}{N} r} \quad (14)$$

Compare to previous equation we got phase shift of  $e^{j2\pi \frac{n}{N} r}$  using IFDMA method. Now from the Figure. 5 the input sequence to the IFFT block is given as

$$\tilde{X}[k] = \begin{cases} X[k] & k = 0, 1, 2, 3 \dots (M-1) \\ 0 & k = M, M+1 \dots (N-1) \end{cases}$$

And output sequence is

$$\begin{aligned} \tilde{X}[n] &= \tilde{X}[Sm + s] = \frac{1}{N} \sum_{k=0}^{N-1} \tilde{X}[k] e^{j2\pi \frac{n}{N} k} \\ &= \frac{1}{S} \frac{1}{M} \sum_{k=0}^{N-1} \tilde{X}[m_1] e^{j2\pi \left(\frac{Sm+s}{SM}\right) k} \end{aligned} \quad (15)$$

If S is taken as zero then above equation becomes.

$$\tilde{X}[n] = \tilde{X}[Sm] = \frac{1}{S} \frac{1}{M} \sum_{k=0}^{N-1} \tilde{X}[m_1] e^{j2\pi \left(\frac{Sm}{SM}\right) k} = \frac{1}{S} \frac{1}{M} \sum_{k=0}^{N-1} \tilde{X}[m_1] e^{j2\pi \left(\frac{m}{M}\right) k}, = \frac{1}{S} \cdot x[m] \quad (16)$$

If S is not taken as zero then we have

$$\tilde{X}[k] = \sum_{p=0}^{M-1} X[p] e^{-j2\pi \frac{p}{N} k}$$

$$\tilde{X}[n] = \tilde{X}[Sm + s]$$

$$\tilde{X}[n] = \frac{1}{S} (1 - e^{j2\pi \frac{s}{S} k}) \frac{1}{M} \sum_{p=0}^{M-1} \frac{X[p]}{(1 - e^{j2\pi \left(\frac{m-p}{M} + \frac{s}{SM}\right) k})} \quad (17)$$

$$\tilde{X}[n] = \frac{1}{S} e^{j2\pi \frac{(M-1)s - Sm}{SM}} \sum_{p=0}^{M-1} \frac{\sin(\pi \frac{s}{S} k)}{M \sin(\pi \frac{(sm+s)}{SM} - \pi \frac{p}{M} k)} \cdot e^{j\pi \left(\frac{p}{M}\right) k} \cdot x[p] \quad (18)$$

From above equation it is clearly shown that the input sequence becomes 1/S scaled copies. Using DFT technique we use N=12, M=4 and S=3. Figure. 6 shows the performance of PAPR using discrete Fourier transform applied for various multiple access techniques.

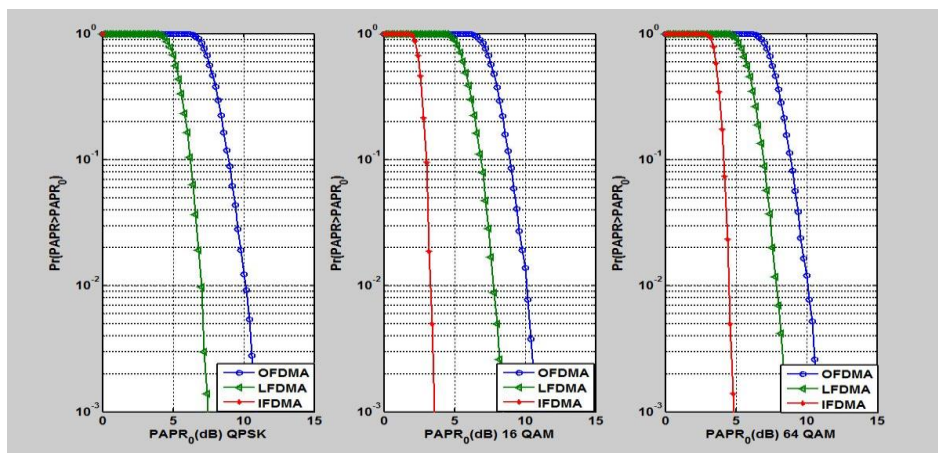
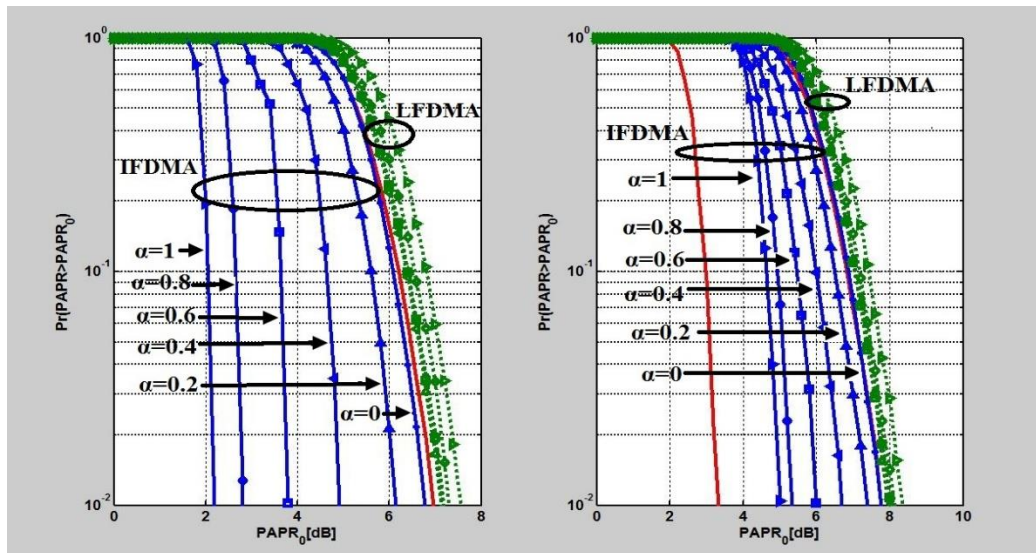


Figure 6. Performance of PAPR Using DFT for Various Multiple Access Technique

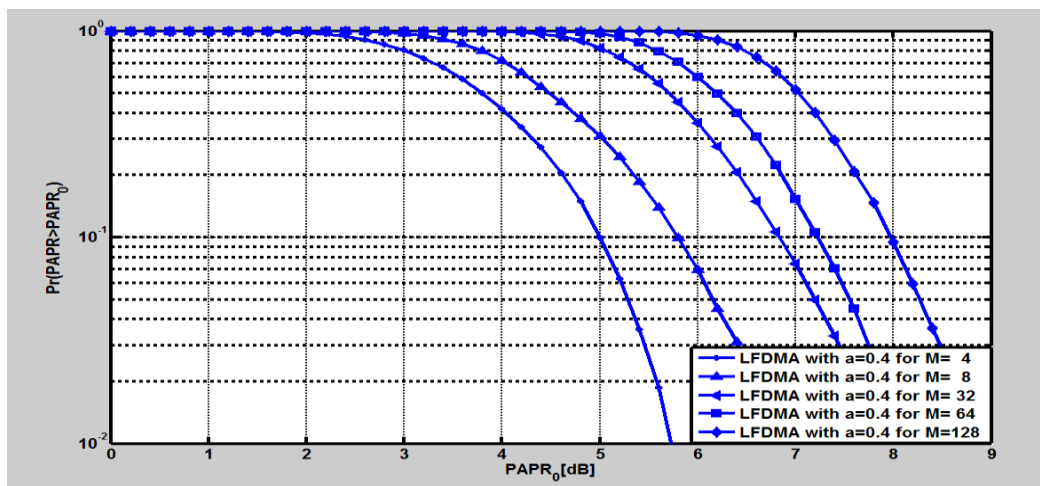
## 6. Results for Analysis of PAPR Performance

Now we consider the pulse shaping effect. By considering pulse shaping effect we observe the PAPR performance of IFDMA and LFDMA using discrete fourier transform technique. In this analysis we vary the value of  $\alpha$  (roll off factor). The value range is taken as 0 to 1 with regular interval of 0.2. from analysis we observe that the PAPR performance is improving if we increase the value of  $\alpha$ . the parameter taken for simulation are  $N=256, M= 64$  and  $S=4$  [12][13][14].



**Figure 7. Analysis of PAPR Performance Using Pulse Shaping**

Figure. 7 clearly shows that the PAPR performance analysis completely depends upon  $\alpha$ . Now we analyze the PAPR performance using DFT technique which is function of  $M$  (Number of subcarrier).



**Figure 8. PAPR Performance Analysis with M**

Subcarrier which are allotted to the user. From this analysis we observe that the PAPR performance is degraded while we increase the value of subcarrier. Where the value of  $M$

is vary from 4 to 128. Figure. 8 shows that the PAPR performance variation with the subcarrier.

## 7. Conclusion

In this paper we describe the method to achieve high rate transmission with low power. Both the analysis done in this paper are based on discrete Fourier transform technique which is very easy and efficient solution for PAPR performance improvement as well as channel estimation in communication. For channel estimation paper describe the least square estimation and discrete Fourier transform technique. It is observed that signal performance is improved by using our discrete Fourier transform technique. The second analysis describe the improvement of PAPR performance using discrete Fourier transform technique.

## References

- [1] J.-J. Van De Beek, O. Edfors, M. Sandell, S. K. Wilson and P. O. Borjesson, "OFDM Channel Estimation by Singular Value Decomposition", *IEEE Trans. on Communications*, vol. 46, no.7, (1998), pp. 931-939.
- [2] C. Fragouli, N. Al-Dhahir and W. Turin, "Training-based channel estimation for multiple-antenna broadband transmissions", *IEEE Transactions on Wireless Communications*, vol. 2, (2003), pp. 384-391.
- [3] M. Hsieh and C. Wei, "Channel estimation for OFDM systems based on comb-type pilot arrangement in frequency selective fading channels", *IEEE Trans. Consumer Electronics*, vol. 44, no. 1, (1998).
- [4] J.-J. van de Beek, "On channel estimation in OFDM systems", *Proc. IEEE VTC*, (1996), pp. 815-819.
- [5] L.C. Cimini, "Analysis and simulation of a digital mobile channel using orthogonal frequency-division multiplexing", *IEEE Trans. Commun.*, vol. 33, (1995), pp. 665-675.
- [6] Zhao, Y. and Huang, "A novel channel estimation method for OFDM mobile communication systems based on pilot signals and transform-domain processing", *IEEE VTC*, vol. 46, (1998), pp. 931-939.
- [7] H. Minn and Bhargava, "An investigation into time-domain approach for OFDM channel estimation", *IEEE Trans. on Broadcasting*, vol. 45, no. 4, (1999), pp. 400-409.
- [8] V. de Beek, "Analysis of DFT-based channel estimators for OFDM", *Personal Wireless Commun*, vol. 12, no. 1, (2000), pp. 55-70.
- [9] F.G. Garcia, "DFT-based channel estimation in 2D-pilot-symbol-aided OFDM wireless systems", *IEEE VTC*, vol.2, (2001).
- [10] H. Atarashi, S. Abeta and M.Sawahashi, "Variable spreading factor orthogonal frequency and code division multiplexing (VSF-OFCDM) for broadband packet wireless access", *IEICE Trans. Comm.*, vol. E86-B, (2003), pp. 291-299.
- [11] R. Kimura and F. Adachi, "Comparison of OFDM and multicode MC-CDMA in a frequency selective fading channel", *IEEE Electronics Letters*, vol. 39, no.3, (2003), pp. 317-318.
- [12] Z. Wang and G.B. Giannakis, "Complex-field coding for OFDM over Fading Wireless Channels", *IEEE Trans. Inform. Theory*, vol. 49, (2003).
- [13] W.G. Jeon, K.H. Chang and Y.S. Cho, "An adapti predistorter for compensation of nonlinear distortion in OFDM system", *IEEE Trans*, (1997), pp. 1167-1171.
- [14] D. Galda and Rohling, "A structure for OFDM-FDMA uplink system", *IEEE*, (2002), pp. 1737-1741.
- [15] "The MathWorks Inc.", *MATLAB User's Guide*, (2012).