Method and Analysis of Medium and Long Term Satellite Orbit Prediction Based on Satellite Broadcast Ephemeris Parameters

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Abstract

Given the lag of IGS precise ephemeris and complexity of dynamic models, in order to solve the problem of long-term satellite orbit prediction in autonomous orbit determination, a new medium and long term orbit prediction method that based on satellite broadcast ephemeris parameters has been put forward. This method is that forecasting the orbit parameters firstly according to broadcast ephemeris history data, and then calculating satellite position according to the forecast ephemeris parameters. In order to verify the feasibility of this method, using 40 days of broadcast ephemeris data, adopting ARMA model combined with sliding window model to predict broadcast ephemeris orbit parameters value in 25 days. Analyzed the parameters prediction error and the influence of parameters prediction error to satellite orbit accuracy, 25 days of orbit prediction error can be controlled in 150 meters. The results show that the predictability of orbit parameters has certain application value for satellite orbit prediction, the medium and long term orbit prediction that based on the satellite broadcast ephemeris parameters is a new and feasible method of orbit prediction.

Keywords: Autonomous orbit determination, Broadcast ephemeris,ARMA model, Sliding window model, Parameters prediction, Orbit prediction

1. Introduction

Currently, short-term prediction result calculated by ground orbit determination system is injected into the satellite, and then GNSS satellite orbit information is broadcast via broadcast ephemeris. Several navigation systems are equipped with perfect ground systems to ensure high ephemeris prediction accuracy of navigation stars. But how to maintain normal operation of the navigation system when the perfect ground end is out of work under attack or destruction remains a problem. In order to prevent wartime ground facilities from being destroyed and strengthen their survivable capability, the United States proposed the autonomous orbit determination conception and realized autonomous orbit determination on the satellite by inter-satellite link and precise ranging. This technology has been applied in the BLOCK IIR and its follow-up satellites. The GALILEO under construction and the upgrading GLONASS also adopt this technology [1-2].

Autonomous navigation needs predicted ephemeris of a certain length of time provided by the ground station to upload them to the satellite [3]. The predicted ephemerides are considered as reference ephemeris which would be corrected according to inter-satellite observable quantities, enabling autonomous orbit determination function [4]. Traditional medium and long term orbit prediction methods made orbit predictions by dynamic models of satellites [1, 5-7]. However, since dynamic modeling process was
too complicated, further exploration of new effective methods for medium and long term orbit prediction was still necessary.

In practical applications, the orbit position of the satellite is calculated by the broadcast ephemeris. Through sufficient statistical analyses of each orbit parameter of the satellite broadcast ephemeris, strong periodicity and tendency were found, thus a new medium and long term orbit prediction method that based on satellite broadcast ephemeris parameters burgeoned. According to long-term statistics, the periods of the orbit parameters are within 30 days and selecting 40-day length of time could be sufficient to reflect periodicity of the orbit parameters. 25 days of broadcast ephemeris data are predicted using 40 days of IGS broadcast ephemeris data and its parameter prediction errors are analyzed, as well as the error between the orbit position calculated based on parameter prediction and the orbit position calculated by actual broadcast ephemeris parameters. The results show that in the case of a satellite navigation system losing ground station control support, adopting the satellite broadcast ephemeris data to long-term orbit prediction is feasible and has a certain reference value for autonomous navigation of satellite navigation system in the future.

2. Differences from the Traditional Orbit Prediction Base on Dynamic Model

The autonomous navigation orbit determination proposed by the United States was based on inter-satellite ranging and its accuracy depended on the accuracy of the mechanical model and inter-satellite ranging [8-9]. Since the force model of the satellite was so complex that its accurate establishment involved astronomy, geodynamics, satellite and a variety of associated interdisciplinary theories and techniques [5-8]. The error of the satellite position calculated based on broadcast ephemeris parameters is limited in meters. All the real-time location requirements can be achieved by broadcast ephemeris. So making satellite orbit prediction from the perspective of broadcast ephemeris is feasible. The proposed method does not involve complex dynamic models, but takes full advantage of relatively mature observable history data of the orbit. Firstly, model and forecast each orbit parameter one by one based on broadcast ephemeris history data of the orbit parameters. Then integrate the forecasted values of all orbit parameters at respective forecast time. Finally, calculate satellite orbit coordinate at reference time according to the forecast broadcast ephemeris. This is the new exploration of satellite orbit prediction approach.

3. Change Analyses of Satellite Orbit Parameters

Orbit parameters broadcast via broadcast ephemeris are shown as follows[10]: $\sqrt{A}$, $e$, $i$, $\omega$, $\Omega$, $M$, $\Delta \nu$, $\dot{\Omega}$, $i$, $C_{ic}$, $C_{is}$, $C_{rc}$, $C_{rs}$, $C_{uc}$, $C_{us}$. Basic Kepler orbit parameters and some orbit perturbation parameters are included. The satellite position in the earth centered earth fixed coordinates could be calculated according to above parameters. Orbit determination takes place in two steps. Firstly, fit a Kepler ellipse by using observations. Then taking this ellipse as reference, improve the elliptical orbit considering the perturbation acceleration. Among the parameters, $\sqrt{A}$, $e$, $i$, $\omega$, $\Omega$, $M$ define a standard ellipse orbit, the six sine-cosine parameters $C_{ic}$, $C_{is}$, $C_{rc}$, $C_{rs}$, $C_{uc}$, $C_{us}$ describe deviation between the actual satellite orbital motion and this standard orbit. Broadcast ephemeris of the GPS satellites is broadcast once every two hours, the satellite orbit coordinate at any time would be calculated with the parameters $\Delta \nu$, $\dot{\Omega}$, $i$ by reference to the reference time, which also called broadcast time.
According to the broadcast ephemeris data provided by the IGS, long time statistical analyses of the satellite orbit parameters found that 15 orbit parameters follow their own changes with strong regularity. 5 days of data changes of Kepler orbit parameter $\sqrt{A}$, with the 2-hour data sampling interval, is illustrated in Figure 1-a. It can be seen that there is a strong periodicity with a period of about 12 hours. MEO satellite acts as the GPS, orbiting the earth every 11 hours and 58 minutes, which explains the cause of periodicity of the parameters to a certain extent. 100 days of data changes of parameter $\sqrt{A}$ are shown in Figure 1-b, which illustrates that the satellite orbit parameter has a dual periodicity, in other words, a small cycle and a large cycle. At the same time there is a certain trend. Since the satellite is acted upon by a variety of forces in rapid motion. So the satellite orbit parameters can be considered as a superposition of periodicity, trend and noise. Such characteristics are found by statistical analyses of satellite orbit parameters, and only the trend of parameter $\sqrt{A}$ is given here due to space limitation. According to these results of statistical analyses, such a hypothesis is proposed: The satellite position coordinate is determined by the satellite orbit parameters sent by the broadcast ephemeris, and one by one analysis a conclusion can be reached that the orbit parameters follow certain trend. Can we predict the satellite orbits by the prediction of satellite orbit parameters?

Figure 1. Trends of Satellite Orbit Parameter $\sqrt{A}$

4. The Prediction Algorithm

Through statistical analyses of the long-term variation characteristics of 15 orbit elements and preliminary comparative analysis of the prediction results of several common models, it is found that adopting ARMA model combined with sliding window model to make predictions is feasible.

4.1. Principle of ARMA Model

ARMA model, founded by Box and Jenkins, is a commonly used stochastic time series model, also known as the B-J method. There are three basic types of ARMA model: Auto-regressive (AR) model, moving average (MA) model and auto-regressive moving average (ARMA) model [11]. As one of the effective modern data processing methods, it is known as the most complex and the most advanced model among the time series prediction methods and has been widely applied for more than 30 years in many areas [12-13].

Mathematical model of ARMA:
Set \( \{ x_t, t = 0, \pm 1, \pm 2, \ldots \} \) as a zero mean stationary series which meets the following Model:

\[
x_t = \varphi_1 x_{t-1} + \varphi_2 x_{t-2} + \cdots + \varphi_p x_{t-p} + \theta_1 \varepsilon_{t-1} + \cdots + \theta_q \varepsilon_{t-q} + \varepsilon_t
\]

(1)

Where \( \varepsilon_t \) represents zero mean, variance represents stationary white noise of \( \sigma^2 \), and \( x_t \) represents autoregressive moving average sequence with the order \( p, q \), abbreviated as ARMA \((p, q)\) sequence. When \( q = 0 \), it is AR \((p)\) sequence; When \( p = 0 \), it is MA \((q)\) sequence.

\( \varphi_1, \varphi_2, \ldots, \varphi_p \) represent the autoregressive coefficient and \( \theta_1, \theta_2, \ldots, \theta_q \) represent the moving average coefficient, both of them are unknown parameters to be estimated.

ARMA model can only handle time series model of stationary process, but the changes of the orbit parameters of broadcast ephemeris are of periodicity and tendency. So it is necessary to take sequence smooth processing of non-stationary time series and the easiest and the most commonly-used method is carrying out a differential processing for the original non-stationary time series [14]. The key of adopting ARMA model is to determine the order of the model. Two main methods are commonly used for order determination: the correlation function method and the partial correlation function method; AIC criteria order determination method and BIC criteria order determination method. AIC and BIC criteria are most widely used. AIC criteria were promoted and developed by the Japanese scholar Akaike to apply to the AR model order identification based on the minimum criteria for the final prediction error (FPE) which is an ARMA model order identification method, known as the Akaike Information Criterion, and then Akaike introduced a new criteria called BIC from the viewpoint of Bayes.

AIC and BIC functions are defined as:

\[
\text{AIC}(p, q) = \ln \hat{\sigma}^2(p, q) + 2(p + q)
\]

\[
\text{BIC}(p, q) = \ln \hat{\sigma}^2(p, q) + 2(p + q) \ln n
\]

(2)

Where \( n \) represents the sample size of the model, \( p, q \) represents the order of the model and \( \hat{\sigma}^2 \) represents the residual sequence variance \( \text{var}(\varepsilon_t) \).

4.2. Prediction Example Based on ARMA Model

The following figure illustrates processing flow of orbit parameter prediction based on ARMA model by an example of orbit parameter \( i_0 \). Because of the small value of \( i_0 \), the several digits after the decimal point have a huge impact on the orbit prediction and are magnified by 1.0E+006 times for convenient observation. The change trend of original modeling data is shown in Figure 2-a. Figure 2-b illustrates the results after periodic removal processing of the data. Figure 2-c illustrates the results after further smooth processing of the data. Comparison between predicted value by ARMA model and actual value is shown in Figure 2-d.

4.3. Sliding Window Principle

According to change characteristics of several orbit parameters, the sliding window smooth prediction method is proposed. Sliding window principle, which based on analysis of data periodicity, is to take the frontal large cycle \( N \) as the window length and take smooth processing along the sliding shaft by a certain sliding step(the sliding step here is \( N \), the window length) as shown below. This prediction method is useful under the precondition of strong periodicity and tendency of the data. A schematic diagram of sliding window smooth prediction is given below.
5. Example and Analyses

GPS broadcast ephemeris parameters are broadcast once every two hours. The satellite orbit coordinate at any time is calculated by reference to the reference time which is also called broadcast time. Accuracy of satellite position calculation in the whole motion process is determined by accuracy of satellite position calculation at the reference time. According to satellite orbit solving equations based on broadcast ephemeris, only 12 orbit parameters including $\sqrt{A}$, $e$, $i_0$, $w$, $\Omega_0$, $C_{ic}$, $C_{is}$, $C_{rc}$, $C_{rs}$, $C_{uc}$, $C_{us}$ are required to calculate satellite position at the reference time. In order to simplify the satellite orbit analysis at the broadcast time, these 12 parameters are analyzed in detail.

5.1. Data Source

Broadcast ephemeris data are selected from September 22, 2012 to November 25, 2012 from brdc****.12n broadcast ephemeris data released by the IGS official website. These 65 days of broadcast ephemeris data are used for modeling, prediction and verification. 40 days of data from September 22, 2012 to October 31, 2012 are used for modeling to predict changes of the satellite orbit in the next 25 days and a comparative analysis of the prediction value and 25 days of actual satellite orbit parameters selected from November 1, 2012 to November 25, 2012 is made to verify the prediction.
accuracy of ARMA model. Broadcast ephemeris data are released once every two
hours, so the interval of data is two hours. Data length of the model is 480 epochs and
data length of the prediction is 300 epochs.

5.2. Prediction Results of ARMA Model

5.2.1. Predictions of Kepler Orbit Parameters

Predictions of six Kepler orbit elements √A, e, i₀, w, Ω₀, M₀ are made
respectively and the results are shown below. Among the orbit elements, √A, e, i₀,
w are predicted based on the ARMA model while Ω₀, M₀ are predicted by the
sliding window principle.

5.2.2. Predictions of Orbit Perturbation Parameters

Prediction of six orbit perturbation parameters, C_{ic}, C_{is}, C_{rc}, C_{rs}, C_{uc}, C_{us} are
made respectively using the ARMA model and the results are shown below.

After being detailed analyses of each orbit parameter, Ω₀ and M₀ are found to have
such strong regularity that their future data changes could be completely predicted by the
sliding window method with its history data and change characteristics. Figure 4 and 5
model, predict and contrast Kepler orbit parameters and orbit perturbation parameters
respectively, where the black lines of data in each sub-module represent modeling data,
the red ones represent the prediction of the model and the blue ones represent actual value
of orbit parameters broadcast by broadcast ephemeris. Thus the orbit parameter prediction
qualities of the models are directly reflected. The change of each parameter follows
certain regularity including large cycles, small cycles and trends as shown above, which
provides precondition and basis for the proposed attempt to predict satellite orbit by
prediction of satellite orbit parameters.

5.3. Prediction Results of ARMA Model

In order to evaluate the prediction accuracy of the model and the feasibility of this new
method of satellite orbit prediction based on orbit parameters prediction, the influences of
accuracy of satellite orbit prediction from prediction error of each parameter and from
their joint action are analyzed under different forecast time of 1 day, 7 days, 15 days and
25 days.

Kepler orbit parameters describe the basic orbit of satellites in motion and orbit
perturbation parameters modify it. In the prediction of orbit parameters, prediction
accuracy of Kepler orbit elements plays a decisive role in the ultimate calculation
accuracy of orbit with a great influence factor. As shown in Table 1 and Table 2, a
seemingly small prediction error of Kepler orbit elements could cause a great deviation of
the satellite orbit. Impacts of orbit perturbation parameters are generally within several
meters to a dozen meters and have little effect on the orbit prediction errors. Prediction
error of Kepler parameters sqrt(A) has a relatively small influence factor with about 10
meters in comparison with several other orbit parameters. Since orbit parameters Ω₀ and
M₀ can be completely controlled within the prediction, the key to orbit parameter
prediction is actually the prediction of orbit elements e, i₀, w and their prediction
accuracy directly determines the prediction accuracy of the satellite orbit. Table 1 shows
prediction error of each orbit parameter under different forecast time of 1 day, 7 days, 15
days and 25 days. Table 2 corresponds to Table 1 and illustrates the error influence of
orbit position from prediction error of each parameter and from the result of their joint
action under above mentioned different forecast time. As can be seen from Table 2,
predicting satellite broadcast ephemeris parameters by using the relevant prediction
models and calculating satellite orbit coordinate based on the prediction achieves
relatively satisfactory results with one-day orbit prediction accuracy of about 20 meters and the 25-day prediction accuracy within 150 meters. Since the broadcast ephemeris parameters themselves are contaminated by errors, there is an error of few meters between the satellite orbit calculated by broadcast ephemeris and the orbit provided by IGS precise ephemeris. So it is understandable that there are errors in the satellite orbit prediction based on the orbit parameters prediction.

At the same time, it can also be seen from Table 2 that the error influence of satellite orbit position from prediction error of orbit element \( w \) alone is greater than that from the result of joint action of error influences of various orbit parameters, which shows that the accuracy of the satellite position coordinates is the result of joint action of various parameters but not the superposition results of position errors brought by errors of various parameters.

**Figure 4. Prediction Results of Kepler Orbit Parameters**
Figure 5. Prediction Results of Orbit Perturbation Parameters

Table 1. Prediction Error of Each Parameter Under Different Forecast Time

<table>
<thead>
<tr>
<th>Orbit parameters</th>
<th>1d (RMS)</th>
<th>7d (RMS)</th>
<th>15d (RMS)</th>
<th>25d (RMS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>sqrt(A)/sqrt(m)</td>
<td>0.0009817</td>
<td>0.001107416</td>
<td>0.001154912</td>
<td>0.001170352</td>
</tr>
<tr>
<td>ε /dimensionless</td>
<td>2.90E-08</td>
<td>2.05E-07</td>
<td>4.44E-07</td>
<td>1.25E-06</td>
</tr>
<tr>
<td>i₀ / radians</td>
<td>2.00E-07</td>
<td>6.07E-07</td>
<td>1.23E-06</td>
<td>6.52E-06</td>
</tr>
<tr>
<td>w / radians</td>
<td>9.03E-07</td>
<td>2.70E-06</td>
<td>4.15E-06</td>
<td>6.06E-06</td>
</tr>
<tr>
<td>Ω₀ / radians</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>M₀ / radians</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>cᵢ₁c / radians</td>
<td>1.89E-07</td>
<td>1.15E-07</td>
<td>9.36E-08</td>
<td>8.20E-08</td>
</tr>
<tr>
<td>cᵢ₁s / radians</td>
<td>2.32E-08</td>
<td>1.42E-08</td>
<td>1.62E-08</td>
<td>2.58E-08</td>
</tr>
<tr>
<td>cᵢ₉c / radians</td>
<td>4.363610548</td>
<td>12.76013499</td>
<td>11.26938001</td>
<td>9.849503608</td>
</tr>
<tr>
<td>cᵢ₉s / radians</td>
<td>2.50249376</td>
<td>3.363472068</td>
<td>5.327543152</td>
<td>6.85431484</td>
</tr>
<tr>
<td>Cᵤc / radians</td>
<td>2.29E-07</td>
<td>2.62E-07</td>
<td>4.08E-07</td>
<td>5.28E-07</td>
</tr>
<tr>
<td>Cᵤs / radians</td>
<td>1.09E-07</td>
<td>5.92E-07</td>
<td>6.72E-07</td>
<td>5.56E-07</td>
</tr>
</tbody>
</table>
Table 2. The Error Influence of Orbit Position from each Parameter Under Different Forecast Time (unit/m)

<table>
<thead>
<tr>
<th>Orbit parameters</th>
<th>1d (RMS)</th>
<th>7d (RMS)</th>
<th>15d (RMS)</th>
<th>25d (RMS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sqrt(A)</td>
<td>10.0641</td>
<td>11.3529</td>
<td>11.8398</td>
<td>11.9981</td>
</tr>
<tr>
<td>e</td>
<td>1.4071</td>
<td>9.9469</td>
<td>21.5435</td>
<td>60.6517</td>
</tr>
<tr>
<td>i0</td>
<td>3.3456</td>
<td>10.1539</td>
<td>20.5755</td>
<td>109.0668</td>
</tr>
<tr>
<td>w</td>
<td>23.8549</td>
<td>71.327</td>
<td>109.6323</td>
<td>160.0895</td>
</tr>
<tr>
<td>Ω0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>M0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>C ic</td>
<td>0.6262</td>
<td>0.381</td>
<td>0.3101</td>
<td>0.2717</td>
</tr>
<tr>
<td>C is</td>
<td>0.3804</td>
<td>0.2328</td>
<td>0.2656</td>
<td>0.423</td>
</tr>
<tr>
<td>C rc</td>
<td>0.8643</td>
<td>2.5275</td>
<td>2.2322</td>
<td>1.951</td>
</tr>
<tr>
<td>C rs</td>
<td>2.4528</td>
<td>3.2968</td>
<td>5.222</td>
<td>6.7185</td>
</tr>
<tr>
<td>C uc</td>
<td>1.1983</td>
<td>1.371</td>
<td>2.1349</td>
<td>2.7628</td>
</tr>
<tr>
<td>C us</td>
<td>2.8224</td>
<td>15.3292</td>
<td>17.4007</td>
<td>14.397</td>
</tr>
<tr>
<td>The result of joint action</td>
<td>21.1843</td>
<td>48.8493</td>
<td>76.5647</td>
<td>142.1994</td>
</tr>
</tbody>
</table>

6. Conclusion

Solving the problem of navigation constellation autonomous orbit involves ephemeris prediction with considerable accuracy, thus the improvement of prediction accuracy is of great significance to navigation accuracy [15]. In order to improve the accuracy and real-time performance of GPS satellite orbit calculation in the absence of ground station support, a lot of useful and important information is obtained from a large amount of long-term statistical analyses on the GPS satellite broadcast ephemeris data. In real-time positioning, satellite position calculation is completed by Kepler orbit parameters and orbit perturbation parameters broadcast by the broadcast ephemeris. Certain periodicity and tendency are found in each orbit parameters through long-term statistical analyses. History data of each orbit parameter can be used to predict corresponding orbit parameters in the future, which makes it possible to predict satellite ephemeresis data at some time in the future by using broadcast ephemeresis history data in the absence of ground tracking station support. With only analysis of the orbit parameters and avoiding the influence of other unknown factors, this method simply and effectively replaces the complex dynamic modeling process which is common in autonomous orbit determination for the satellite orbit prediction.

In order to verify the feasibility and accuracy of this method, 12 parameters which jointly determine broadcast ephemeresis orbit position at the reference time, including Kepler orbit parameters $\sqrt{A}$, $e$, $i0$, $w$, $\Omega0$, $M0$ and orbit perturbation parameters $C_{ic}$, $C_{is}$, $C_{rc}$, $C_{rs}$, $C_{uc}$, $C_{us}$ are used to make ARMA model prediction for a certain time. Satellite position errors generated by prediction error of each parameter are counted. Prediction error of each parameter and its resulting satellite position errors are analyzed under different forecast time, as well as the satellite position error due to joint action of various parameters. How to adopt different algorithms to improve the accuracy of satellite positioning in exceptional circumstances has been a subject of great concern [16]. A new satellite orbit prediction method is proposed to predict satellite ‘broadcast ephemeris’ data at some time in the future by using broadcast ephemeresis history data based on the analyses of long-term change characteristics of orbit parameters. The accuracy and feasibility of the method are verified by abundant reliable data analyses. This attempt of orbit prediction method has a certain theoretical value and reference value for orbit prediction of navigation satellite for a certain period of time and deserves further study.
References


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Yunhang Zhu, received her B. S. degree in Electronic Engineering (1992) from Beijing Information Science & Technology University and M.S. degree in information management (2007) from Graduate Wuhan University. Now she is a associate professor in Department of Information Engineering of Hunan College Information. Her current main research interests include information processing and large scale integrated circuit design.