

Optimal Threshold of LTE-Femtocell Network Based Bayes-Nash Equilibrium Theory

Hao Chen¹, Kan Liu^{2*} and Wu Tunhua³

¹*School of Computer and Information Engineering, Tianjin Chengjian University, Tianjin 300384, China*

²*Department of electronic business, College of information technical, Hebei Normal University, No.20, East of 2nd Ring South Road, Shijiazhuang City, Hebei Province, P. R. China*

³*School of Information and Engineering, Wenzhou Medical University, Wenzhou 325035, China*

¹*haochen11144@163.com*, ²*liucau@126.com*, ³*wth@zjut.edu.cn*

Abstract

To increase LTE (long time evolution) networks spectrum utilization and interference mitigation, a LTE system overlaid with femtocells is studied. This paper will focus a self-optimized power control scheme for LTE-femtocell networks, in which the transmitted power of a femtocell base station is adjusted based on the optimal SINR threshold. It is known that game theory is a useful tool for analyzing outage probabilities and optimal power in wpszireless networks. In this paper, Bayes-Nash equilibrium theory is used to derive a optimal SINR (signal-interference-noise-ratio) threshold from each femtocell. The power control scheme can be applied to realistic LTE-femtocell networks to enable robust communication against cross-tier interference thereby obtaining a substantial link quality.

Keywords: *LTE-femtocell, game theory, Bayes-Nash equilibrium*

1. Introduction

LTE is being standardized by 3GPP to provide multi-megabit bandwidth, more efficient use of the radio network, latency reduction, improved mobility, and potentially lower cost per bit[1]. Femtocells are low-power access points that operate in licensed spectrum and provide mobile coverage and capacity over internet-grade backhaul. In order to improve the LTE network throughputs and spectrum efficiency, LTE-femtocell two-tiered networks[2-5] have been studied. Conventional power control work ties in cellular networks and prior work on utility optimization based on game theory. Results in Foschini [6], Zander [7], Grandhi [8] and Bambos [9].

2. Contribution

Prior work about femtocell power control has proposed to use the utility-based non-cooperative femtocell SINR adaptation [10]. In that literature, SINR threshold of each femtocell is pre-established. And then Nash equilibrium can be calculated. But in reality, much information is incomplete information and games are asymmetry. The key contributions in our paper is that use incomplete information games theory—Bayes-Nash equilibrium theory to study two-tiered femtocell power control. Bayes-Nash equilibrium theory is employed to find optimal SINR threshold. Owing to the Bayes-Nash equilibrium theory, the adaptation minimum SINR targets can be found. An optimal channel-dependant SINR threshold is obtained at each femtocell.

3. System Model

The system consists of a single central macrocell B_0 serving a region C , providing a cellular coverage radius R_c . B_i , $1 \leq i \leq N$. The LTE macrocell is underlaid with N co-channel femtocells APs. The system module is shown in Figure 1.

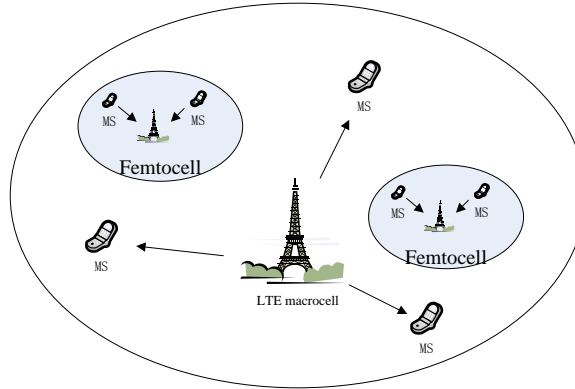


Figure 1. System Module

Femtocell users are located on the circumference of a disc of radius R_f centered at their femtocell AP. Orthogonal uplink signaling is assumed in each slot (1 scheduled active user per cell during each signaling slot), where a slot may refer to a time or frequency resource (the ensuing analysis leading up to Theorem 1 apply equally well over the downlink). During a given slot, let $i \in \{1, 2, \dots, N\}$ denote the scheduled user connected to its BS B_i . Designate user i 's transmitting power to be p_i Watts. Let σ^2 be the variance of AWGN (Additive White Gaussian Noise) at B_i .

Definition 1 The received SINR γ_i of user i at B_i is given as

$$\gamma_i = \frac{p_i g_{i,i}}{\sum_{j \neq i} p_j g_{i,j} + \sigma^2} \geq \Gamma_i \quad (1)$$

Where Γ_i represents the SINR threshold for user i at B_i . The term $g_{i,j}$ denotes the channel gain between user j and BS B_i , but it really is interference term for user i at B_i . The term $g_{i,i}$ can also account for post-processing SINR gains.

Definition 2 The term $I_i(p_{i-})$ represents the interference value of user j ($j \neq i$) at B_i . In order to accord with the terms of game theory, $i-$ denotes element sets other than i .

$$I_i(p_{i-}) \triangleq \sum_{j \neq i} p_j g_{i,j} + \sigma^2 \quad (2)$$

Using Eq.(1) and Eq.(2), the received SINR γ_i can be rewritten in Eq.(3)

$$\gamma_i = \frac{p_i g_{i,i}}{I_i(p_{i-})} \quad (3)$$

4. The Optimal SINR Threshold with Incomplete Information

Auction game is one type of Bays-NE theory. It will be applied to find the optimal SINR threshold solution, that is Bayes-NE solution. The incomplete information factors mainly include: whether one user transmitting signals or not and transmitting signal power. The two factors completely are random. In order to conveniently analyze, we assume that a few conditions that are also very close to the actual conditions. Each femtocell BS transmitted power P_i is a random variable. The term p_i denotes a numerical value with a random variable mappings to a given sample space. Γ_i^* is a optimal solution of Γ_i . We need to find an optimal SINR threshold Γ_i^* .

When Bayes-NE can be arrived, strategy function S_i is given as

$$S_i(p_i) = \Gamma_i^* \tag{4}$$

In the section, our task is to calculate the strategy function set $S_{set} = \{S_1, S_2, \dots, S_N\}$, when all elements of LTE-femtocell networks have arrived Bayes-NE.

Assuming 1: within any FAP coverage area, the probability that every user whether transmitting signals or not is independent identically distributed (i.i.d). However, for different femtocell systems (FAP and femtocell users), the distribution function may be different. The distribution function is user transmitting signal power function.

$$\begin{cases} \text{Distribution Function: } F(p_i) & (\text{don't transmitting signals}) \\ \text{Distribution Function: } 1 - F(p_i) & (\text{transmitting signals}) \end{cases}$$

Definition 3: The term $P_r(\cdot)$ denotes the probability. The term $F(\cdot)$ denotes distribution function. The relation of two functions is given as

$$P_r(X \leq x) = F_x(x) \tag{5}$$

Assumption 2: In order to maximize the interference mitigation, in every unit time, only one femtocell BS of max power is in transmitting state.

Assumption 3: For any user, only when its transmitting power is maximum of all of FAP receiving signals, this user can take optimization effect on FAP receiver SINR threshold.

The optimization model of FAP receiver threshold is based auction game model. If all the users transmit signal, and the user transmitting power maximum, this event probability as follows:

$$Y = \underset{j \in i-}{Max} P_j \tag{6}$$

$$P_r(p_i > Y) = [1 - F(p_i)]^{N-1} \tag{7}$$

Eq.(7) indicates that the i^{th} transmitting power is bigger than other users transmitting power.

Definition 4: Strategy function of femtocell users

$$\text{Strategy Function} = \begin{cases} 0 & \text{The user can not take optimization effect on FAP receiver threshold} \\ \gamma_i - \Gamma_i & \text{The user can take optimization effect on FAP receiver threshold} \end{cases}$$

Combining strategy function with Eq.(7), the utility function of i^{th} user optimizing FAP receiver threshold can be expressed on Eq.(8).

Definition 4: The term U_i represents Bays-NE utility function, and it is given as

$$U_i = [1 - F(p_i)]^{N-1} (\gamma_i - \Gamma_i) \quad 1 \leq i \leq N \quad (8)$$

Eq.(8) physical meaning is: The utility function is expressed that difference of the real FAP receiver SINR and the FAP receiver SINR threshold. The bigger the difference is, the better the FAP receiver communication quality is. Because the more difference can provide greater redundancy for outage communication.

For all $1 \leq i \leq N$ users within any FAP area, the process that they transmit signals is incomplete information game .Eq.(9) and Eq.(10) can optimize the FAP receiver threshold.

$$Max U_i \quad 1 \leq i \leq N \quad (9)$$

$$p_i^* = \arg Max U_i \quad (10)$$

When the power control game reaches Bays-NE, p_i^* is as a symbol for optimal power solution. For all users within any FAP area, their transmitting signal probability distribution is i.i.d. So, when all femtocell system is in equilibrium, optimal power p_i^* is the same value. However, for different FAP, the optimal solution may be different. Because the probability distribution may be different. For example, for different femtocell users, the probability that whether users transmitting signals or not is Bernoulli distribution or Poisson distribution. So, for different FAP, the optimal threshold is also different. Our objective is to deduce the optimal SINR threshold Γ_i^* function by using p_i^* . Using Eq. (3), Eq.(8) and Eq.(10), Eq.(11) and Eq.(12) are derived as follows:

$$p_i^* = \arg Max [1 - F(p_i)]^{N-1} \left(\frac{g_{i,i} p_i}{I_i(p_{i-})} - \Gamma_i \right) \quad p_i \in [0, p_{Max}] \quad (11)$$

$$p_i^* = \arg Max [1 - F(p_i)]^{N-1} (G_i p_i - \Gamma_i) \quad p_i \in [0, p_{Max}] \text{ and } G_i \triangleq \frac{g_{i,i}}{I_i(p_{i-})} \quad (12)$$

Eric Maskin and John Riley have proved Eq.(12) that Bayes-Nash equilibrium exists only for optimal solution[11-12]. Taking the first-order derivative of U_i with respect to p_i and applying Eq.(8), the optimal solution can be derived. Moreover, when $p_i = p_i^*$, Eq.(4) will be set up. At the same time, it means that differentiating with respect to p_i of Γ_i^* can be achieved

$$\frac{\partial u_i}{\partial p_i} = (N-1)[1 - F(p_i^*)]^{N-2} (-1) f(p_i^*) [G_i p_i^* - S(p_i^*)] + [1 - F(p_i^*)]^{N-1} [G_i - S'(p_i^*)] = 0 \quad (13)$$

where $f(p_i^*)$ denotes pdf of p_i and $S'(p_i^*)$ represents the first-order derivative of Γ_i^* w.r.t p_i . Since $[1 - F(p_i^*)]^{N-1} > 0$, this term can be removed from both sides of Eq.(13), yields

$$S'(p_i^*) + M(p_i^*) S(p_i^*) = G_i [M(p_i^*) p_i^* + 1] \quad (14)$$

$$\text{and } M(p_i^*) \triangleq (1 - N) [1 - F(p_i^*)]^{-1} f(p_i^*)$$

Lemma 1: (The general solution of first-order linear non-homogeneous differential equation)

If the normal form of differential equation: $y' + p(x)y = q(x)$, then the general solution

is as follows:

$$y = e^{-\int p(x)dx} \left(\int q(x)e^{\int p(x)dx} dx + C \right) \quad (15)$$

where C is a constant coefficient.

The lemma 1 provides a most important method to solve Eq.(14). So Eq.(16) shows as follows, that is the solution of Eq.(14).

$$\Gamma_i^* = S(p_i^*) = G_i \{ [(1 - F(p_i^*))^{-1} + [1 - F(p_i^*)]^{1-N} \int_0^{p_i^*} [1 - F(t)]^{N-1} dt + C \} \quad (16)$$

According to physical meanings, it is assumed as $p_i^* = 0, S(p_i^*) = 0$. $C = 1$ can be derived. Therefore, the strategy set function Γ_i^* is rewritten as follows :

$$\Gamma_i^* = S(p_i^*) = G_i \{ [(1 - F(p_i^*))^{-1} + [1 - F(p_i^*)]^{1-N} \int_0^{p_i^*} [1 - F(t)]^{N-1} dt + 1 \} \quad (17)$$

As a result, when Eq.(4) has been explicitly determined ,the strategy function set S_{set} will consists of different values of Eq.(17) according to parameters of each femtocell . It will ensure each femtocell further to optimize their power control, spectrum utilization and mitigate interference.

5. Simulation Result and the Analysis

In this section, we will simulate the result of utilizing optimal threshold to power control in matlab 7.0 platforms. We list the parameters Table, give the results of Bays-Nash based experiments. Firstly, some parameters are given. Secondly, simulation result is illustrated.

Table 1. System Parameters

Variable Signal	Parameter (Unit)	Value
R	Femtocell Radius (m)	30
f	FAP carrier frequency(GHz)	2
P_{max}	Femtocell user transmission power(W)	10
M	pseudo random cycle number By Monte Carlo method	100
K	power iterative number	100
T	FAP number in two-tier networks	20
N	femtocell user number in a FAP area	10
F(x)	Whether femtocell user transmission signal or not	uniform distribution
n	Number of every femtocell user transmission signal	10

q	Failure possibility of transmission signal	0.3
k	Failure Number of transmission signal of every user in one FAP area	4
Γ_{\max}	Max SINR threshold	$\Gamma^*+2\text{dB}$
Γ_{\min}	Min SINR threshold	$\Gamma^*-2\text{dB}$
β_i	cost factor	0.1
σ^2	additive gaussian white noise power (W)	1×10^{-4}

Assuming that each FAP user sends a signal is uniform distribution, and each user is independent and identically distribution.

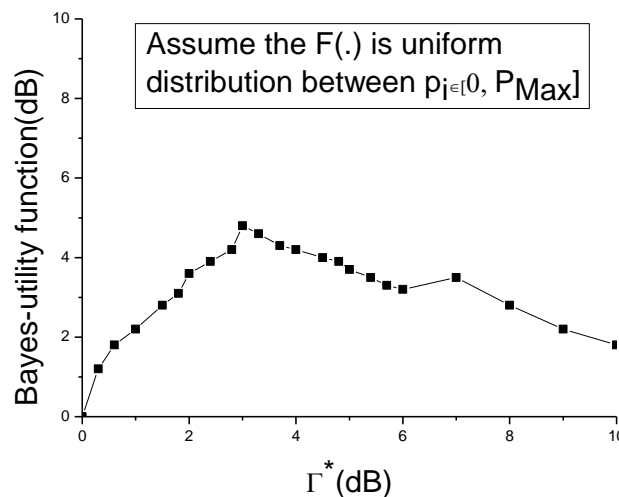


Figure 2. The Optimal Sinr Threshold $p_i \in [0, P_{\max}]$

Following the experiments – Monte Carlo simulation, we use the Bayes-utility function to solve the optimal SINR threshold. From Fig.2, it shows that the Γ^* exists only value, when the parameter is given, such as G_i , $F(\cdot)$ and N .

6. Conclusion

In this paper, the power control and interference mitigation issues of femtocells in two-tier LTE macro-femto networks are discussed. A novel power control scheme is proposed based on Bayes-Nash equilibrium and iterative algorithm. When Bayes-Nash equilibrium are used in femtocells, the optimal SINR target and optimal transmit power can be derived, which is critical to mitigate interference between neighboring femtocells and improve iterative algorithm efficiency. The simulation results show that, by using the presented scheme, optimal SINR target and optimal transmit power of femtocells can be obtained and sufficient SINR to mitigate interference can be provided. In conclusion, the suggested scheme can make femtocell power control more efficient than that of the power

control without using Bayes-Nash equilibrium.

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Authors



Hao Chen received Ph.D degree from Tianjin University at School of Electronic and Information Engineering, in 2012. His current research interests include wireless communication, interference of different communication system and power control, at the same time, he is also a teacher of School of Computer and Information Engineering, Tianjin Chengjian University.

Kan Liu received B.S degree and M.S in College of Management & Economics from China Agriculture University in 2006 and 2008, respectively. At present, she is a lecturer at Hebei Normal University. Her current research interests are focus on E-business and Network theory.

Wu Tunhua is a teacher of School of Information and Engineering, Wenzhou Medical University.

