Performance Analysis of SEC-SC System

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Abstract

In order to achieve the balance between implementation complexity and system performance, a new hybrid combining technique named SEC-SC is proposed in this paper, which is composed of switch-and-examine combining (SEC) and selection combining (SC). The average symbol error probability (ASEP) and outage probability (OP) of the SEC-SC system over N-Nakagami fading channels is investigated. The exact ASEP expressions are derived for several modulation schemes, including phase shift keying (PSK), quadrature amplitude modulation (QAM), and pulse amplitude modulation (PAM). The exact closed-form OP expressions are also presented. Then the ASEP and OP performance under different conditions is evaluated through numerical simulations to verify the analysis. The simulation results showed that the performance of the SEC-SC system is improved with the diversity branches and the fading coefficient increased, but the level of improvement is declined as the number of cascaded components increased.

Keywords: hybrid combining; switch-and-examine combining; selection combining; N-Nakagami fading channels; average symbol error probability; outage probability

1. Introduction

In recent years, mobile application development is swiftly expanding because users prefer to continue their social, entertainment, and business activities while on the go. The demand for higher capacity in mobile communication systems is motivated by the presence of service hot-spots and the widespread of content-rich web-based applications. Mobile-to-mobile (M2M) communication has attracted great attention from both academic and industrial fields [1]. It is widely employed in many popular wireless communication networks, such as mobile ad-hoc networks, vehicle-to-vehicle networks, and LTE-Advanced (LTE-A) cellular network [2]. However, the classical Rayleigh, Rician, or Nakagami fading channels have been found not to be applicable in M2M communication. When both the transmitter and receiver are in motion, the double-Rayleigh fading model has been found to be applicable [3]. Moreover, this model has been extended to double-Nakagami fading model in [4]. Afterwards, N-Nakagami channel is adopted to provide a realistic description of the M2M channel in [5].

Multiple-input multiple-output (MIMO) technology can improve the spectral efficiency and reliability of wireless communications over fading channels, and has been widely employed in M2M communication systems [6-8]. It has been actively studied and
considered in the standardization process of next-generation Broadband Wireless Access Networks (BWANs) such as Third Generation Partnership Project (3GPP) Long Term Evolution (LTE)-Advanced and IEEE 802.16m [9]. MIMO technology includes maximal ratio combining (MRC), equal gain combining (EGC), and selection combining (SC). Using MRC and EGC techniques, channel state information is required, which is difficult to obtain. SC is impractical for systems that have uninterrupted transmission, such as frequency division multiple access (FDMA) systems. Hence, switched diversity combining (SDC) is often implemented to solve this problem [10]. SDC techniques include switch-and-stay combining (SSC) and switch-and-examine combining (SEC). Using the moment generating function (MGF) approach, the average bit error rate (BER) performance of a dual SSC diversity receiver over correlated generalized-K (KG) fading channels was analyzed in [11]. In [12], exact closed-form analytical expressions for the average bit error probability (BEP) of multi-branch switched diversity systems over independent and identically Nakagami-m distributed fading channels were derived. Using the MGF method, the average BER performance of a dual-branch SSC diversity system over k-μ fading channels was investigated in [13].

SSC is feasible for practical application, but this is at the expense of system performance. In order to meet the trade-off between performance and complexity, a new hybrid diversity strategy named SSC-SC was designed in [14]. The closed-form expressions for BER of the SSC-SC system over Rayleigh fading channels were derived. However, SSC merely applies to dual-branch systems, restricting its practicability. To overcome the above drawbacks, we extend the SSC scheme to multi-branch systems. A new hybrid combining technique named SEC-SC is proposed in this paper, which is composed of SEC and SC. To the best knowledge of the author, the performance of SEC-SC system over N-Nakagami fading channels is not available in the literature. In the present work, using the MGF method, the exact average symbol error probability (ASEP) expressions are derived. The exact closed-form outage probability (OP) expressions are also presented. The main contributions are listed as follows:

1. Closed-form expressions are provided for the probability density function (PDF) and cumulative density functions (CDF) of the signal-to-noise ratio (SNR) over N-Nakagami fading channels. These are used to derive exact ASEP and OP expressions for SEC-SC system.
2. The accuracy of the analytical results under different conditions is verified through numerical simulation. Results are presented which show that the performance of SEC-SC system is improved with the diversity branches and the fading coefficient increased, but the level of improvement is declined as the number of cascaded components increased.
3. The derived ASEP and OP expressions can be used to evaluate the performance of the vehicular communication systems such as inter-vehicular communications, intelligent highway applications and mobile ad-hoc applications.

The rest of the paper is organized as follows. The SEC-SC system model is presented in Section 2. Section 3 provides the exact ASEP expressions for several modulation schemes, including phase shift keying (PSK), quadrature amplitude modulation (QAM), and pulse amplitude modulation (PAM). The exact closed-form OP expressions are presented in Section 4. Section 5 conducts Monte Carlo simulations to illustrate the ASEP and OP performance. Concluding remarks are given in Section 6.

2. **System Model**

\[ Z \sim N-Nakagami \]  

Z follows N-Nakagami distribution, which is given as [5]

\[ Z = \prod_{i=1}^{N} a_i \]  

(1)
where \( N \) is the number of cascaded components, and \( a_i \) is a Nakagami distributed random variable with PDF as
\[
f(a) = \frac{2m^m}{\Omega^m \Gamma(m)} a^{2m-1} \exp\left(-\frac{m}{\Omega} a^2\right)
\]  
(2)

\( \Gamma(\cdot) \) is the Gamma function, \( m \) is the fading coefficient and \( \Omega \) is a scaling factor.

According to [5], the PDF of \( Z \) is given as
\[
f(z) = \frac{2}{\zeta \prod \Gamma(m_i)} G_{N,0,0}^{N,0} \left[ z^2 \prod_{i=1}^{N} \frac{m_i}{\zeta \Omega_i} \right]
\]  
(3)

where \( G[\cdot] \) is Meijer’s \( G \)-function.

We will assume that there are \( L_1 \times L_2 \) diversity branches, they are subject to independently and identically distributed (i.n.i.d) \( N \)-Nakagami fading. The instantaneous SNR for each branch is given by
\[
\gamma = \left| Z \right|^2 \frac{E_s}{N_0}
\]  
(4)

where \( E_s \) is the transmitted symbol’s average energy and \( N_0 \) is the single-sided additive white Gaussian noise (AWGN) power spectral density.

The corresponding average SNR is given as
\[
\gamma = E\left(\left| Z \right|^2 \frac{E_s}{N_0}\right)
\]  
(5)

where \( E() \) denotes expectation.

The CDF of \( r \) can be derived as [5]
\[
F(r) = \frac{1}{\zeta \prod \Gamma(m_i)} G_{1,N+1,0}^{N,1,0} \left[ \frac{r}{\zeta} \prod_{i=1}^{N} m_i \right]
\]  
(6)

The corresponding PDF can be obtained as [5]
\[
f(r) = \frac{1}{\zeta \prod \Gamma(m_i)} G_{0,N,0}^{N,0,0} \left[ \frac{r}{\zeta} \prod_{i=1}^{N} m_i \right]
\]  
(7)

Let us consider a SEC-SC system operating over the \( N \)-Nakagami fading channels. Firstly, \( L_1 \) diversity branches separated by \( L_2 \) antennas are through SEC, for example, \( i, i+L_2, \ldots, i+(L_1-1)L_2, (i=1,2,\ldots, L_2) \). There are \( L_2 \) SEC combiners. The CDF of \( \gamma_{SEC} \) can be given as [10]
\[
F_{SEC}(r) = \begin{cases} 
[F(r_{th})]^{L_{SEC}-1} F(r) & 0 < r < r_{th} \\
[F(r_{th})]^{L_{SEC}} [F(r) - F(r_{th})] + [F(r_{th})]^{L_{SEC}} & r \geq r_{th}
\end{cases}
\]  
(8)

where \( r_{th} \) is a given switching threshold.

The PDF of \( \gamma_{SEC} \) can be given as [10]
\[ f_{\text{SEC}}(r) = \begin{cases} \left[ F(r_{th}) \right]^{L_z-1} f(r) & 0 < r < r_{th} \\ \frac{[F(r_{th})]^{L_z-1} - 1}{F(r_{th})} f(r) & r \geq r_{th} \end{cases} \] (9)

After SEC, there are \( L_2 \) diversity branches. The \( L_2 \) diversity branches are through SC. The SNR \( \gamma_{\text{SEC}} \) at the output of the SC combiner can be given as [15]

\[ \gamma_{\text{SEC}} = \max (\gamma_1, \gamma_2, \ldots, \gamma_{L_2}) \] (10)

So the CDF of \( \gamma_{\text{SEC-SC}} \) can be given as

\[ F_{\text{SEC-SC}}(r) = \left( F_{\text{SEC}}(r) \right)^{L_2} \]

\[ = \begin{cases} \left[ F(r_{th}) \right]^{L_2 - L_z} \left[ F(r) \right]^{L_z-1} f(r), & 0 < r < r_{th} \\ \left[ \frac{[F(r_{th})]^{L_z-1} - 1}{F(r_{th})} \right] \left( F(r) - F(r_{th}) \right) + [F(r_{th})]^{L_z-1} f(r), & r \geq r_{th} \end{cases} \] (11)

By taking the first derivative of (11) with respect to \( r \), the corresponding PDF of \( \gamma_{\text{SEC-SC}} \) can be given as

\[ f_{\text{SEC-SC}}(r) = \begin{cases} L_2 \left[ F(r_{th}) \right]^{L_2-L_z} \left[ F(r) \right]^{L_z-1} f(r), & 0 < r < r_{th} \\ L_2 \left[ \frac{[F(r_{th})]^{L_z-1} - 1}{F(r_{th})} \right] \left( F(r) - F(r_{th}) \right) + [F(r_{th})]^{L_z-1} f(r), & r \geq r_{th} \end{cases} \] (12)

The MGF of \( \gamma_{\text{SEC-SC}} \) can be given as

\[ M_{\text{SEC-SC}}(s) = \int_0^\infty e^{-sr} f_{\text{SEC-SC}}(r) dr \]

\[ = I_1 + I_2 \]

Next, the \( I_1 \) is evaluated.

\[ I_1 = \left( \frac{L_2}{\prod_{i=1}^S \Gamma(m_i)} \right) \left[ G_{S,S}^{N,N,0} \left[ \begin{array}{c} r_{th} \\ \frac{r}{T} \prod_{i=1}^N m_i \end{array} \right] \right]^{L_2-L_z} \times \]

\[ \int_0^{r_{th}} e^{-sr} \left[ G_{S,S}^{N,N,0} \left[ \begin{array}{c} r \frac{N}{T} \prod_{i=1}^{N-1} m_i \end{array} \right] \right]^{L_z-1} dr \]

\[ = \left( \frac{L_2}{\prod_{i=1}^S \Gamma(m_i)} \right) \left[ G_{S,S}^{N,N,0} \left[ \begin{array}{c} \frac{r_{th}}{T} \prod_{i=1}^N m_i \end{array} \right] \right]^{L_2-L_z} \]

\[ \times \left( \frac{L_2}{\prod_{i=1}^S \Gamma(m_i)} \right) \left[ G_{S,S}^{N,N,0} \left[ \begin{array}{c} \frac{r}{T} \prod_{i=1}^{N-1} m_i \end{array} \right] \right]^{L_z-1} \]

\[ = \left( \frac{L_2}{\prod_{i=1}^S \Gamma(m_i)} \right) \left[ G_{S,S}^{N,N,0} \left[ \begin{array}{c} \frac{r_{th}}{T} \prod_{i=1}^N m_i \end{array} \right] \right]^{L_2-L_z} \]

(14)
Next, the $I_2$ is evaluated.

$$I_2 = s \int \frac{1}{\Gamma (m_i)} G_{N+1}^{k} \left[ \frac{r_k}{\gamma} \prod_{i=1}^{N} m_i \left| \frac{w_{1}}{w_{2}} \right| \right] e^{-t} dt$$

$$- 1$$

$$\times \left[ G_{N+1}^{k} \left[ \frac{r_k}{\gamma} \prod_{i=1}^{N} m_i \left| \frac{w_{1}}{w_{2}} \right| \right] - G_{N+1}^{k} \left[ \frac{r_k}{\gamma} \prod_{i=1}^{N} m_i \left| \frac{w_{1}}{w_{2}} \right| \right] \right]$$

$$+ \frac{1}{\Gamma (m_i)} G_{N+1}^{k} \left[ \frac{r_k}{\gamma} \prod_{i=1}^{N} m_i \left| \frac{w_{1}}{w_{2}} \right| \right] e^{-t} dt$$

$$\left( \begin{array}{c} 1 \frac{1}{\Gamma (m_i)} G_{N+1}^{k} \left[ \frac{r_k}{\gamma} \prod_{i=1}^{N} m_i \left| \frac{w_{1}}{w_{2}} \right| \right] \end{array} \right)$$

(15)

3. Average Symbol Error Probability

The ASEP obtained by the MGF method is given as [16]

$$P_{\text{ASEP}} = \sum_{d=1}^{D} E_d \int_{0}^{\theta_d} M_{\text{SEC-SC}} \left( \frac{\varphi_d}{V_d - 2 \Lambda_d \sin^2 \theta} \right) d \theta$$

(16)

For $q$-ary PAM modulation, the ASEP is given by

$$P_{\text{ASEP}} = \frac{2(q-1)}{\pi q} \int_{0}^{\pi/2} M_{\text{SEC-SC}} \left( \frac{3}{q^2-1} \sin^2 \theta \right) d \theta$$

(17)

For $q$-ary PSK modulation, the ASEP is given by

$$P_{\text{ASEP}} = \frac{1}{\pi} \int_{0}^{(q-1)\pi/q} M_{\text{SEC-SC}} \left( \frac{\sin^2 \left( \frac{\pi}{q} \right)}{\sin^2 \theta} \right) d \theta$$

(18)

For $q$-ary QAM modulation, the ASEP is given by

$$P_{\text{ASEP}} = \frac{4q}{\pi q} \int_{0}^{\pi/2} M_{\text{SEC-SC}} \left( \frac{3}{2(q-1)\sin^2 \theta} \right) d \theta$$

(19)
4. Outage Probability

The OP is given by

$$P_{\text{out}} = \Pr \left[ 0 \leq r \leq r_T \right] = \int_{0}^{r_T} f_{\text{SEC-SC}}(r) \, dr$$

(20)

where \( r_T \) is a given threshold for correct detection.

Substituting (12) into (20) results in

$$P_{\text{out}} = \left\{ \begin{array}{ll}
\left[ F(t_0) \right]^{l_1} F(t_1) & .0 < r_T < r_b \\
\left[ F(t_0) \right]^{l_1} - \frac{1}{\Gamma(\alpha)} \prod_{\gamma=1}^{\alpha} \left[ \frac{\gamma}{\gamma-1} \right] & , r_1 \geq r_b
\end{array} \right.$$  

$$\left[ F(t_1) - \left[ F(t_0) \right]^{l_1} \right]^{l_1} . \left[ \frac{\gamma}{\gamma-1} \right] = \frac{1}{\Gamma(\alpha)} \prod_{\gamma=1}^{\alpha} \left[ \frac{\gamma}{\gamma-1} \right]$$

5. Numerical Results

In this section, we present Monte-Carlo simulations to confirm the derived analytical results. Additionally, random number simulation was done to confirm the validity of the analytical approach. All the computations were done in MATLAB and some of the integrals were verified through MAPLE. The fading coefficient is \( m=1, 2, 3 \), the number of cascaded components is \( N=2, 3, 4 \).
Figure 1. The Effect of the Diversity Branches on the OP Performance

Figure 1 presents the impact of the diversity branches on the OP performance of the SEC-SC system over $N$-Nakagami fading channels. The number of cascaded components is $N=2$. The fading coefficient is $m=2$. The switching threshold $r_T$ is 5dB. The given threshold $r_B$ is 2dB. The diversity branches are $(L_1=3, L_2=1)$, $(L_1=3, L_2=2)$, $(L_1=4, L_2=2)$. Simulation results show that the OP performance is improved with the diversity branches increased. For example, when $SNR=8$dB, $(L_1=3, L_2=1)$, the OP is $3.6 \times 10^{-2}$, $(L_1=4, L_2=1)$, the OP is $1.5 \times 10^{-2}$, $(L_1=3, L_2=2)$, the OP is $1.3 \times 10^{-3}$. When the diversity branches are fixed, with the increase of $SNR$, the OP is reduced gradually.

Figure 2 presents the impact of the fading coefficient $m$ on the OP performance of the SEC-SC system over $N$-Nakagami fading channels. The number of cascaded components is $N=2$. The diversity branches are $(L_1=3, L_2=2)$. The fading coefficient is $m=1, 2, 3$. Simulation results show that the OP performance is improved with the fading coefficient $m$ increased. For example, when $SNR=8$dB, $m=1$, the OP is $1.5 \times 10^{-2}$, $m=2$, the OP is $1.3 \times 10^{-3}$, $m=3$, the OP is $1.8 \times 10^{-4}$. This is because the fading severity of an $N$-Nakagami channel is reduced with a larger $m$. When the $m$ is fixed, with the increase of $SNR$, the OP is reduced gradually.

Figure 2. The Effect of the Fading Coefficient $m$ on the OP Performance
Figure 3. The Effect of the Number of Cascaded Components $N$ on the OP Performance

Figure 3 presents the impact of the number of cascaded components $N$ on the OP performance of the SEC-SC system over $N$-Nakagami fading channels. The number of cascaded components is $N=2$, 3, 4, which respectively denotes the double-Nakagami, 3-Nakagami, 4-Nakagami fading channels. The diversity branches are $(L_1=3, L_2=2)$. The fading coefficient is $m=2$. Simulation results show that the OP performance is degraded as $N$ increased. For example, when SNR=8dB, $N=2$, the OP is $1.3 \times 10^{-3}$, $N=3$, the OP is $6.6 \times 10^{-3}$, $N=4$, the OP is $1.7 \times 10^{-2}$. This is because the fading severity of the cascaded channels increases as $N$ increased. When the $N$ is fixed, with the increase of SNR, the OP under different fading channels is reduced gradually.

Figure 4. The OP Performance Comparison of SEC, SEC-SC, SC

Figure 4 presents the OP performance comparison of SEC, SEC-SC, SC over $N$-Nakagami fading channels. The number of cascaded components is $N=2$. For SEC-SC, the diversity branches are $(L_1=3, L_2=2)$; for SEC and SC, the diversity branches are $L=6$. The fading coefficient is $m=2$. The switching threshold $r_{th}$ is taken to be 5dB. The given threshold $r_T$ is taken to be 2dB. Simulation results show that the OP performance of SEC-
SC is better than that of SEC. With the increase of SNR, the OP difference between SEC-SC and SC is reduced gradually. But, SEC-SC is more practical than SC.

6. Conclusion

The ASEP and OP performance of the SEC-SC system over $N$-Nakagami fading channels is investigated in this paper. The exact ASEP and OP expressions are presented. The simulation results show that the diversity branches $L_1 \times L_2$, the fading coefficient $m$, and the number of cascaded components $N$ have an important influence on the ASEP and OP performance. The derived ASEP and OP expressions can be used to evaluate the ASEP and OP performance of the vehicular communication systems such as intervehicular communications, intelligent highway applications and mobile ad-hoc applications. In the future, we will consider the impact of the correlated channels on the ASEP and OP performance of the SEC-SC system.

Acknowledgments

The authors would like to thank the referees and editors for providing very helpful comments and suggestions. This project was supported by National Natural Science Foundation of China (No. 61304222, No. 61301139), Natural Science Foundation of Shandong Province (No.ZR2012FQ021), Shandong Province Outstanding Young Scientist Award Fund (No. 2014BSE28032), Fundamental Research Funds for the Central Universities (No. 14CX02139A).

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