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Research on PN Code Acquisition Strategy

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Abstract

PN code acquisition is the key to synchronization in direct-sequence spread-spectrum(DSSS) systems. Aiming at the problems in the process of PN code acquisition, the block diagram of acquisition strategy is illustrated firstly. Then, with the choice of different search mode, determination mode and verification mode in the block diagram, the detection probabilities and false alarm probabilities are presented separately, and the false alarm probability of threshold detection method, the detection probability and false alarm probability of maximum-selection determination method are deduced, and all kinds of acquisition strategies are given. Finally, Simulation results shows that the detection probabilities and false alarm probabilities in theory and experiment is consistent with different acquisition strategy, and so does the validation of detection probabilities by maximum-selection method in multiple dwell times.

Keywords: *PN Code Acquisition; Acquisition Strategy; Detection Probability; False Alarm Probability*

1. Introduction

As the spread spectrum communication has the advantages of anti-jamming, low intercept and code division multiple access(CDMA) communications etc, it is widely used in mobile communication, navigating positioning, and satellite communication. The synchronization of PN code consists of two steps, acquisition and tracking. PN code acquisition is a process of coarse synchronization to limit phase difference within half of one code chip interval, while tracking is performed to achieve code synchronization more accurately. PN code acquisition is the prerequisite of synchronization in DSSS systems, because only when PN code acquisition is completed, would that track the code and demodulate the data. With the rapid development of FPGA devices, matched filters are used to detect signal fast. After the process of correlation, it is a prior problem to detect correlative value fast and accurately. Generally speaking, acquisition strategies includes search mode, determination mode, and verification mode. The optimum method of PN code detection can be achieved with the best acquisition strategy on certain structure. The paper [1] presents the uniform framework of acquisition process, and analyses specific search stage and verification stage. Whereas, the framework is not complete enough, and its arithmetic expressions of detection probability in maximum-selection method is complex and unable to get integrable expressions. The paper [2] gives all kinds of acquisition strategies with simple comparison but analyses systemically. The paper [3] makes comparison of 3 acquisition verification strategies, without analyzing systematically combined with search stage. Therefore, starting from acquisition strategies,

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this paper summarizes detection probabilities and false-alarm probabilities of different PN code acquisition strategies, and analyzes its performance.

This paper is organized as follows: Section 2 describes the principle of PN code acquisition. In Section 3, The PN code acquisition strategies and the required probability expressions are given. Section 4 presents the numerical results, while the last section is devoted to the findings and conclusions.

2. The Principle of PN Code Acquisition

As is illustrated in Figure 1, the intermediate frequency signal $r(t)$ is transformed into Inphase-Quadrature phase baseband signals through digital down-conversion and sent to matched filter to complete correlation operation to obtain orthogonal signals $I(j)$ and $Q(j)$, from which energy signal $Z(j)$ is obtained. If the signal of energy satisfies decision-making criterion through some search mode, the acquisition process is successful.

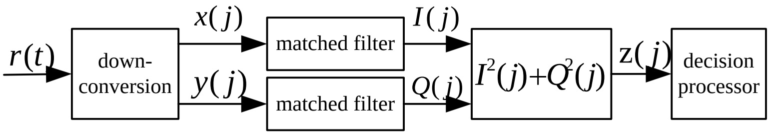

Figure 1. Model of Non-Correlative Detection in BPSK-DSSS Receiver

As is shown by Figure 2, a complete PN code acquisition block diagram is introduced. The block diagram will be analyzed detailedly in the next chapter.

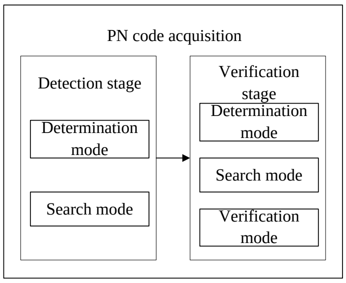

Figure 2. Block Diagram of PN Code Acquisition Strategy

3. PN Code Acquisition Strategy

3.1. Detection Stage

3.1.1. Determination Mode

In general, acquisition decision modes include threshold decision, maximum-selection decision and hybrid of threshold and maximum-selection decision. Each of these decision modes are to be analyzed in the following passage.

(1) Threshold decision (TD). If the energy value exceeds the threshold, acquisition is declared. The key to the method is the selection of threshold. The setting of the decision threshold can be either fixed or adaptive. Theoretically, in a specific spectrum spread system, there must be an optimum threshold, but it is difficult to find it with a clear expression, and the signal levels are unknown and varying. By adjusting threshold in accord with the variable environment, the adaptive detection solves the problem through estimating of Signal to Noise Ratio (SNR) directly or indirectly.

The threshold θ, detection probability P_d and false alarm probability P_f [7] are as follows:

$$P_d = P(z > \theta \mid H_1) = Q_1\left(\sqrt{2M^2/V}, \sqrt{2\theta/V}\right) \tag{1a}$$

$$P_f = P(z > \theta \mid H_0) = \exp\left(-\frac{\theta}{V}\right) \tag{1b}$$

H_1 means that the timing error between received signal and local PN sequence is within a chip, whereas H_0 means greater than a chip. Where $V = 2L\sigma^2$, the $L\sigma^2$ is the equivalent variance of cumulated chips after dispreading in Additive Gaussian White Noise(AWGN) channel. Where $M^2 = L^2 E_c$, L is the length of PN code, E_c is the energy per chip. The Marcum Q-function $Q_1(a,b) = \int_b^\infty x \cdot \exp\left(\frac{-(x^2+a^2)}{2}\right) \cdot I_0(ax)dx$. $I_0(x) = \frac{1}{2\pi}\int_0^{2\pi} \exp(x\cos\theta)d\theta$, Where $I_0(x)$ is 0^{th}-order Bessel function. The SNR is μ , where $\mu = \frac{M^2}{V}$.

Suppose the phase of PN code synchronization is in chip *i*, with a same distribution in a period, then the detection probability of threshold detection is [1]:

$$P_{d\_v} = \frac{\left(1-(1-P_f)^L\right)}{L \cdot P_f} \cdot P_d \tag{2}$$

The false alarm probability in chip *i* is :

$$P_{fi} = \begin{cases} \left(1-(1-P_f)^{i-1}\right) + \\ (1-P_f)^{i-1}(1-P_d)\left(1-(1-P_f)^{L-i}\right), & 1 \le i \le L-1 \\ 1-(1-P_f)^{i-1}, & i = L \end{cases} \tag{3}$$

And the false alarm probability is :

$$P_{f\_v} = \frac{1}{L}\sum_{i=1}^{L} P_{fi} \tag{4}$$

(2) Maximum-selection decision(MSD). In this method, the greatest correlative peak is found in a period of PN code and regarded as the synchronous phase. The status of synchronization must be verified, otherwise the noise will be falsely considered as detection case.

Suppose the signal *x*(*j*) and *y*(*j*) in AWGN channel, with $x(j) \sim N(0, L\sigma^2)$ and $y(j) \sim N(0, L\sigma^2)$, then:

$$\begin{aligned} I_{H_0} &\sim N(0, L\sigma^2) \\ I_{H_1} &\sim N(L\sqrt{E_c}\cos\phi, L\sigma^2) \\ Q_{H_0} &\sim N(0, L\sigma^2) \\ Q_{H_1} &\sim N(L\sqrt{E_c}\sin\phi, L\sigma^2) \end{aligned} \tag{5}$$

The normalized form is as follows:

$$\begin{aligned}
&\frac{I_{H_0}}{\sqrt{L}\sigma} \sim N(0,1) \\
&\frac{I_{H_1}}{\sqrt{L}\sigma} \sim N\left(\frac{\sqrt{LE_c}\cos\phi}{\sigma},1\right) \\
&\frac{Q_{H_0}}{\sqrt{L}\sigma} \sim N(0,1) \\
&\frac{Q_{H_1}}{\sqrt{L}\sigma} \sim N\left(\frac{\sqrt{LE_c}\sin\phi}{\sigma},1\right)
\end{aligned} \tag{6}$$

So that:

$$\begin{aligned}
&\left(\frac{I_{H_0}}{\sqrt{L}\sigma}\right)^2 \sim \chi^2(0) \\
&\left(\frac{I_{H_1}}{\sqrt{L}\sigma}\right)^2 \sim \chi^2\left(\frac{LE_c\cos^2\phi}{\sigma^2}\right) \\
&\left(\frac{Q_{H_0}}{\sqrt{L}\sigma}\right)^2 \sim \chi^2(0) \\
&\left(\frac{Q_{H_1}}{\sqrt{L}\sigma}\right)^2 \sim \chi^2\left(\frac{LE_c\sin^2\phi}{\sigma^2}\right)
\end{aligned} \tag{7}$$

The paper [8] indicates non-central χ^2-distribution with n degrees of freedom, from which the non-central parameter λ can be deduced as follows:

$$\lambda = \left(\frac{\sqrt{LE_c}\cos\phi}{\sigma}\right)^2 + \left(\frac{\sqrt{LE_c}\sin\phi}{\sigma}\right)^2 = 2\mu \tag{8}$$

The paper [9] realizes quick acquisition of PN code in low SNR by maximum-selection with multi-symbol accumulation. However, the derivate maximum-selection method detection probability formula is not integrable, thus cannot give an explicit value. Suppose the interdependent detection of different phase signal, regardless of the effect of frequency offset, there are L decision variable $z_1, z_2, \cdots, z_L$ with continuous phase in one PN code. The z_i is the decision value at the synchronization acquisition phase, while z_k is not, and $i \neq k$, so the detection probability is:

$$\begin{aligned}
P_{d_max} &= P\left(z_i = \max(z_1, z_2, \cdots, z_L) \mid H_1\right) \\
&= \prod_{k=1, k\neq i}^{L} P(z_i > z_k \mid H_1) \\
&= \left(P\left(\frac{\left(z_i / L\sigma^2\right)/2}{\left(z_k / L\sigma^2\right)/2} > 1 \mid H_1\right)\right)^{L-1}
\end{aligned} \tag{9}$$

The detailed deduction is in the appendix, from which we can result:

$$P\left(\frac{z_i}{z_k} > 1 \mid H_1\right) = 1 - \frac{1}{2}\cdot\exp\left(-\frac{\lambda}{4}\right) \tag{10}$$

The detection probability of maximum-selection method is:

$$P_{d\_max} = \left(1 - \frac{1}{2}\cdot \exp\left(-\frac{\lambda}{4}\right)\right)^{L-1} \quad (11)$$

The false alarm probability of maximum-selection method is:

$$P_{f\_max} = 1 - P_{d\_max} = 1 - \left(1 - \frac{1}{2}\cdot \exp\left(-\frac{\lambda}{4}\right)\right)^{L-1} \quad (12)$$

(3) Maximum-selection threshold decision (MSTD). In this method, the maximum of decision variables in the length of the window is found and compared to the threshold. If the maximum is greater than the threshold, it is in the H_1 state. Else, it is not synchronized.

The detection probability of maximum-selection threshold method is [10]:

$$P_{d\_h} = \sum_{i=0}^{L-1}\left(C_i^{L-1}\cdot\frac{(-1)^i}{i+1}\cdot\exp\left(-\frac{i}{(i+1)}\cdot\frac{M^2}{V}\right)\cdot Q_1\left(\sqrt{\frac{2M^2}{V(i+1)}},\sqrt{\frac{2(i+1)\theta}{V}}\right)\right) \quad (13)$$

The false dismissal probability [10] is:

$$P_M = \left(1 - Q_1\left(\sqrt{2M^2/V},\sqrt{2\theta/V}\right)\right)\cdot\left(1 - \exp\left(-\frac{\theta}{V}\right)\right)^{L-1} \quad (14)$$

The false alarm probability is:

$$P_{f\_h} = 1 - P_M - P_{d\_h} \quad (15)$$

3.1.2. Search Mode

Point-by-point search and window search are two familiar types of search mode. Point-by-point search, which means search synchronization acquisition phase by phase continuously, is mainly used together with threshold decision. Window search is a block processor. A phase point with the maximum correct detection probability within a window is chosen as the synchronization acquisition phase, so it is usually combined with verification mode.

3.2. Verification Stage

3.2.1. Verification Mode

Under the worse environment, the false alarm probability in single dwell time is greater in the process of acquisition, therefore an acquisition method with united-decision, namely detection in multiple dwell times, is required. But acquisition means no verification in a single dwell time. There are three types of acquisition verification: continuous decision detector in multiple dwell times, M of N form of detector and Tong detector.

① Continuous decision detector in multiple dwell times. The correlative values in N periods are compared with threshold to decide whether to discard the entire group of data. If it is met in every dwell time, the acquisition is reliable.

The detection and false alarm probabilities are:

$$P_D = (P_d)^N \quad (16a)$$

$$P_F = (P_f)^N \quad (16b)$$

② M of N form of detector. If there are M or greater than M detection variables in the same phase of PN sequence in N dwell times, the acquisition is successful. The detection and false alarm probabilities are:

$$P_D = \sum_{i=M}^{N} C_i^N P_d^{\,i} (1 - P_d)^{N-i} \tag{17a}$$

$$P_F = \sum_{i=M}^{N} C_i^N P_f^{\,i} (1 - P_f)^{N-i} \tag{17b}$$

③ Tong detector. In this detector, we set the counter K with initial value B. If the decision criterion is satisfied, K plus 1, otherwise K minus 1. If K=0, those data are abandoned, K=A, signal exists, and the verification is accomplished. The detection and false-alarm probabilities are [1]:

$$P_D = \frac{\left(\frac{1 - P_d}{P_d}\right)^B - 1}{\left(\frac{1 - P_d}{P_d}\right)^A - 1} \tag{18a}$$

$$P_F = \frac{\left(\frac{1 - P_{fa}}{P_{fa}}\right)^B - 1}{\left(\frac{1 - P_{fa}}{P_{fa}}\right)^A - 1} \tag{18b}$$

3.2.2. Search Mode in the Verification Stage

Search mode includes detection position point search and window search. These two types are almost the same as the ones in decision mode, but the only difference is that detection position point searches needs the verification of acquisition position after finishing detection.

3.3. Conclusion of Acquisition Strategies

2 types of detection in single dwell time and acquisition strategy in multiple dwell times with 9 types of decision modes and 3 types of verification strategies are listed in Table 1 and Table 2 respectively.

Table 1. Acquisition Strategy in Single Dwell Time

Scheme	Decision mode	Search Mode
A	MSTD	Window Search
B	TD	Point by Point Search

Table 2. Acquisition Strategy in Multiple Dwell Times

Scheme	Decision in detection stage	Search mode in decision stage	Decision in Verification stage	Search mode in Verification stage
A	TD	Point by Point Search	TD	Detection Position Point Search
B	MSTD	Window Search	TD	Detection Position Point Search
C	MSTD	Window Search	MSTD	Window Search
F	TD	Point by Point Search	MST	Window Search
D	MST	Window Search	TD	Detection Position Point Search
E	MST	Window Search	MST	Window Search
F	MSTD	Window Search	MST	Window Search
H	TD	Point by Point Search	MSTD	Window Search
I	MST	Window Search	MSTD	Window Search

4. Numerical Results

In this section, the performance of the acquisition scheme is evaluated by means of computer simulations. We consider that the simulation is conducted in AWGN channel, with the choice of *m* sequence, and the length is L=63,127 and 255. E_c=1, the number of simulation experiments is 10^5.

4.1. Detection in Single Dwell Time

Figure 3 and Figure 4 show that the experiment results go near to the theoretical curve, which indicates the derivation of the detection and false alarm probabilities with maximum-selection determination method are right. But contrasting between simulation results and theoretical values, there are a few differences. The reason may come from two factors: Firstly, because short PN code has worse pseudo randomness, larger correlation values are likely in the presence of non-synchronous acquisition position; Secondly, the zero mean value of correlation is assumed in theoretical derivations, while this kind of ideal PN code does not exist in simulations, which leads to the performance loss.

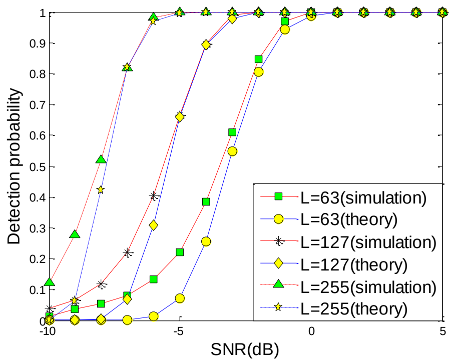

Figure 3. Detection Probability of Maximum-Selection Determination Method

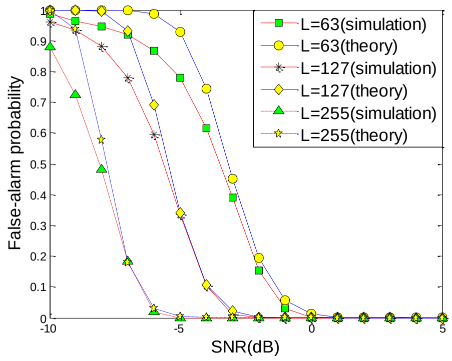

Figure 4. False Alarm Probability of Maximum-Selection Determination Method

4.2. Detection in Multiple Dwell Times

Case E is considered, selecting M of N form of detector, with N=5 and M=4.

Figure 5 shows that the simulation value almost accords with theoretical curves, with greater errors when PN codes are short.

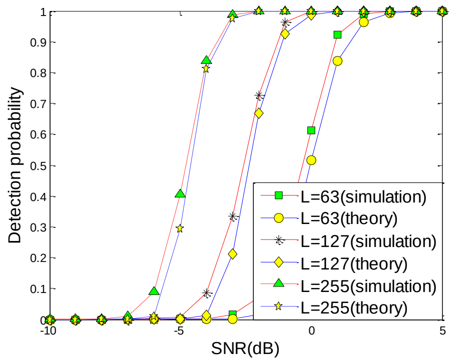

Figure 5. Detection Probability

5. Conclusions

In this paper, we summarize the strategies of acquisition, with the analysis of acquisition and false alarm probabilities from the aspect of acquisition determination, search mode and verification mode. The acquisition methods of different acquisition strategies are given, with 2 types of detection method in single dwell time, 9 types of determination and 3 types of verification strategies. Theoretical derivation is verified by simulating and analyzing detection and false alarm probability of maximum-selection determination acquisition strategies. The conclusion derived from the formula of the paper can be extended to all acquisition schemes, provided important theoretical reference for PN code acquisition.

Appendix

A non-central Γ distribution is derived through dividing a non-central χ^2 random variable by a centralized χ^2 one, *i.e.*,

$$x = \frac{x_1 / \nu_1}{x_2 / \nu_2} \tag{19}$$

Where $x_1 \sim \chi^2_{\nu_1}(\lambda)$, $x_2 \sim \chi^2_{\nu_2}$, and x_1、 x_2 are independent of each other, so that *x* is of non-central Γ probability distribution functions(pdf). The pdf is as follows:

$$p(x)=\exp\left(-\frac{\lambda}{2}\right)\sum_{k=0}^{\infty}\frac{\left(\frac{\lambda}{2}\right)^{k}}{k!}\frac{\left(\frac{v_1}{v_2}\right)^{\frac{1}{2}v_1+k}}{B\left(\frac{v_1+2k}{2},\frac{v_2}{2}\right)}\cdot x^{\frac{v_1}{2}+k-1}\cdot\left(1+\frac{v_1}{v_2}x\right)^{-\frac{1}{2}(v_1+v_2)-k} \quad (20)$$

Where $B(u,v)=\frac{\Gamma(u)\Gamma(v)}{\Gamma(u+v)}$, $\Gamma(u)=\int_0^{\infty}t^{u-1}\exp(-t)dt$, $\Gamma(u)=(u-1)\Gamma(u-1)$,

$\Gamma(u)=(u-1)!$, $\Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}$ 。

Let $v_1=2$, $v_2=2$, $z=\frac{\lambda x}{2(1+x)}$, so the pdf is reduced to:

$$p(x)=\exp\left(-\frac{\lambda}{2}\right)\cdot\left(1-\frac{2z}{\lambda}\right)^{2}\cdot\exp(z)\cdot(z+1) \quad (21)$$

$$P\left(\frac{z_i}{z_k}>1\,|\,H_1\right)=1-\int_0^{1}p(x)dx=1-\frac{1}{2}\cdot\exp\left(-\frac{\lambda}{4}\right) \quad (22)$$

Acknowledgments

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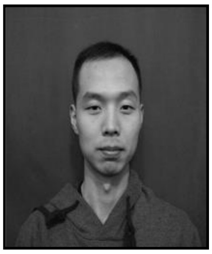

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