Adaptive Synchronization of Chaotic Systems with known Response System Parameters

Cheng Gan\textsuperscript{1,A}, Yuan Cheng\textsuperscript{1,B}, Ruiqi Wang\textsuperscript{2,C} and Junwei Lei\textsuperscript{3,D}

\textsuperscript{2}Department of Armament Engineering, Naval Aeronautical and Astronautical University, Yantai Shandong 264001, China
\textsuperscript{3}Department of Control Engineering, Naval Aeronautical and Astronautical University, Yantai Shandong 264001, China
\textsuperscript{A}gancheng@zzuli.edu.cn, \textsuperscript{B}chengyuan@zzuli.edu.cn, \textsuperscript{C}richkey1980@gmail.com, \textsuperscript{D}leijunwei@126.com

Abstract

A kind of adaptive synchronous control method was proposed to solve a special synchronization problem between two chaotic systems, where the response system is totally known without uncertainty but the driven system contains both unknown parameters and uncertain nonlinear functions. An update law of estimation of unknown parameters of driven system by constructing a proper Lyapunov energy function and the stability of the whole system was guaranteed by Lyapunov stability theorem. What is worthy pointing out is that the chaotic systems are not required to satisfy the Lipschitz condition. At last, detailed numerical situation was done to show the rightness and effectiveness of the proposed method.

Keywords: Chaotic System; Synchronization; Adaptive Control; Nonlinear Function; Robustness

1 Introduction

Chaotic system has complex behavior and it is widely used in secure communication. Synchronization of two chaotic systems is a key technology of secure communication and that is why it attracted many researchers’ interest. Rongwei Guo designed a kind of simple nonlinear adaptive control law to realize synchronization of two chaotic systems by using LaSalle theorem in paper [1]. It is a novel method but the unknown parameters and other uncertain situations are not considered. And the chaotic system are assumed to satisfy the so-called Lipschitz condition to guarantee the stability of the whole system. Although there are many chaotic system can be assumed to be bounded and satisfied the local Lipschitz condition. But the global Lipschitz condition is strict and many chaotic systems can not satisfy this global Lipschitz condition [2-9].

Since now most of the current synchronization are realized on the assumption that the chaotic systems used can satisfy the so-called global Lipschitz condition, Wei Lin [2] proposed a kind of adaptive synchronous strategy for chaotic systems can only satisfy the local Lipschitz condition or even the chaotic system is unbounded. But the unknown parameter and uncertain nonlinear functions are not considered simultaneously.

So in this paper, a kind of general chaotic system is taken as an example and the parameters of response system are assumed to be known, but the there are unknown parameters and uncertain nonlinear functions in driven chaotic system. A new kind of robust adaptive synchronous controller of uncertain chaotic systems was designed based on Lyapunov stability theorem. What is worthy pointing out is that the chaotic systems are not required to satisfy the Lipschitz condition. At last, detailed numerical situation was done to show the rightness and effectiveness of the proposed method [4-6].
2. Problem Description

Consider the below driven chaotic system and response chaotic system, where parameters of response system is known, the driven system with unknown parameters and nonlinear functions can be written as follows:

The driven chaotic system model can be written as

\[ \dot{x} = f(x) + F(x)\dot{\theta} + \Delta(x,t) \quad (1) \]

The response chaotic system model can be written as

\[ \dot{y} = f(y) + b u \quad (2) \]

Taken a four dimension chaotic system as an example, the driven system can be expanded as

\[ \dot{x}_1 = f_{x_1}(x_1,\ldots,x_4) + \sum_{i=1}^{p_1} F_{x_1i}(x_1,\ldots,x_4)\dot{\theta}_{1i} + \sum_{i=1}^{p_1} \Delta_{x_1i}(x,t) \quad (3) \]

\[ \dot{x}_2 = f_{x_2}(x_1,\ldots,x_4) + \sum_{i=1}^{p_1} F_{x_2i}(x_1,\ldots,x_4)\dot{\theta}_{2i} + \sum_{i=1}^{p_1} \Delta_{x_2i}(x,t) \quad (4) \]

\[ \dot{x}_3 = f_{x_3}(x_1,\ldots,x_4) + \sum_{i=1}^{p_1} F_{x_3i}(x_1,\ldots,x_4)\dot{\theta}_{3i} + \sum_{i=1}^{p_1} \Delta_{x_3i}(x,t) \quad (5) \]

\[ \dot{x}_4 = f_{x_4}(x_1,\ldots,x_4) + \sum_{i=1}^{p_1} F_{x_4i}(x_1,\ldots,x_4)\dot{\theta}_{4i} + \sum_{i=1}^{p_1} \Delta_{x_4i}(x,t) \quad (6) \]

And the slave response system can be expanded as

\[ \dot{y}_1 = f_{y_1}(y_1,\ldots,y_4) + b_1 u_1 \quad (7) \]

\[ \dot{y}_2 = f_{y_2}(y_1,\ldots,y_4) + b_2 u_2 \quad (8) \]

\[ \dot{y}_3 = f_{y_3}(y_1,\ldots,y_4) + b_3 u_3 \quad (9) \]

\[ \dot{y}_4 = f_{y_4}(y_1,\ldots,y_4) + b_4 u_4 \quad (10) \]

where \( \dot{\theta}_i \) is unknown parameter and \( \Delta_i \) is unknown nonlinear function, so the number of unknown parameters is \( n* p_1 \), and the number of nonlinear function is \( n* p_2 \) [7-9].

The objective of synchronization problem of chaotic system with unknown parameters and uncertain nonlinear functions is to design a control law \( u = u(x,y,\dot{\theta},\dot{\varphi}) \) such that the state of response system can track the state of driven system, then it means \( y \rightarrow x \).

3. Assumption

Assumption 1: The response system has the same structure of the driven system and they have the same dimension \( f_{ui} = f_{yi} \).

Assumption 2: Some parts of driven system are known, it means \( F_{xij} \) and \( f_{yi} \) are known.
Assumption 3: The response system is totally known, it means that \( f_{yi} \) and \( b_i \) are known.

Assumption 4: The nonlinear function of driven system satisfies following conditions. It means:

\[
\left| \Delta_{uj} (X, t) \right| \leq q_{uj}(X)
\]  

(11)

where \( d_{uj} \) is a known constant and \( q_{uj}(X) \) is a known positive smooth function [10-15].

4. Design of Robust Adaptive Synchronization Controller

Define an error variable as \( z_i = y_i - x_i \), then the above drive - response system can be transformed as an error response of system as

\[
\dot{z}_i = f_{yi}(y_i, \ldots, y_4) - f_{xi}(x_i, \ldots, x_4)
\]

\[
- \sum_{j=1}^{p_i} F_{uj}(x_i, \ldots, x_4) \theta_{uj} - \sum_{j=1}^{p_i} \Delta_{uj}(x, t) + b_i u_i
\]

(12)

The control law can be designed as follows.

\[
u_i = f_{zi}(x)[-f_{ji}(y_i, \ldots, y_4) + f_{xi}(x_i, \ldots, x_4)]
\]

\[
+ \sum_{j=1}^{p_i} F_{uj}(x_i, \ldots, x_4) \dot{\theta}_{uj} + \sum_{j=1}^{p_i} \dot{q}_{uj}(X) - f_{uj}(z_i)]
\]

(13)

where

\[
f_{zi}(x) = b_i^{-1}
\]

(14)

\[
f_{zi}(z_i) = k_{i1} z_i + k_{i2} \left( \frac{z_i}{\varepsilon_i} + \frac{3}{2} z_i^{1/3} \right) \exp(z_i^{2/3}) + k_{i4} sgn(z_i)
\]

(15)

Then:

\[
z_i \dot{z}_i = z_i[-f_{zi}(z_i) - \sum_{j=1}^{p_i} F_{uj}(x_i, \ldots, x_4) \dot{\theta}_{uj} + \sum_{j=1}^{p_i} \dot{q}_{uj}(x) - \Delta_{uj}(x, t)]
\]

(16)

Where \( \dot{\theta}_{uj} \) is defined as

\[
\dot{\theta}_{uj} = \theta_{uj} - \dot{\theta}_{uj}
\]

(17)

Considering that

\[
z_i \sum_{j=1}^{p_i} \left\{ \dot{q}_{uj}(x) - \Delta_{uj}(x, t) \right\} \leq \sum_{j=1}^{p_i} \left[ z_i \dot{q}_{uj}(x) + q_{uj} \right] \left| \dot{\psi}_{uj}(X) \right|
\]

(18)

Define a new variable \( \ddot{q}_{uj} \) as
\[
\ddot{q}_{ij} = q^*_{ij} + \text{sign}(z_i) \dot{q}_{ij}
\]  

Then it holds

\[
\tau \sum_{j=1}^{p_i} \{ \dot{\theta}_{ij} \psi_{ij}(x) - \Delta_{ij}(x,t) \} \leq \sum_{j=1}^{p_i} \{ |\dot{z}_i| |\psi_{ij}(x)\dot{q}_{ij}| \}
\]

Considering that

\[
\dot{\theta}_{ij} = \dot{\theta}_{ij} - \ddot{\theta}_{ij} = -\ddot{\theta}_{ij}
\]

Design the adaptive adjustment law as

\[
\dot{\theta}_{ij} = -z_i F_{ij}(x_1, \ldots, x_4)
\]

In the same way, it is easy to get

\[
\dot{q}_{ij} = \text{sign}(z_i) \dot{q}_{ij}
\]

Design the adaptive adjustment law as

\[
\dot{q}_{ij} = -z_j \psi_{ij}(x)
\]

Choose a Lyapunov function as

\[
V = \sum_{i=1}^{n} \dot{z}_i^2 + \sum_{i=1}^{p} \sum_{j=1}^{p_i} \frac{1}{2} (\dot{\theta}_{ij})^2 + \sum_{i=1}^{n} \sum_{j=1}^{p_i} \frac{1}{2} (\dot{q}_{ij})^2
\]

Solve its derivative and it is easy to get

\[
\dot{V} \leq \sum_{i=1}^{n} -z_i f_{ij}(z_i) < 0
\]

According to the Lyapunov stability theorem [16-22], it is easy to prove that the system is stable and \( z_i \) can converged to zero, then the synchronization is realized.

5: The Analysis of Numerical Simulation

Taking a four dimension hyper-chaotic system as an experiment object, the model of drive system is as follows.

\[
\dot{x}_1 = a(x_2 - x_1) + k_{ab} x_4 \cos x_2
\]

\[
\dot{x}_2 = b x_1 - k x_2 x_3 + k_{ab}(1 + \sin(x_2 x_3)) x_2
\]

\[
\dot{x}_3 = -c x_1 + b x_2^2 + k_{ab}(2 - \cos(x_2 x_3 x_4)) x_3
\]

\[
\dot{x}_4 = -d x_1 + k_{ab} x_3 (3 + \sin(x_2 x_3))
\]

where \( a, b, c, d \) are unknown parameters. The assumption conditions that uncertain nonlinear functions satisfy are the same as above. The structure of response system is known. Its model is shown as below:
\[ \dot{y}_1 = a_1 (y_2 - y_1) + u_1 \]  \hspace{1cm} (31)

\[ \dot{y}_2 = b_1 y_4 - k_1 y_1 + u_2 \]  \hspace{1cm} (32)

\[ \dot{y}_3 = -c_1 y_3 + h y_1^2 + u_3 \]  \hspace{1cm} (33)

\[ \dot{y}_4 = -d_1 y_1 + u_4 \]  \hspace{1cm} (34)

System parameters are set as \((a, b, c, d) = (9, 39, 2.4, -10.5)\). The initial states of drive system are set as \((x_1, x_2, x_3, x_4) = (1, -1.2, -2)\). And the initial states of response system are set as \((y_1, y_2, y_3, y_4) = (-3, 3, -5.5)\).

The error of system is shown as below.

\[ \dot{e}_1 = a_1 y_2 - a_1 (x_2 - x_1) - k_1 a_1 x_2 + u_1 \] \hspace{1cm} (35)

\[ \dot{e}_2 = b_1 y_4 - k_1 y_1 + b_1 y_4 x_2 - k_1 b_1 (1 + \sin(x_2 x_4)) x_2 + u_2 \] \hspace{1cm} (36)

\[ \dot{e}_3 = -c_1 y_3 + h y_1^2 - (-c_1 x_3 + h x_1^2) + k_1 h (2 - \cos(x_2 x_4)) x_4 + u_3 \] \hspace{1cm} (37)

\[ \dot{e}_4 = -d_1 y_1 - d_1 x_4 + k_1 d_1 x_3 + 3 + \sin(x_2 x_4)) + u_4 \] \hspace{1cm} (38)

The designed control law is as follows

\[
  u_i = [- f_{ii}(y_1, \cdots, y_4) + f_{ii}(x_1, \cdots, x_4) \\
  + \sum_{j=1}^{p_1} F_{ij}(x_1, \cdots, x_4) \dot{\theta}_{ij} + \sum_{j=1}^{p_2} \dot{q}_{ij} \varphi_{ij}(x) - f_{ii}(z_i)]
\]  \hspace{1cm} (39)

Definition and selection of controller parameters are shown as above. The simulation results are shown in following figures.

Figure 1. Trajectory of Uncontrolled Chaotic Systems(1)
Figure 1 to Figure 4 shows the free movement trajectory of drive system and response system without control. Comparison of trajectory between drive system and response system without control can be shown from Figure 5 to Figure 8. It is obvious that states of two systems cannot synchronize with each other.
Figure 5. Trajectory of State X1 and Y1

Figure 6. Trajectory of State X2 and Y2

Figure 7. Trajectory of State X3 and Y3

Figure 8. Trajectory of State X4 and Y4
By adopting the proposed method of this paper, states of response chaotic system can track the state of driven system. The synchronization of each states can be shown from Figure 9 to Figure 12.

**Figure 9. Tracing Curve of State X1**

**Figure 10. Tracing Curve of State X2**

**Figure 11. Tracing Curve of State X3**
In summary, the drive system and response system can achieve fast synchronization by using method proposed in this paper for the drive system with unknown parameters and uncertain nonlinear functions.

6. Conclusions

A kind of robust adaptive synchronous strategy was proposed to solve the synchronization problem of chaotic systems. Uncertainties were considered and adaptive strategy was adopted to solve the unknown parameters. The update law was designed by constructing a Lyapunov energy function. And the bad affection caused by uncertain strategy was adopted to solve the unknown parameters. The update law was designed by using the method proposed in this paper.

References


Authors

Cheng Gan (1981-) was born in Pingxiang city of Jiangxi province of China. He graduated from Zhengzhou University of Light Industry, Zhengzhou of China in 2002 and received bachelor’s degree with the major of computer science and technology. He received his master’s degree in 2006 from Huazhong University of Science and Technology and his major is computer application technology. He is now a lecturer in the Computer Department of Zhengzhou University of Light Industry. He published more than 10 academic papers, and 5 of them were retrieved by EI. His current research interests are the computer application technology and control theory.

Ruiqi Wang (1980-) was born in Xinxiang City, Henan province of China. He received his Doctor degree in Guidance, Navigation and Control in 2011 from Naval Aeronautical and Astronautical University, Yantai of China. He graduated from Naval Aeronautical and Astronautical University, Yantai of China in 2001 and received his bachelor’s degree with the major of missile control and test. After that he continued his study in this school and received his master’s degree and doctor’s degree in 2006 and 2011 respectively. He has published more than 20 papers, where 7 papers was indexed by EI. His present interests are control theory, missile control and bilateral control.

Junwei Lei (1981) was born in Chibi of Hubei province of China and received his Doctor degree in Guidance, Navigation and Control in 2010 from Naval Aeronautical and Astronautical University, Yantai of China. Her present interests are control theory, chaotic system control, aircraft control and adaptive control.

He was promoted to be a lecture of NAAU in 2010. His typical book named Nussbaum gain control technology of supersonic missiles was published in 2013 in China.