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# A High Spectral Efficiency CDMA System Based on Expanded Generalized Complementary Orthogonal Code Groups

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***Abstract***

*A new CDMA system which uses fractional chip shift expanded generalized complementary orthogonal code groups as spectrum spreading code sequences and offer high spectral efficiency is introduced in the paper. This new CDMA system is totally different with conventional CDMA system in access codes designing and spectrum spreading process. In this CDMA system, the access codes of different cells keep orthogonality, which eliminates inter cells interferences (ACI) by aperiodic cross-correlation function (CCF). The spreading process of new CDMA system introduces inter symbols interference like overlap time division multiplexing (OVTDM) deliberately so as to improve spectral efficiency. The construction of expanded generalized complementary orthogonal code groups, shift method, principle of new CDMA system are explained respectively. Analysis and simulation results verify the validity of this new CDMA system.*

**Keywords:** *Expanded Generalized Complementary Orthogonal Code Groups, Fractional Chip Shift, Multi Codes Maximum Likelihood Detection*

## 1. Introduction

In conventional code division multiple access (CDMA) system, each user will be assigned a unique code sequence and signals from various users are separated at the receiver by cross correlation of the received signal with each of the possible user code sequence. [1] Theoretically, CDMA system requires such perfect code sequence set of which auto correlation (except the origin) and cross correlation are zero for any relative time shift. However, Welch [2] and Sarwate [3] concluded that such perfect code sequence set does not exist. Besides adjacent cell interference (ACI), multiple access interference (MAI) and inter symbol interference (ISI) are also caused by imperfect code sequence set, which make conventional CDMA system have to introduce multi-user detection [4] to overcome MAI and ISI. Furthermore, the spectrum spreading of the conventional CDMA system can not improve spectral efficiency itself because it requires fully orthogonality both in adjacent symbols and in different code sequences.

From Golay's concept of "complementary series" [4] and Schweitzer's concept "generalized complementary code sets"[4], the perfect complete complementary orthogonal code sequence set with ideal cross correlation and auto correlation can be constructed and the perfect complete complementary orthogonal code sequence set can be expanded by a matrix to form expanded generalized complementary orthogonal code groups. In general, cross correlation of code sequence from different expanded generalized complementary orthogonal code groups is still ideal and auto correlation of code sequences from same expanded generalized complementary orthogonal code groups is not ideal. Different cells will be assigned different expanded generalized complementary orthogonal code group with ideal cross correlation to eliminate ACI and

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each cell sequence code in same expanded generalized complementary orthogonal code groups group will be fractional chip shifted not only to generate more access codes but also to improve spectral efficiency.

The rest of this paper is organized as follows: Section II gives an introduction of expanded generalized complementary orthogonal code groups. Section III describes the principle and model of new CDMA system. Section IV includes simulation result and analysis. Section V is the conclusion of the paper.

## 2. Introduction of Expanded Generalized Complementary Orthogonal Code Groups

Expanded generalized complementary orthogonal code groups are derived from perfect complete complementary orthogonal code pairs mate. Before introducing expanded generalized complementary orthogonal code groups, perfect complete complementary orthogonal code pairs mate should be explained first for it is the base of all other complementary orthogonal code. Then definition of perfect complete generalized complementary orthogonal code groups and expanded generalized complementary orthogonal code groups are given respectively.

### 2.1. Perfect Complete Complementary Orthogonal Code Pairs Mate

In the formula:

$$\mathbf{b}_k = \mathbf{C}_k [+] \mathbf{S}_k, k = 0,1$$

$\tilde{b}_k \triangleq [\tilde{b}_{k,0}, \cdots, \tilde{b}_{k,N_0-1}]$ is a normalized code vector and its dimensions is $N_0$, the meaning of normalized is:

$$\|\mathbf{b}_k\| = \|\mathbf{C}_\mathbf{k}\| + \|\mathbf{S}_\mathbf{k}\| = 1$$

The symbol $\|\ \|$ means $\|\mathbf{A}\|^2 \triangleq \sum_{i=0}^{N-1} a_i a_i^H$ where $\mathbf{A} \triangleq [a_0, a_1, \cdots, a_{N-1}]$. Symbol $[+]$ should be noted that it is not traditional addition but the complementary addition. The complementary addition means if there is an operation of $\mathbf{b}_k (k = 0,1)$ itself or between $\mathbf{b}_k (k = 0,1)$, the different name of component codes should be calculated respectively, which means complementary addition not allowing any operation between component code C and S. The operation result is the sum of the operation of component code C and S.

The definition of perfect complete complementary orthogonal code pairs mate is given as follow:

Considering the following formula:

$$\mathbf{b}_k \mathbf{b}_{k'}^H(l) \triangleq \mathbf{C}_k \mathbf{C}_{k'}^H(l) [+] \mathbf{S}_k \mathbf{S}_{k'}^H(l) = \delta_{kk'} . \delta(l)$$

where,

$$\begin{aligned} \delta_{kk'} &\triangleq \begin{cases} 1 & ,k = k' \\ 0 & ,k \neq k' \end{cases} \\ & \qquad\qquad\qquad k, k' = 0,1 \\ \delta(l) &\triangleq \begin{cases} 1 & ,l = 0 \\ 0 & ,l \neq 0 \end{cases} \end{aligned}$$

and

 

$$\mathbf{b}_k(l) \triangleq \begin{cases} [\underbrace{0,\cdots,0}_{l},\tilde{b}_{k,0},\tilde{b}_{k,1},\cdots,\tilde{b}_{k,N_0-l-1}] & l \geq 0 \\ [\tilde{b}_{k,|l|},\tilde{b}_{k,|l|+1},\cdots,\tilde{b}_{k,N_0-1},\underbrace{0,\cdots,0}_{|l|}] & l \leq 0 \end{cases} \qquad |l| = 0,1,\cdots,N_0-1$$

is the shift code vector with aperiodic $l$ time.

That is to say, in the sense of complementary, the aperiodic auto-correlation function (ACF) and aperiodic cross-correlation function (CCF) of $\mathbf{b}_k(k=0,1)$ are all ideal.

### 2.2. Perfect Complete Generalized Complementary Orthogonal Codes

In fact, complementary orthogonal code not only can be composed of two component codes but also can consist of several component codes, which is the concept of generalized complementary orthogonal codes.

Similar to perfect complete complementary orthogonal code pairs mate, the definition of perfect complete generalized complementary orthogonal codes is:

Considering the follow code groups:

$$\tilde{\mathbf{b}}_k = \tilde{\mathbf{b}}_k^0[+]\tilde{\mathbf{b}}_k^1[+]\cdots[+]\tilde{\mathbf{b}}_k^{K-1} = [\sum_{k''=0}^{K-1}]\tilde{\mathbf{b}}_k^{k''},\ k=0,1,\cdots,K-1$$

where $\tilde{\mathbf{b}}_k^{k''} = [\tilde{b}_k^{k''}(0),\tilde{b}_k^{k''}(1),\cdots,\tilde{b}_k^{k''}(N_0-1)]$

$k''=0,1,\cdots,K-1$ are $N_0$ dimensions normalized code vectors, normalized means:

$$\left\|\tilde{\mathbf{b}}_k\right\|^2 = \tilde{\mathbf{b}}_k\tilde{\mathbf{b}}_k^H = \sum_{k''=0}^{K-1}\left\|\tilde{\mathbf{b}}_k^{k''}\right\|^2 = 1\cdot$$

Symbol $[+]$ and $[\Sigma]$ also represent generalized complementary addition.

$$\tilde{\mathbf{b}}_k\tilde{b}_{k'}^H(l) = \sum_{k''=0}^{K-1}\tilde{\mathbf{b}}_k^{k''}\tilde{\mathbf{b}}_{k'}^{k'',H}(l) = \delta_{k.k'}.\delta(l), k,k'=0,1,\cdots,K-1, \quad l=0,1,\cdots,N_0-1$$

where $\delta_{kk'} = \begin{cases} 1 & ,k=k' \\ 0 & ,k\neq k' \end{cases}$ $\quad \delta(l) = \begin{cases} 1 & ,l=0 \\ 0 & ,l\neq 0 \end{cases}$

$$\tilde{\mathbf{b}}_k^{k''}(l) = \begin{cases} [\underbrace{0,0,\cdots\cdots,0}_{l},\tilde{b}_{k,0}^{k''},\tilde{b}_{k,1}^{k''},\cdots,\tilde{b}_{,N_0-l-1}^{k''}] & l \geq 0 \\ [\tilde{b}_{k,|l|}^{k''},\tilde{b}_{k,|l|+1}^{k''},\cdots\cdots,\tilde{b}_{k,N_0-1}^{k''},\underbrace{0,0,\cdots,0}_{|l|}] & l \leq 0 \end{cases} \qquad |l| = 0,1,\cdots,N_0-1$$

The codes mentioned above are perfect complete generalized complementary orthogonal codes.

### 2.3. Expanding Theorem 1 and Perfect Complete Generalized Complementary Orthogonal Code Groups

It is clear that more access codes are required in CDMA system rather than perfect code pairs mate. In sequence design books[7][8]so many methods are introduced to generate perfect complete complementary orthogonal code pairs mate rather than perfect complete generalized complementary orthogonal code groups. The way of generating perfect complete generalized complementary orthogonal code groups is based on following theorem.

Theorem 1: Suppose $G_{N\times N}$ is a Hadamard matrix:

$$G_{N\times N} = \begin{pmatrix} g_{11} & \cdots & g_{1N} \\ \vdots & \ddots & \vdots \\ g_{N1} & \cdots & g_{NN} \end{pmatrix}$$

And $B_2$ is a perfect complete complementary orthogonal code pairs mate (both C and S sequence have $\forall l > 1$ chips):

$$B_2 = \begin{pmatrix} C_1 & S_1 \\ C_2 & S_2 \end{pmatrix}$$

In the following operation, $C_1$, $C_2$, $S_1$ and $S_2$ are treated as basic elements of matrix $B_2$ and $B_2$ is a $2\times 2$ matrix.

$$\mathbf{B}_{2N} = \mathbf{B}_2 \otimes G = [M_1 \quad \cdots \quad M_i \quad \cdots \quad M_{2N}]^{\mathrm{T}} = \begin{bmatrix} g_{11}C_1 \cdots g_{1N}C_1 \quad g_{11}S_1 \cdots g_{1N}S_1 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ g_{N1}C_1 \cdots g_{NN}C_1 \quad g_{N1}S_1 \cdots g_{NN}S_1 \\ g_{11}C_2 \cdots g_{1N}C_2 \quad g_{11}S_2 \cdots g_{1N}S_2 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ g_{N1}C_2 \cdots g_{NN}C_2 \quad g_{N1}S_2 \cdots g_{NN}S_2 \end{bmatrix}$$

$\otimes$ is the symbol of Kronecker product. We get a perfect complete generalized complementary orthogonal code group which comprises $2N$ element codes $M_1 \ \cdots \ M_i \ \cdots \ M_{2N}$. The sequences number of each element code is even and in the sense of complementary, the aperiodic ACF and aperiodic CCF of these codes are perfect. The proof is given as follows:

Proof: $M_i$ and $M_j$ $(1 \le i < j \le N)$ are chosen arbitrarily from upper half rows of $B_{2N}$ and $M_i$ denotes the i-th row of $B_{2N}$ $(1 \le i \le 2N)$

$$M_i = g_{i1}C_1 \cdots g_{iN}C_1 \quad g_{i1}S_1 \cdots g_{iN}S_1$$

$$M_j = g_{j1}C_1 \cdots g_{jN}C_1 \quad g_{j1}S_1 \cdots g_{jN}S_1$$

Then the aperiodic CCF can be calculated as:

$$R_{M_i,M_j}(\tau) = \sum_{q=1}^{N} \{R_{g_{iq}C_1, g_{jq}C_1}(\tau) + R_{g_{iq}S_1, g_{jq}S_1}(\tau)\}.$$

When $0 < \tau \le l-1$,

$$R_{g_{iq}C_1, g_{jq}C_1}(\tau) = \sum_{a=0}^{l-1-\tau} g_{iq}C_{1,a} * g_{jq}^{*}C_{1,a+\tau}^{*} = g_{iq}g_{jq}^{*}R_{C_1,C_1}(\tau)$$

$$R_{g_{iq}S_1, g_{jq}S_1}(\tau) = \sum_{a=0}^{l-1-\tau} g_{iq}S_{1,a} * g_{jq}^{*}S_{1,a+\tau}^{*} = g_{iq}g_{jq}^{*}R_{S_1,S_1}(\tau)$$

As mentioned above, the aperiodic ACF of $(C_1, S_1)$ is perfect,

$$R_{M_i,M_j}(\tau) = 0, \quad 0 < \tau \le l-1.$$

Similarly, we can prove $R_{M_i,M_j}(\tau) = 0$ when $1-l \le \tau < 0$.

Considering $\tau = 0$,

$$R_{M_i,M_j}(\tau) = R_{C_1,C_1}(0)\sum_{q=1}^{N} g_{iq} * g_{jq}^{*} + R_{S_1,S_1}(0)\sum_{q=1}^{N} g_{iq} * g_{jq}^{*} .$$

$G$ is a Hadamard matrix and the follow formula can be got based on properties of Hadamard matrix,

$$\sum_{q=1}^{N} g_{iq} * g_{jq}^{*} = 0 ,$$

then

$$R_{M_i,M_j}(0) = 0 ;$$

If $M_i$ and $M_j$ are not in same half of $B_{2N}$ ,that is to say one in upper half rows, the other in lower half rows of $B_{2N}$ :

$$\begin{cases} M_i, M_j \quad 1 \le i \le N, N+1 \le j \le 2N \\ or \\ M_i, M_j \quad 1 \le j \le N, N+1 \le i \le 2N \end{cases}$$

The process of the proof for $M_i$ and $M_j$ which are not in same half of $B_{2N}$ follows along the proof above and is omitted.

In the process of proving perfect aperiodic CCF, $M_j$ is replaced by $M_i$ ($1 \le i \le 2N$ ), then the following conclusion can be easily got

$$R_{M_i,M_i}(\tau) = 0 \text{ for } \tau \ne 0 \text{ and } R_{M_i,M_i}(0) \ne 0$$

which means the aperiodic ACF of each element codes of $B_{2N}$ is perfect.

Based on theorem 1, certain number of perfect complete generalized complementary orthogonal code groups can be generated.

## 2.4. Expanding Theorem 2 and Expanded Generalized Complementary Orthogonal Code Groups

But in practice, there is limitation of the length of access codes and the number of ideal access codes is not enough to meet practical requirement. In fact, the aperiodic CCF plays the main role in ACI and if we stress perfect aperiodic CCF and imperfect ACF, more access codes will be generated based on theorem 2.

Theorem 2: For any given matrix $\mathbf{A}_{A_{row}\times A_{col}}$ :

$$\mathbf{A}_{A_{row}\times A_{col}} \square \begin{bmatrix} \tilde{\mathbf{a}}_0 \\ \tilde{\mathbf{a}}_1 \\ \vdots \\ \tilde{\mathbf{a}}_{A_{row}-1} \end{bmatrix}$$

$$\tilde{\mathbf{a}}_m = [a_m(0), a_m(1), \cdots, a_m(A_{col.}-1)],\ m = 0,1,\cdots, A_{row}-1$$

and perfect complete generalized complementary orthogonal code groups:

$$\tilde{\mathbf{b}}_k = \tilde{\mathbf{b}}_k^0 [+] \tilde{\mathbf{b}}_k^1 [+] \cdots [+] \tilde{\mathbf{b}}_k^{K-1} = [\sum_{k''=0}^{K-1}]\ \tilde{\mathbf{b}}_k^{k''},\ k = 0,1,\cdots, K-1$$

Let

$$\mathbf{B}_k(\mathbf{A}) \triangleq \begin{bmatrix} \mathbf{b}_k(\tilde{\mathbf{a}}_0) \\ \mathbf{b}_k(\tilde{\mathbf{a}}_1) \\ \vdots \\ \mathbf{b}_k(\tilde{\mathbf{a}}_{A_{row}-1}) \end{bmatrix} = \mathbf{b}_k^0(\mathbf{A})[+]\mathbf{b}_k^1(\mathbf{A})[+]\cdots[+]\mathbf{b}_k^{K-1}(\mathbf{A})$$

where $k = 0,1,\cdots,K-1$,

$$\mathbf{b}_k^{k''}(\mathbf{A}) \triangleq \mathbf{b}_k^{k''} \otimes \mathbf{A} = [\tilde{b}_k^{k''}(0), \tilde{b}_k^{k''}(1), \cdots, \tilde{b}_k^{k''}(N_0 - 1)] \otimes \mathbf{A}$$
$$\tilde{\mathbf{b}}_k^{k''}(\tilde{\mathbf{a}}_m) \triangleq \tilde{\mathbf{b}}_k^{k''} \otimes \tilde{\mathbf{a}}_m = [\tilde{b}_k^{k''}(0), \tilde{b}_k^{k''}(1), \cdots, \tilde{b}_k^{k''}(N_0 - 1)] \otimes \tilde{\mathbf{a}}_m$$

and $\otimes$ denotes the Kronecker product.

The conclusion is: $K$ groups access codes are generated based on the procedure above, and each group has $A_{row}$ access codes. The most important property of $K$ groups access codes is: the aperiodic ACF and aperiodic CCF between rows from same group are not ideal (the aperiodic ACF and aperiodic CCF are up to the property of $\mathbf{A}$ ) , but if access codes are coming from different groups, their aperiodic CCF is still ideal. i.e.:

$$\mathbf{b}_k[\tilde{\mathbf{a}}_m]\mathbf{b}_{k'}^H[\tilde{\mathbf{a}}_{m'}(l)] \equiv 0, \quad \forall k \neq k', \forall m, m'$$

where $k, k' = 0,1,\cdots,K-1$; $k, k' = 0,1,\cdots,K-1$, and $m, m' = 0,1,\cdots,A_{row}-1$.

Proof: similar to the proof process of theorem 1.

The code groups mentioned in theorem 2 is expanded generalized complementary orthogonal code groups.

### 2.5. Examples and Validating Properties of Expanded Generalized Complementary Orthogonal Code Groups

The new kind of complementary codes can be constructed on the base of theorem 1 and 2: the simplest perfect complete complementary orthogonal code pairs mate is easy to generate and then perfect complete generalized complementary orthogonal code groups can be generated based on theorem 1 and more access codes can be generated through enlarging perfect complete generalized complementary orthogonal code groups based on theorem 2.

In practice, adjacent cells should be assigned different codes according to four-color principle and at least four perfect complete generalized complementary orthogonal codes are needed.

For example: $B_4$ includes four perfect complete generalized complementary orthogonal codes and its four element codes are listed in following matrix:

$$\begin{array}{cc} & \begin{array}{cc} \text{C} & \text{S} \end{array} \\ B_4 = & \begin{bmatrix} ++-+ & +--- \\ ++++ & +-++ \\ -+++ & --+- \\ -+-- & ---+ \end{bmatrix} \end{array}$$

where“ + ”and“ - ”denotes “1”and“-1” respectively.

Matrix $\tilde{A} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ is the expand matrix and if code $B_4^1 = [++-+ \ \ +---]$ is assigned to cell 1and after expanding process the component code $C$ of cell 1 access code groups is

 

$$C_{cell1} = \begin{bmatrix} 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 \\ 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 \end{bmatrix}.$$

In order to generate more access codes, we can shift the codes at fractional chip level or chip level, which do not change the aperiodic CCF of two codes from different groups [9]. For example, $C_{cell1}$ is shifted at the level of two chips, then the new access codes are:

$$C_{cell1}^{shift} = \begin{bmatrix} 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & & & & \\ 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & & & & \\ & & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & & \\ & & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & & \\ & & & & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 \\ & & & & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & & & \end{bmatrix}$$

(the blank part are all zeros).

In order to validate that there is no ACI if different expanded generalized complementary orthogonal code groups are assigned to different cells, a simulation with QPSK modulation scheme in AWGN channel is carried out and simulation results are shown in Figure 1and Figure 2.

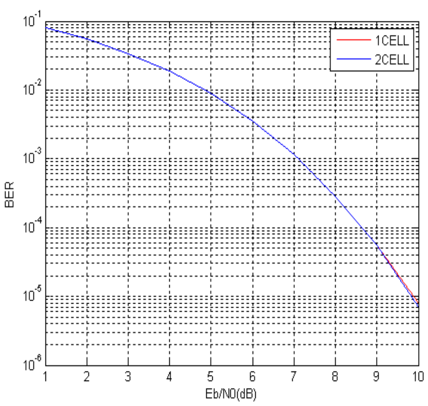


**Figure 1. Bit Error Rate**

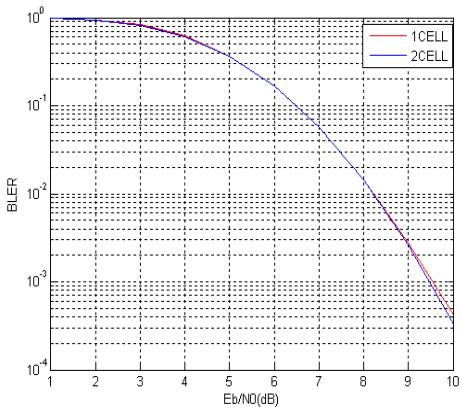


**Figure 2. Block Level Error Rate**

Figure 1 and Figure 2 show and the performance of one cell and that of two cells are same, which prove that access codes of different cells are orthogonal and ACI is eliminated. If the new CDMA system adopts this kind of codes, it can overcome some fatal problems of conventional CDMA, such as near-far effect.

## 3. New CDMA System Model and Relationship between New CDMA System and OVTDM

The core technologies of new CDMA system are access codes designing and spectrum spreading process. The former has been introduced above and the latter is related with a new time division multiplexing technology. The following content gives system model of CDMA system and introduces the new time division multiplexing technology.

### 3.1. New CDMA System Model

Figure 3 illustrates the new CDMA system structure in AWGN channel. Data bits of each user are mapped into symbols and then the symbols are encoded by their unique rate-one convolutional spreading encoder. After that, all users' signals form one signal and are corrupted by white Gaussian noise. At the receiver, multi code maximum likelihood detection (MC-MLD) which can be implemented by Viterbi algorithm will fulfill decoding process.

Figure 4 shows the unique rate-one convolutional spreading encoder structure for each user. In Figure 4, the code sequence is the weighted vector on the taps and the output of encoder is the sum of each tap output which is the multiplication of code sequence and delayed symbols.

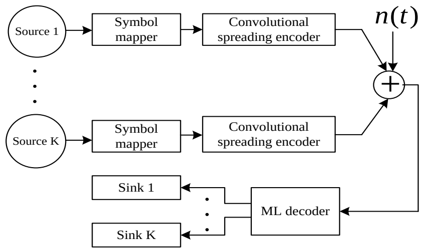


**Figure 3. Structure of New CDMA in AWGN**

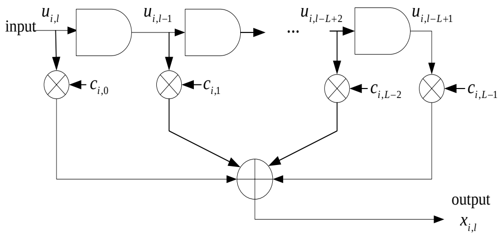


**Figure 4. Structure of Convolutional Spreading Encoder**

Suppose $\mathbf{c_i} = [c_{i,0}, c_{i,1}, \cdots, c_{i,L-1}]$ , $\mathbf{u_i} = [u_{i,0}, u_{i,1}, \cdots u_{i,F-1}]$ , $\mathbf{x_i} = [x_{i,0}, x_{i,1}, \cdots x_{i,F+L-2}]$ are spreading code sequence, symbols vector and the output of convolutional encoder respectively of user i, where L and F are length of code sequence and length of symbol vector. The output of convolutional encoder $\mathbf{x_i} = [x_{i,0}, x_{i,1}, \cdots x_{i,F+L-2}]$ is the result of encoding $\mathbf{u_i}$ with code sequence $\mathbf{c_i}$. From Figure 2, it is easy to find that $\mathbf{x_i}$ can be expressed by $\mathbf{u_i}$ and $\mathbf{c_i}$ as Eq. 3:

$$x_{i,l} = \begin{cases} \sum_{n=0}^{l} u_{i,l-n} c_{i,n} & 0 \le l \le L-1 \\ \sum_{n=0}^{L-1} u_{i,l-n} c_{i,n} & L \le l \le F-1 \\ \sum_{n=l-F+1}^{L-1} u_{i,l-n} c_{i,n} & F \le l \le F+L-2 \end{cases}$$

and the output of AWGN channel is:

$$y_l = x_{1,l} + x_{2,l} + \cdots x_{K,l} + n_l \quad 0 \le l \le F+L-2$$

where $n_l$ is white Gaussian noise.

Because all the users send their signals simultaneously, the channel is equivalent to inter symbol channel and Viterbi algorithm can be used in the MC-MLD decoder.

In this new CDMA system, different code sequence group will be assigned to different cell and the new code sequences can be generated by the way of shift original sequence to meet the system requirement of much more code sequences. If the shift is chip level, the cross correlation and auto correlation of cell code sequences are kept as before, but do not improve spectral efficiency. If the shift is fractional chip level, spectral efficiency will be improved at the cost of higher decoding complexity of MC-MLD caused by not ideal cross correlation and auto correlation of cell code sequences. In fact, the latter is equivalent to overlap time division multiplexing (OVTDM) [10].

### 3.2. Introduction of OVTDM and Relationship between New CDMA System and OVTDM

OVTDM is proposed to meet the requirement of higher and higher spectral efficiency and introduce inter symbol interference (ISI) as at the transmitter as a natural encoding constraints. Analysis and simulations have proved that the OVTDM scheme can achieve notable coding gain and compared with conventional modulation schemes under the condition of same spectral efficiency, OVTDM has lower cardinality of signal output constellation set, which brings lower decoding complexity at the receiver. The OVTDM principle diagram and system model are shown in Figure 5and Figure 6 respectively:

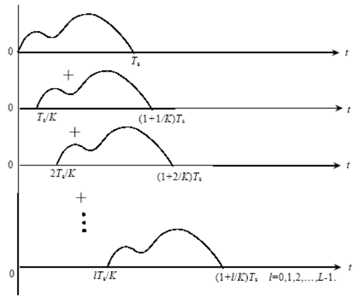


**Figure 5. Principle Diagram of OVTDM System**

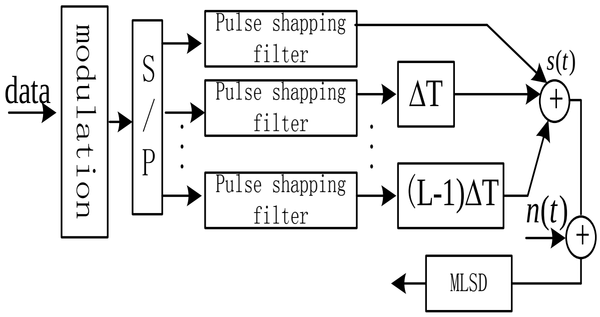


**Figure 6. OVTDM System Model**

The mathematical model of OVTDM signal in equivalent lowpass waveform is:

$$s(t) = \sum_{l=0}^{L-1} u(l)\, a(t - l\Delta T)$$

where $a(t)$ is pulse shaping waveform and if $T_s$ is symbol duration and $t \notin [0, T_s)$, then $a(t)=0$; $L$ and $u(l)$ denote the symbols number of one frame and the transmitted data sequence respectively; The time shift of adjacent transmitted signals is denoted by $\Delta T$ and the value of $\Delta T$ is equal to $T_s / K$ where $K \in Z^+$. In addition, $K$ is also called constraint length for its being the number of overlapped symbols within one symbol and OVTDM is the same as interference free system when $K$ is equal to 1.

If the signal passes through AWGN channel, the received signal is:

 

$$r(t) = s(t) + n(t)$$

where $n(t)$ is white Gaussian noise and its power spectral density is $N_0/2$.

At the receiver, the entire received signal $r(t)$ is detected and the maximum likelihood data sequence $u$ whose corresponding wave $s(t)$ is closest to $r(t)$ can be expressed as:

$$\mathbf{u} = \arg[\underset{\mathbf{u}}{Min} \int_0^{T_a} \left| r(t) - \sum_{l=0}^{L-1} \hat{u}(l) a(t - l\Delta T) \right|^2 dt]$$

So the maximum-likelihood sequence detection (MLSD) can be adopted to minimize the Euclidean distance based on above formula.

Figure 5 shows that OVTDM system can also treated as a convolution system, therefore the received signal can be expressed as:

$$v(l) = \sum_{k=0}^{K-1} h(k) u(l-k) + n(l)$$

where $h(k) \quad (k = 0, 1, \cdots K-1)$ denotes tap coefficient which combines both channel gain and pulse shaping waveform. Then the metric in decoding algorithm is:

$$M_V = \sum_{l=0}^{L-1} \left\| v(l) - \sum_{k=0}^{K-1} h(k) \widehat{u}(l-k) \right\|^2$$

It has been proved that there is a linear relationship between the spectral efficiency of OVTDM and overlap times in one symbol period [10].

$$\eta = \frac{L \cdot \log_2 M}{B \cdot T} = \frac{L \cdot \log_2 M}{B \cdot \frac{L+K-1}{K} T_s} = \frac{K \cdot \log_2 M}{B T_s} \cdot \frac{L}{L+K-1} \qquad (2)$$

where $L$ is the number of symbols in one frame, $B$ is bandwidth of OVTDM signal, $M$ is modulation level, $T_s$ is symbol duration.

From Eq. 2, the spectral efficiency of OVTDM can be approximated as Eq. 3 if $L$ is large enough.

$$\lim_{L \to \infty} \eta = \frac{K \cdot \log_2 M}{B T_s} = \eta_{max} \qquad (3)$$

Figure 7 gives the simulation results which shows the performance of OVTDM with QPSK modulation scheme is better than the performance of the same spectral efficiency QAM modulation.

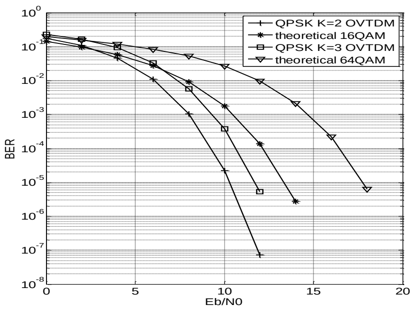


**Figure 7. Performance of QPSK-OVTDM**

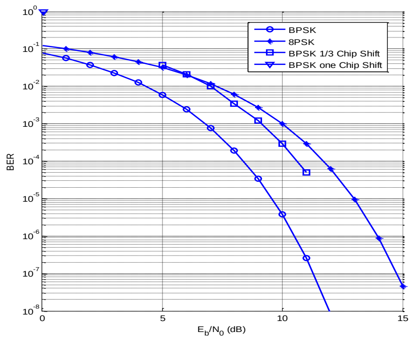


**Figure 8. Simulation Results**

Based on analysis about OVTDM and new CDMA system, if the access codes from same code group are generated at fractional chip level shift and are assigned to different users, then there are several signals which are overlapped within one chip duration time after pulse shaping filter of CDMA system, which is equivalent to OVTDM. The denominator of fraction about fractional chip level shift is just equal to the overlap time $K$ from the angle of OVTDM. From Eq. 3, the bigger denominator of fraction is, the higher spectral efficiency of new CDMA system is.

## 4. Simulation and Analysis

Figure 8 is the simulation results of new high spectral efficiency CDMA system with BPSK modulation scheme in AWGN channel. In simulation, we choose two cells and each cell is assigned one different group code sequences and some of each cell code sequences have been shifted with one chip and one-third chip to produce new code sequences. At receiver, Viterbi algorithm is used in MC-MLD decoder.

From the Figure 8, BPSK theoretical bit error ratio (BER) curve overlaps that of new CDMA system using one chip shifted code sequences, which indicates that there is no any ACI between two cells and the new CDMA system using one chip shifted code sequences does not improve spectral efficiency. On the contrary, the BER performance of new CDMA system using one-third chip shift code sequences is better than that of 8PSK under same spectral efficiency, which indicates that using factional chip shifted code sequences in the new CDMA system can improving spectral and obtain extra coding gain simultaneously.

## 5. Conclusion

The new CDMA system which has high spectral efficiency through novel spectrum spreading technique is presented and analyzed in the paper. The spectrum spreading process of this new CDMA system is just equivalent to OVTDM which introduces ISI deliberately as inter symbols constraints from the perspective of coding theory.

The good performance of new CDMA system is at the cost of high decoding complexity at receiver and if the channel is ISI channel, the decoding complexity will grows rapidly and the transmitter should be modified to guarantee no any operation between different names of component codes.

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