

Optimal Routing Strategy on Scale-free Networks with Heterogeneous Delivering Capacity

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Abstract

We proposed the information traffic dynamics on scale-free networks considering the heterogeneous delivering capacity. In the previous researches, the delivering capacity is the same for all nodes in the system, which obviously contradicts the real observations. In this paper, a heuristic algorithm for the optimization of transport is proposed to enhance traffic efficiency in complex networks, where each node's capacity is set as $c_i = k_i$ and k_i is the degree of node i . Our algorithm balances traffic on a scale-free network by minimizing the maximum effective betweenness, which can avoid or reduce the overload in some busy nodes. The simulation result shows that the network capacity can reach a very high value, which is four times more than that of the efficient routing strategy. The distribution of traffic load is also studied and it is found that our optimal routing strategy can make a good balance between hub nodes and non-hub nodes, resulting in high network capacity.

Keywords: routing strategy; heuristic algorithm; heterogeneous capacity; scale-free networks; network capacity

1. Introduction

Complex network theory can describe many natural and social systems with nodes representing individuals and links representing connections. Researches on complex networks have been flourishing since the discovery of small-world phenomenon by Watts and Strogatz [1] in year 1998 and scale-free property by Barabasi and Albert [2] one year later at the end of last century [3, 4]. One important research field is the network traffic problem.

In the past few years, the phase-transition phenomena [5, 6], the scaling of traffic fluctuations [7, 8], and the routing strategies [9-19] of networked traffic have been widely studied. And Models are proposed to mimic the traffic on complex networks by introducing packets generating rate as well as randomly selected sources and destinations of each packet. In these models, the capacity of networks is defined by a critical packet generating rate R_c at which a continuous transition occurs from free flow to congestion. In the free state, the numbers of created and delivered packets are balanced, leading to a steady state. However, in the jammed state, the number of accumulated packets increases with time due to the limited delivering capacity of each node.

Recent research has focused on two ways to maintain network efficiency: (1) modifying the network structure; (2) developing reliable routing strategies. Since it is normally more expensive to modify the network topology, adopting efficient routing

strategies is preferred. Consequently, various routing strategies are proposed, including random walk, shortest path, efficient path [9], local routing strategy [11], pheromone routing strategy the global dynamic routing strategy [12-13], static optimal packet routing strategy and traffic resource allocation for complex networks [14-15].

However, previous routing strategies, such as the famous efficient routing strategy and the commonly used local routing strategy, did not consider the heterogeneous delivering capacity for each nodes, but simply set a const value to all nodes. In this paper, we consider each nodes with a different delivering capacity as $c_i = k_i$, where k_i is the degree of node i . The heuristic algorithm aims at balancing traffic load on complex networks by minimizing the maximum node effective betweenness. The resulting routing strategy (optimal routing strategy) can avoid or reduce the overload of crucial hub nodes, leading to a very high network capacity.

This paper is organized as follows. In Section 2, we describe the network model and traffic rules in detail. In Section 3, the simulation results are presented and discussed. We conclude this paper in Section 4.

2. Network Traffic Model

Following common practices, we adopt the well-known Barabasi-Albert (BA) model to represent the underlying network. This model starts with m_0 fully connected nodes, a new node with m edges ($m \leq m_0$) is added to the existing graph at each time step according to the preferential attachment rule, *i.e.*, the probability \prod_i of being connected to the existing node i is proportional to the degree k_i of the node,

$$\prod_i = \frac{k_i}{\sum_j k_j} \quad (1)$$

In our system, the packets are delivered from sources to destinations following a given routing strategy. The detailed traffic model is described as follows: At each time step, there are R packets enter the system with randomly chosen sources and destinations. In each time step, one node can deliver at most C packets to its neighboring nodes. In this paper, each node's C is set as $C = k$, where k is degree of node. The packets will be removed from the system once they arrive at their destinations. The queue length of each node is assumed to be unlimited and the FIFO (first-in-first-out) discipline is applied to each node.

In order to characterize the network capacity which describes the phase transition of traffic flow, we use the order parameter presented in Reference [20].

$$\eta(R) = \lim_{t \rightarrow \infty} \frac{C \langle \Delta Np \rangle}{R \Delta t} \quad (2)$$

Where $\Delta Np = Np(t + \Delta t) - Np(t)$, $\langle \dots \rangle$ indicates the average over time windows of width Δt , and $Np(t)$ is the total number of packets within the network at time t . The order parameter represents the balance between the inflow and outflow of packets. In the free flow state, due to the balance of created and removed packets, η is around zero. With increasing packet generation rate R , there will be a critical value of R_c that characterizes the phase transition from free flow to congestion. When R exceeds R_c , the packets accumulate continuously in the network, and η will become larger than zero. Thus the network capacity can be measured by the maximal generating rate R_c at the phase-transition point.

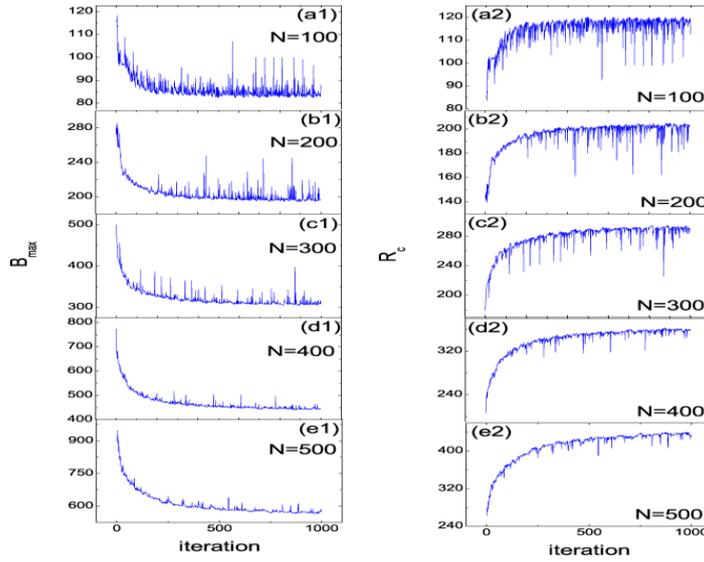


Figure 1. (Color online) (a1-e1) Maximum Effective betweenness B_{max}^{eff} as a Function of the Number of Iterations for Network with 100,200,300,400,500 Nodes. (a2-e2) the Network Capacity R_c as a Function of the Number of Iterations for Network with 100,200,300,400,500 Nodes

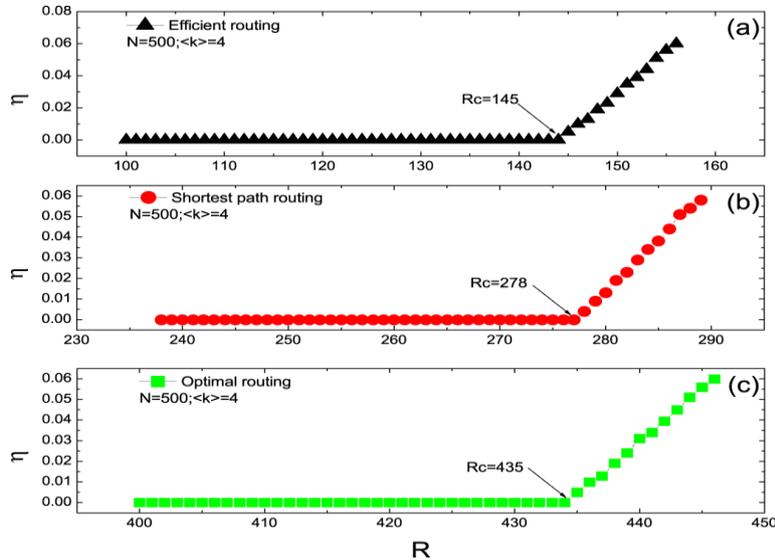


Figure 2. (Color online) the Order Parameter η VS R under the Three Routing Strategies

Next, we will introduce the optimal routing strategy considering the node delivering capacity C . In a standard model where each node owns the same fixed capacity C , the congestion will first occur at the node with the largest betweenness B_{max} . In our model with heterogeneous delivering capacity, to find the bottleneck of a network traffic system, we introduce an indicator of effective betweenness for nodes:

$$B_i^{eff} = \frac{B_i}{C_i} \quad (3)$$

Where C_i is the delivering capacity of node i , and B_i is the betweenness of node i . Obviously, the congestion will first occur at the node with the largest effective betweenness, and the network's capacity R_c can be estimated as [9]:

$$R_c = \frac{N(N-1)}{B_{\max}^{eff}} \quad (4)$$

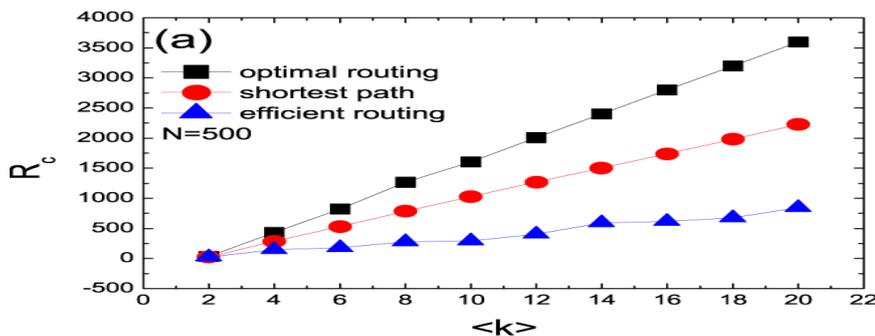
where B_{\max}^{eff} is the largest effective betweenness of the network, and N is the size of the network. Therefore, the goal of the optimization process is to minimize the largest effective betweenness in the system. The optimal routing strategy can be described as follows [13]:

- (1) Assign weight 1 to every link and find the shortest paths between all pairs of nodes.
- (2) Compute the effective betweenness of each node.
- (3) Find the node which has the largest effective betweenness B_{\max}^{eff} , and add 1 to the weight of every link that connects it to the other nodes.
- (4) Recompute the shortest paths, go back to Step 2.

3. Simulation Results

Figure 1(a1)-1(e1) show the evolution of the maximum effective betweenness as a function of the number of iterations in the scale-free networks. As is shown, for each network, the maximum effective betweenness B_{\max}^{eff} is not monotonic with the increase of iteration, instead of decreasing to a constant value. From Equation 4, one can see, the network's capacity R_c is inversely proportional to the maximum effective betweenness B_{\max}^{eff} . As is expected, the network's capacity R_c increases with the increase of iteration to an almost constant value. This is exemplified in Figure 1 (a2)-1(e2). In this way, the routing paths change from the shortest path to the optimal routing strategy.

Figures 2(a)-(c) show the evolution of the order parameter η with the increase of R under the efficient, shortest, and optimal routing path with the network size $N = 500$ and average degree $\langle k \rangle = 4$. As is shown, the network capacity R_c under optimal routing strategy can reach a very high value ($R_c = 435$), compared with that of shortest path routing strategy ($R_c = 278$), and the efficient routing strategy ($R_c = 145$). As far as we know, for all static routing strategies, the network capacity under optimal routing strategy is the highest.



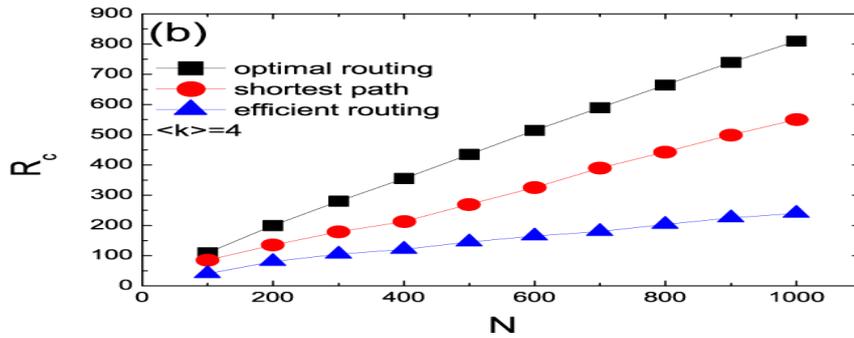


Figure 3. (Color Online) (a) The Network Capacity R_c VS Average Degree $\langle k \rangle$ with the Same Network Size $N = 500$ under the Three Different Routing Strategies. (b) The Network Capacity R_c VS Network Size N with the Same Average Degree $\langle k \rangle = 4$ Under the Three Routing Strategies

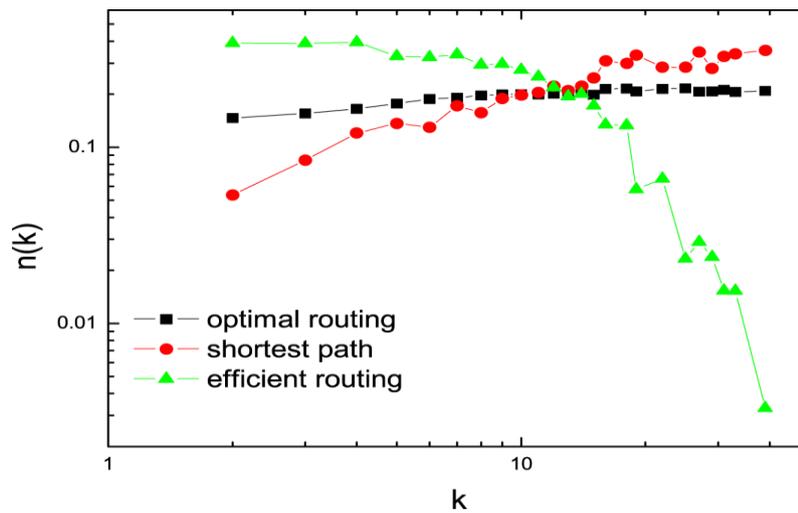


Figure 4. (Color Online) Distribution of Average Number of Packets on Nodes with Degree k under the Three Routing Strategies

In Figure 3, we show the relation of network's capacity R_c VS the average degree $\langle k \rangle$, and VS the network size N under three routing strategies. Figure 3(a) shows that with the same network size, the network's capacity increases with the average degree $\langle k \rangle$ under three routing strategies. The rank of network capacities is optimal routing > shortest path routing > efficient routing. In Figure 3(b), one can also see that the network's capacity increases with the increasing of the network size N . Again, with the same average degree or network size, the network's capacity under the optimal routing strategy is the largest.

To better understand why the optimal routing strategy can achieve the higher network capacity, we investigate the average packet number $n(k)$ of nodes as a function of its degree k under three routing strategies in the free-flow state. As is shown in Figure 4, when all packets are delivered by the shortest path strategy, the average packet number $n(k)$ follows a power-law distribution: $n(k) \propto k^\alpha$ with $\alpha > 0$, which reflects the high heterogeneous traffic on each node. Compared with the nodes with small degree, the nodes with large degree are more burdened. In efficient routing strategy, the $n(k)$ also follows a power-law distribution: $n(k) \propto k^\alpha$ but with $\alpha < 0$, which indicates that the

nodes with small degree are more burdened. Compared with the shortest path and the efficient routing strategies, $n(k)$ is essentially independent of k under optimal routing strategy. This means that, in the packet delivering process, every node, either high-degree or low-degree, undertakes the same burden, which leads to a larger network capacity.

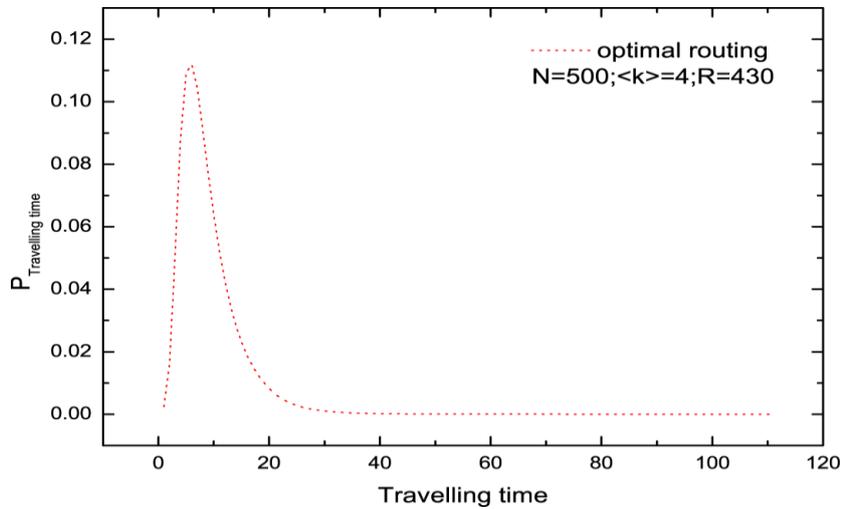


Figure 5. (Color Online) The Probability Distribution of Travel Time under the Optimal Routing Strategies

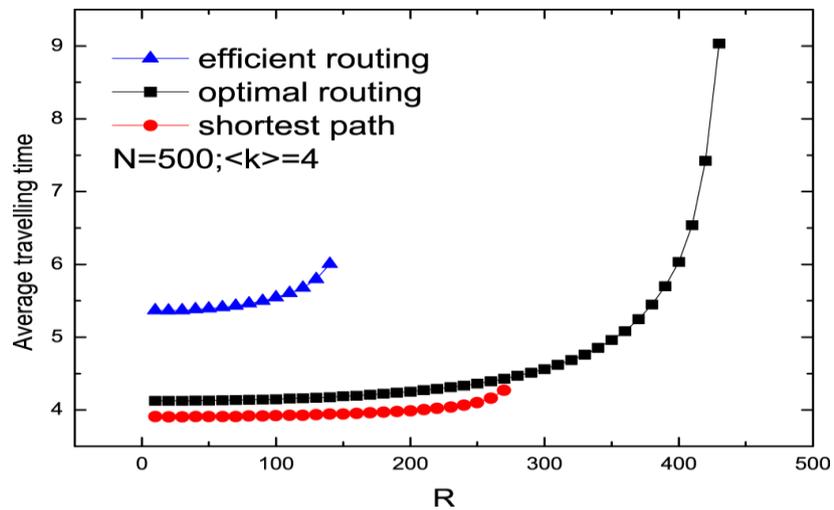


Figure 6. (Color Online) the Average Packets Travel Time $\langle T \rangle$ that Packets Spend in Free-flow State, $R < R_c$

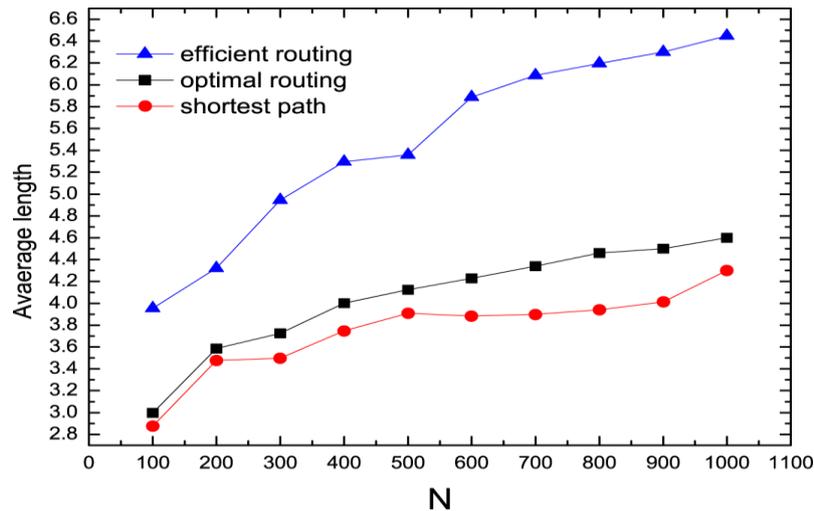


Figure 7. (Color Online) the Average path Length VS Network Size N for Optimal Routing Strategy

The efficiency of the routing strategy can be characterized not only by the network capacity but also by the packets transmission speed. We investigated the probability distribution of traveling time and average traveling time respectively under optimal routing strategy in the free-flow state. Packet traveling time is also an important factor for characterizing the network's behavior. The traveling time is the time that a packet spends on traveling from a source to a destination. As is shown in Figure 5, the probability of traveling time approximately follows a Poisson distribution under optimal routing strategy. It indicates that, the time spent on the path from sources to destinations are approximately the same for most packets. In Figure 6, we investigate the average packets travel time $\langle T \rangle$ that packets spend in the network in free-flow state ($R < R_c$). As is shown, for each free-flow state, when all packets are delivered by the efficient routing strategy, the average travel time spend in the network is largest, while shortest by the shortest path strategy.

Finally, we investigate the average path length for optimal routing strategy with different network sizes. The path length is defined as the number of nodes that the packet travels from the sources to destinations. As is shown in Figure 7, for each network size, the rank of the average path length is efficient routing $>$ optimal routing $>$ shortest path routing. This phenomenon is consistent with the result of Figure 6, *i.e.*, for the same N , the longer of path length, the longer of travel time.

4. Conclusion

In summary, we introduced an optimal routing strategy on scale-free networks with heterogeneous node capacity. Compared with the traditional routing strategies, our strategy can well balance the traffic load by minimizing the maximum effective betweenness, which can avoid or reduce the overload in some busy nodes. The simulation result shows that the network capacity is four times more than that of the famous efficient routing strategy. Because the heterogeneous node capacity is universal in many real communication or transportation systems, our study may be useful for understanding the behavior of network traffic system and for evaluating the overall efficiency of the system.

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