

## Synchronization of Uncertain Chaotic System with Unknown Response System Parameters

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### Abstract

*Synchronization of chaotic system is meaningful and widely used in secure communication. A kind of robust adaptive synchronization method was proposed to solve a special chaotic synchronization problem that both response system and driven system have unknown parameters and uncertain nonlinear functions. And the difficulty is also caused by the different structures between master chaotic system and slave chaotic system. A kind of update law of estimation of unknown parameters were designed by choose a proper Lyapunov energy function. And the stability of the whole system is guaranteed by Lyapunov stability theorem. At last, detailed numerical simulation was done to show the rightness of the proposed method.*

**Keywords:** *chaotic system; synchronization; adaptive control; nonlinear function; the Lipschitz condition; robustness;*

### 1. Introduction

Synchronization of chaotic system was studied by many researchers for its possible application in secure communication<sup>[1-23]</sup>. The synchronization of chaotic systems with different structures was researched by Jian Huang<sup>[1]</sup>. Considering the situation that the parameters of driven system and response system are totally unknown, synchronization between LS chaotic system and CYQY chaotic system, and synchronization between LS chaotic system and Chen chaotic system are realized respectively with adaptive method. But only unknown parameters are considered and the situation of chaotic system with uncertain nonlinear functions are not discussed in this paper.

The situation that the driven chaotic system has the same structure with the response system is discussed by Ju H. Park in paper<sup>[2,3]</sup>. A kind of feedback synchronization method based on Lyapunov function and LMI method are proposed for Genesio-Tesi chaotic systems with three unknown parameters. And the criterion for the existence of control law are solved by LMI method. So the disadvantage of this paper is that the above method is designed for the special Genesio-Tesi systems with the same structure. And the uncertainties of chaotic system are not fully discussed so it is not easy to be applied in a general chaotic system with both unknown parameters and other uncertainties.

Xianyong Wu<sup>[4]</sup> discussed the synchronization problem of chaotic system with different structure but the response system does not contain any unknown parameter. And a kind of adaptive chaotic synchronous controller was designed for the synchronization between special Chen chaotic system and Henon-Heiles chaotic system. And the update law of estimation of unknown parameters are constructed based on Lyapunov stability

theorem. But also the system uncertainties are assumed to be simple and complex situation of system uncertainties are not considered.

In this paper, synchronization problem of a kind of general chaotic systems was discussed and the response system has the same dimension but different structure with the driven system. Both the unknown parameters and uncertain nonlinear functions are considered in driven system and response system, and a kind of robust adaptive synchronous controller were designed based on Lyapunov stability theorem to fulfill the synchronization. At last, a four dimension chaotic system was taken as an example to do the numerical simulation and the simulation result testified the rightness and effectiveness of the proposed method[4-6].

## 2. Problem Description

Consider the below driven chaotic system and response chaotic system[7-10], the response system has unknown parameters and uncertain nonlinear functions, and the driven system with unknown parameters and nonlinear function can be described as follows:

The driven chaotic system model can be written as

$$\dot{x} = f_x(x) + F_x(x)\theta_x + \Delta_x(x, t) \quad (1)$$

The response chaotic system model can be written as

$$\dot{y} = f_y(y) + F_y(y)\theta_y + \Delta_y(y, t) + bu \quad (2)$$

Taken a four dimension chaotic system as an example, the driven system can be expanded as

$$\dot{x}_1 = f_{x1}(x_1, L, x_4) + \sum_{j=1}^{p_1} F_{x1j}(x_1, L, x_4)\theta_{x1j} + \sum_{j=1}^{p_2} \Delta_{x1j}(x, t) \quad (3)$$

$$\dot{x}_2 = f_{x2}(x_1, L, x_4) + \sum_{j=1}^{p_1} F_{x2j}(x_1, L, x_4)\theta_{x2j} + \sum_{j=1}^{p_2} \Delta_{x2j}(x, t) \quad (4)$$

$$\dot{x}_3 = f_{x3}(x_1, L, x_4) + \sum_{j=1}^{p_1} F_{x3j}(x_1, L, x_4)\theta_{x3j} + \sum_{j=1}^{p_2} \Delta_{x3j}(x, t) \quad (5)$$

$$\dot{x}_4 = f_{x4}(x_1, L, x_4) + \sum_{j=1}^{p_1} F_{x4j}(x_1, L, x_4)\theta_{x4j} + \sum_{j=1}^{p_2} \Delta_{x4j}(x, t) \quad (6)$$

And the slave response system can be expanded as

$$\dot{y}_1 = f_{y1}(y_1, L, y_4) + \sum_{j=1}^{p_3} F_{y1j}(y_1, L, y_4)\theta_{y1j} + \sum_{j=1}^{p_4} \Delta_{y1j}(y, t) + b_1u_1 \quad (7)$$

$$\dot{y}_2 = f_{y2}(y_1, L, y_4) + \sum_{j=1}^{p_3} F_{y2j}(y_1, L, y_4)\theta_{y2j} + \sum_{j=1}^{p_4} \Delta_{y2j}(y, t) + b_2u_2 \quad (8)$$

$$\dot{y}_3 = f_{y3}(y_1, L, y_4) + \sum_{j=1}^{p_3} F_{y3j}(y_1, L, y_4)\theta_{y3j} + \sum_{j=1}^{p_4} \Delta_{y3j}(y, t) + b_3u_3 \quad (9)$$

$$\dot{y}_4 = f_{y4}(y_1, L, y_4) + \sum_{j=1}^{p_3} F_{y4j}(y_1, L, y_4)\theta_{y4j} + \sum_{j=1}^{p_4} \Delta_{y4j}(y, t) + b_4u_4 \quad (10)$$

where  $\theta_x$  and  $\theta_y$  are unknown parameter and  $\Delta_x$  is unknown nonlinear function, so the number of unknown parameters is  $n^*(p_1 + p_3)$ , and the number of nonlinear function is  $n^*(p_2 + p_4)$ .

For the situation that both driven system and response system have unknown parameters and uncertain nonlinear functions, the objective of synchronization problem of chaotic system is to design a control law  $u = u(x, y, \hat{\theta}_x, \hat{q}_x, \hat{\theta}_y, \hat{q}_y)$ ,  $\hat{\theta}_x = f(x, y, \hat{\theta}_x)$ ,

$\hat{q}_x = f(x, y, \hat{q}_x)$ ,  $\hat{\theta}_y = f(x, y, \hat{\theta}_y)$ ,  $\hat{q}_y = f(x, y, \hat{q}_y)$  such that the state of response system can track the state of driven system, then it means  $y \rightarrow x$ .

### 3. Assumption

Assumption 1: The response system has the same structure of the driven system and they

have the same dimension  $f_{xi} = f_{yi}$ .

Assumption 2: Some parts of driven system are known, it means  $F_{xij}$  and  $f_{xi}$  are known.

Assumption 3: The response system is totally known, it means that  $f_{yi}$  and  $b_i$  are known<sup>[11-14]</sup>.

Assumption 4: The nonlinear function of driven system satisfies following conditions. It means : for  $1 \leq i \leq n$ ,  $1 \leq j \leq p_2$ , there exists a unknown constant  $q_{ij}^* \leq d_{ij}$  such that

$$|\Delta_{xij}(X, t)| \leq q_{ij}^* \psi_{ij}(X) \quad (11)$$

where  $d_{ij}$  is a known constant and  $\psi_{ij}(X)$  is a known positive smooth function.

Assumption 5: The nonlinear function of response system satisfies following conditions. It means : for  $1 \leq i \leq n$ ,  $1 \leq j \leq p_4$ , there exists a unknown constant  $q_{yij}^* \leq d_{yij}$  such that

$$|\Delta_{yij}(X, t)| \leq q_{yij}^* \psi_{yij}(X) \quad (12)$$

where  $d_{yij}$  is a known constant and  $\psi_{yij}(X)$  is a known positive smooth function<sup>[15-17]</sup>.

### 4. The Design of Robust Adaptive Synchronization Controller

Define an error variable as  $z_i = y_i - x_i$ . The error response system of the above drive-response system can be written as

$$\begin{aligned} \dot{z}_i = & f_{yi}(y_1, L, y_4) - f_{xi}(x_1, L, x_4) + \sum_{j=1}^{p_3} F_{yij}(y_1, L, y_4) \theta_{yij} + \sum_{j=1}^{p_4} \Delta_{yij}(y, t) \\ & - \sum_{j=1}^{p_1} F_{xij}(x_1, L, x_4) \theta_{xij} - \sum_{j=1}^{p_2} \Delta_{xij}(x, t) + b_i u_i \end{aligned} \quad (13)$$

Design control law as follows

$$\begin{aligned} u_i = & f_{2i}(x) [-f_{yi}(y_1, L, y_4) + f_{xi}(x_1, L, x_4) + \sum_{j=1}^{p_1} F_{xij}(x_1, L, x_4) \hat{\theta}_{xij} \\ & + \sum_{j=1}^{p_2} \hat{q}_{xij} \psi_{xij}(x) - \sum_{j=1}^{p_3} F_{yij}(y_1, L, y_4) \hat{\theta}_{yij} - \sum_{j=1}^{p_4} \hat{q}_{yij} \psi_{yij}(y) - f_{zi}(z_i)] \end{aligned} \quad (14)$$

Where

$$\begin{aligned} f_{2i}(x) = & b_i^{-1} \\ f_{zi}(z_i) = & k_{i1} z_i + k_{i2} \frac{z_i}{|z_i| + \varepsilon_{i1}} + k_{i3} \frac{3}{2} z_i^{1/3} \exp(z_i^{2/3}) + k_{i4} \text{sign}(z_i) \end{aligned} \quad (15)$$

Then

$$z_i \dot{\theta}_{xij} = z_i [-f_{z_i}(z_i) - \sum_{j=1}^{p_1} F_{xij}(x_1, L, x_4) \theta_{xij}^0 + \sum_{j=1}^{p_2} \{\hat{q}_{xij} \psi_{ij}(x) - \Delta_{xij}(x, t)\} + \sum_{j=1}^{p_3} F_{yij}(y_1, L, y_4) \theta_{yij}^0 + \sum_{j=1}^{p_4} \{\Delta_{yij}(y, t) - \hat{q}_{yij} \psi_{yij}(x)\}] \quad (16)$$

Where

$$\theta_{xij}^0 = \theta_{xij} - \hat{\theta}_{xij}, \quad \theta_{yij}^0 = \theta_{yij} - \hat{\theta}_{yij} \quad (17)$$

Considering that

$$z_i \sum_{j=1}^{p_2} \{\hat{q}_{xij} \psi_{ij}(x) - \Delta_{xij}(x, t)\} \leq \sum_{j=1}^{p_2} \{z_i \hat{q}_{xij} \psi_{xij}(x) + q_{xij}^* |z_i| \psi_{xij}(x)\} = \sum_{j=1}^{p_2} \{|z_i| \psi_{xij}(x) [\text{sign}(z_i) \hat{q}_{xij} + q_{xij}^*]\} \quad (18)$$

$$z_i \sum_{j=1}^{p_4} \{\Delta_{yij}(y, t) - \hat{q}_{yij} \psi_{yij}(x)\} \leq \sum_{j=1}^{p_4} \{-z_i \hat{q}_{yij} \psi_{yij}(y) + q_{yij}^* |z_i| \psi_{yij}(y)\} = \sum_{j=1}^{p_4} |z_i| \psi_{yij}(y) \{-\text{sign}(z_i) \hat{q}_{yij} + q_{yij}^*\} \quad (19)$$

Define

$$q_{xij}^0 = q_{xij}^* + \text{sign}(z_i) \hat{q}_{xij} \quad (20)$$

$$q_{yij}^0 = q_{yij}^* - \text{sign}(z_i) \hat{q}_{yij} \quad (21)$$

Then

$$z_i \sum_{j=1}^{p_2} \{\hat{q}_{xij} \psi_{xij}(x) - \Delta_{xij}(x, t)\} \leq \sum_{j=1}^{p_2} \{|z_i| \psi_{xij}(x) q_{xij}^0\} \quad (22)$$

$$z_i \sum_{j=1}^{p_4} \{\hat{q}_{yij} \psi_{yij}(y) - \Delta_{yij}(y, t)\} \leq \sum_{j=1}^{p_4} \{|z_i| \psi_{yij}(y) q_{yij}^0\} \quad (23)$$

Considering that

$$\dot{\theta}_{xij}^0 = \dot{\theta}_{xij} - \dot{\theta}_{xij}^0 = -\dot{\theta}_{xij} \quad (24)$$

Design the adaptive adjustment law as

$$\dot{\theta}_{xij}^0 = -z_i F_{xij}(x_1, L, x_4) \quad (25)$$

In the same way, it holds

$$\dot{q}_{xij}^0 = \text{sign}(z_i) \dot{q}_{xij} \quad (26)$$

$$\dot{q}_{yij}^0 = -\text{sign}(z_i) \dot{q}_{yij} \quad (27)$$

And design the adaptive adjustment law as

$$\dot{q}_{xij}^0 = -z_i \psi_{xij}(x) \quad \dot{q}_{yij}^0 = z_i \psi_{yij}(y) \quad (28)$$

Select a Lyapunov function as

$$V = \sum_{i=1}^n z_i^2 + \sum_{i=1}^n \sum_{j=1}^{p_1} \frac{1}{2} (\theta_{xij}^0)^2 + \sum_{i=1}^n \sum_{j=1}^{p_2} \frac{1}{2} (q_{xij}^0)^2 \quad (29)$$

Solve its derivation and it is easy to get:

$$\dot{V} \leq \sum_{i=1}^n -z_i f_{z_i}(z_i) \leq 0 \quad (30)$$

According to Lyapunov stability theory, the system is stable and then  $z_i \rightarrow 0$ , so the synchronization of chaotic system can be realized.

### 5. Numerical Simulation

Take a four dimension hyper-chaotic system as an experiment object, The model of drive chaotic system can be written as follows.

$$\dot{x}_1 = a(x_2 - x_1) + k_{lb}x_4 \cos x_2 \tag{31}$$

$$\dot{x}_2 = bx_1 - kx_1x_3 + k_{lb}(1 + \sin(x_2x_3))x_2 \tag{32}$$

$$\dot{x}_3 = -cx_3 + hx_1^2 + k_{lb}(2 - \cos(x_1x_2x_3x_4))x_1 \tag{33}$$

$$\dot{x}_4 = -dx_1 + k_{lb}x_3(3 + \sin(x_1x_3)) \tag{34}$$

where  $a, b, c, d$  are unknown parameters. The assumption conditions that uncertain nonlinear functions satisfy are the same with above. The parameters of response system are known. Its structure is shown as below<sup>[18-20]</sup>.

$$\dot{y}_1 = a_y(y_2 - y_1) + k_{lb}(1 + \sin(y_2y_3))y_2 + u_1 \tag{35}$$

$$\dot{y}_2 = b_yy_1 - ky_1y_3 + k_{lb}y_4 \cos y_2 + u_2 \tag{36}$$

$$\dot{y}_3 = -c_yy_3 + hy_1^2 + k_{lb}y_3(3 + \sin(y_1y_3)) + u_3 \tag{37}$$

$$\dot{y}_4 = -d_yy_1 + k_{lb}(2 - \cos(y_1y_2y_3y_4))y_1 + u_4 \tag{38}$$

Unknown parameters in the program are set as  $(a_y, b_y, c_y, d_y) = (9, 39, 2.4, -10.5)$ . The initial states of drive system are set as  $(x_1, x_2, x_3, x_4) = (1, -1, 2, -2)$ . And the initial states of response system are set as  $(y_1, y_2, y_3, y_4) = (-3, 3, -5, 5)$ . The error system is shown as below.

$$\dot{e}_1 = a_y(y_2 - y_1) - a(x_2 - x_1) - k_{lb}x_4 \cos x_2 + u_1 \tag{39}$$

$$\dot{e}_2 = b_yy_1 - k_1y_1y_3 - \{bx_1 - kx_1x_3 + k_{lb}(1 + \sin(x_2x_3))x_2\} + u_2 \tag{40}$$

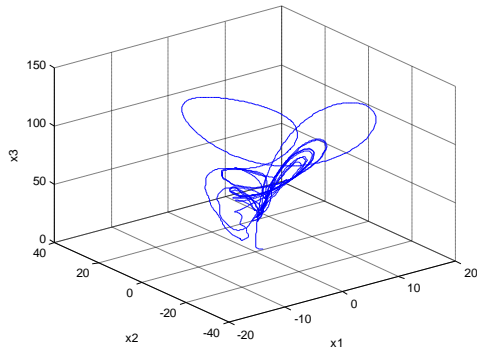
$$\dot{e}_3 = -c_yy_3 + hy_1^2 - \{-cx_3 + hx_1^2 + k_{lb}(2 - \cos(x_1x_2x_3x_4))x_1\} + u_3 \tag{41}$$

$$\dot{e}_4 = -d_yy_1 - \{-dx_1 + k_{lb}x_3(3 + \sin(x_1x_3))\} + u_4 \tag{42}$$

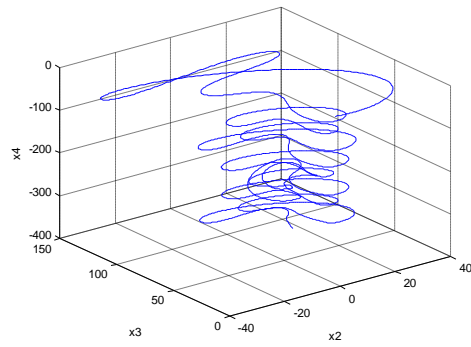
The designed control law is as follows.

$$u_i = f_{2i}(x)[-f_{yi}(y_1, L, y_4) + f_{xi}(x_1, L, x_4) + \sum_{j=1}^{p_1} F_{xij}(x_1, L, x_4)\hat{\theta}_{xij} + \sum_{j=1}^{p_2} \hat{q}_{xij}\psi_{xij}(x) - \sum_{j=1}^{p_3} F_{yij}(y_1, L, y_4)\hat{\theta}_{yij} - \sum_{j=1}^{p_2} \hat{q}_{yij}\psi_{yij}(y) - f_{zi}(z_i)] \tag{43}$$

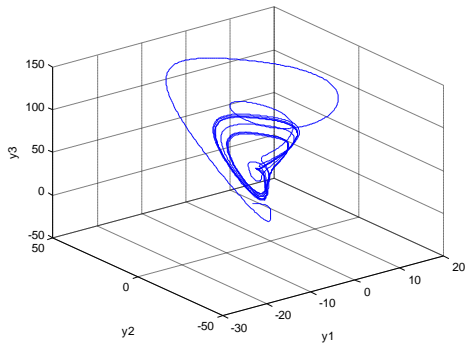
Definition and selection of controller parameters are shown above. The simulation results are shown in following figures<sup>[21-23]</sup>.



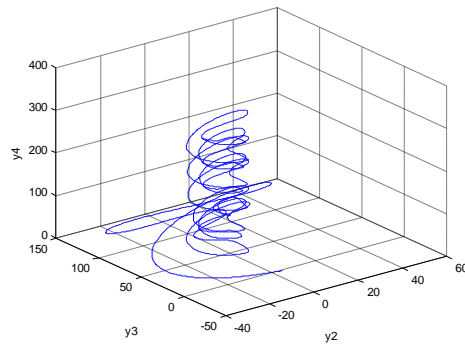
**Figure 1. Trajectory of Uncontrolled Chaotic Sysmtes (1)**



**Figure 2. Trajectory of Uncontrolled Chaotic Sysmtes (2)**

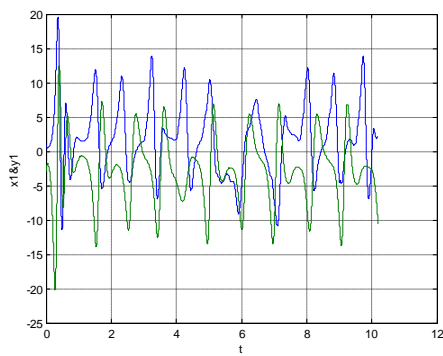


**Figure 3. Trajectory of Uncontrolled Chaotic Sysmtes (3)**

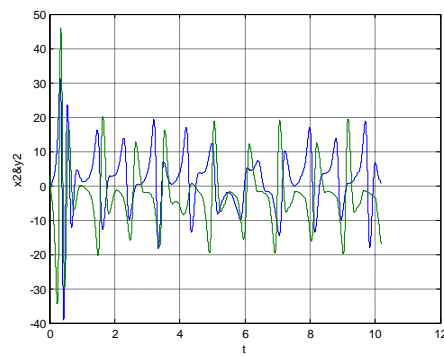


**Figure 4. Trajectory of Uncontrolled Chaotic Sysmtes (4)**

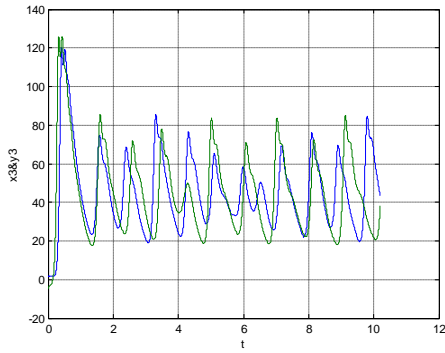
Freedom movement trajectory of drive system and response system without control are shown by figure 1 to figure 4. Comparison of two chaotic trajectory between drive system and response system without control are shown from figure 5 to figure 8. It can be found from the diagram that two systems can not synchronize with each other.



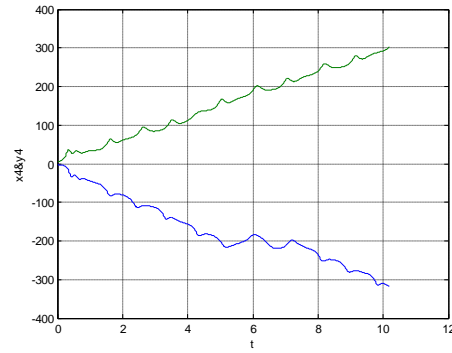
**Fig.5 Trajectory of State x1 and y1**



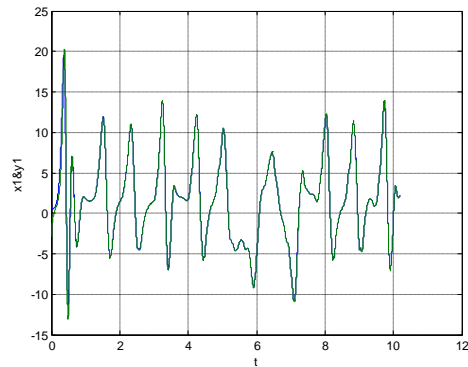
**Figure 6. Trajectory of State x2 and y2**



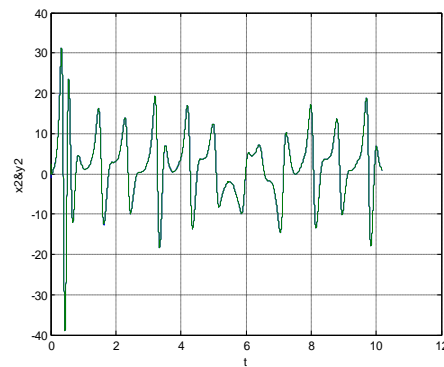
**Fig.7 Trajectory of states  $x_3$  and  $y_3$**



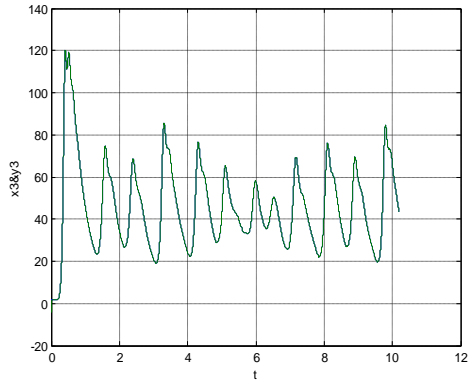
**Figure 8. Trajectory of States  $x_4$  and  $y_4$**



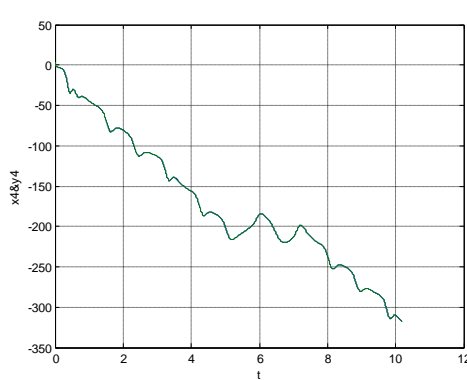
**Figure 9. Tracing Curve of State  $x_1$**



**Figure 10. Tracing Curve of State  $x_2$**

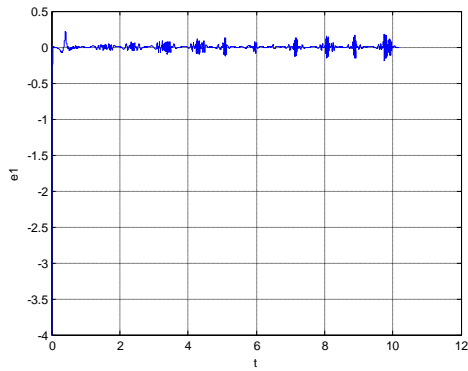


**Figure 11. Tracing Curve of State  $x_3$**

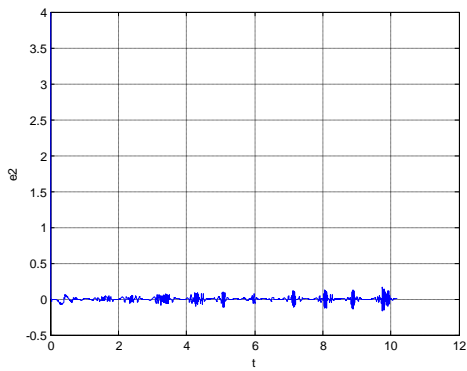


**Figure 12. Tracing Curve of State  $x_4$**

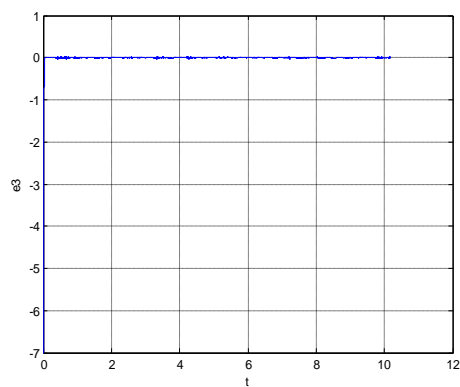
It can be shown from figure 9 to figure 12 that response system can track drive system in the effect of synchronous controller.



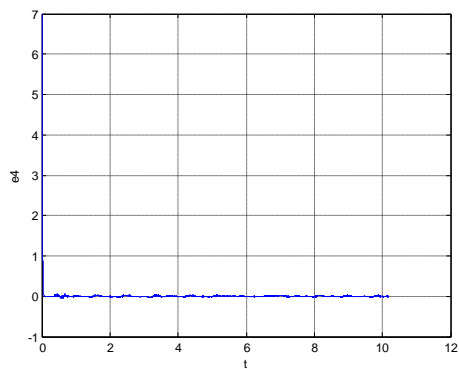
**Figure 13. Error Curve of e1**



**Figure 14. Error Curve of e2**



**Figure 15. Error Curve of e3**



**Figure 16. Error Curve of e4**

In summary, the drive system and response system can achieve fast synchronization by using method in this paper in the drive systems with unknown parameters and uncertain nonlinear functions. In the same time, it can be shown from figure 13 to figure 16 that synchronization error can be stable in a small area near zero.

## 6. Conclusions

The synchronization problem of chaotic systems with unknown parameters and uncertain nonlinear functions in driven and response system was discussed. And the difficulty is also caused by the different structures between master chaotic system and slave chaotic system. Adaptive law was designed to cope with the unknown parameters and update law was designed to estimate unknown parameters. Also the robust strategy was adopted to treat the nonlinear functions. And the stability of the whole system is guaranteed by Lyapunov stability theorem. At last, numerical simulation of four dimension chaotic system was done to testify the rightness of the proposed method. So this method is proved to be effective in theory but in real engineering application, main difficulty will face is the nonlinear function introduced by the controller. It will lead the selection of simulation step to be difficult or the simulation should better be done with variable step simulation method or the system will be unstable. This is the main disadvantage caused by the introduction of terminal sliding mode items. But it also has obvious advantage that if the simulation step is set properly, the synchronization error will be small and the synchronization speed will be very quick. So it means that if those nonlinear control items are used in the control strategy, the simulation program and the realization of those algorithm should be done more carefully and exquisitely. Also, a



more complex and more advance simulation program and simulation method is needed in order to realize those more complex nonlinear control items.

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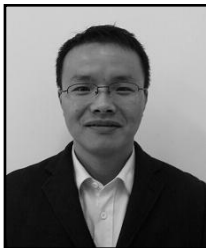
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