International Journal of Future Generation Communication and Networking
Vol.7, No.4 (2014), pp.119-126
http://dx.doi.org/10.14257/ijfgcn.2014.7.4.11

Around of Modeling Complex Network via Graph Theory

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Abstract

A complex network is a interaction network of entities where global behavior is not deductible from the individual behaviors of each entities, leading to new properties emergence. Our problem is the network analysis ad modeling. Network analysis needs a formalism to assemble together the structure (static approach) and the function (dynamic approach), and to have a better understanding of the networks characteristics. In this paper, we introduce common used network modeling based on graph theory, having the role to simulate complex networks.

Keywords: *Graph Theory, Small Word, Algorithmic, Complex Network, Clustering Coefficient, Assortativity*

1. Introduction

1.1. Definition

A complex system is a set consisting of a large number of interacting entities that prevent the observer to provide, via calculation, its feedback, behavior or evolution.

Thus, a chemical reaction is simple because we know in advance the outcome. Indeed, a few equations are used not only to describe the evolutionary process, but the future or final states of the system. It is not necessary to attend to the phenomenon or realize an experiment to see what will result in reality. Contrariwise, the nerve cells in our brain or colony of ants are complex systems because the only way to know the evolution of the system is to experience, possibly on a reduced model.

In other words, when one wants to model a system, we design a number of rules of evolution, then we can simulated the system by iterating these rules until obtain a structured result. A system is expressed complex if the final result is not directly predictable by knowing the rules that say how the system changes.

In the mathematical, this results by the inability to model the system by solvent predictive equations. What is important is not so much the number of factors or dimensions (parameters, variables), but the fact that each of them indirectly influences the other, which themselves in turn influence, making the behavior of a system irreducible whole. To predict this behavior, it is necessary to take them all into account, which is to perform a simulation of the system studied.

ISSN: 2233-7857 IJFGCN

1.2. Complex Networks Behavior

A complex network covers most of the following behaviors. Thereby reciprocally define a complex network: it is a system with a large number of the following behaviors. It is unusual to define a class of objects to study from their behavior rather than from their constitution.

- self-organization and emergence of properties or coherent structures, pattern appearance. This characteristic is often required to describe a complex network.
- Local robustness and fragility: since there are many links, if an element is affected by an event outside, its neighbors will be too. It follows that the system is often more robust to a small local disturbance than it would be without the links. But at the same time, modify the overall system can be done through a smaller disturbance than in the system without links.
- symmetry breaking: the knowledge of a part of the system does not allow to say that the rest of the system is on average in the same state.
- several possible behaviors are competing, some are simple, others chaotic. The system is often at the border between the two and alternates these two types of behavior.
- multiple temporal and spatial scales appear, there is therefore a hierarchy of structures.

2. Complex Networks Classification

According to M.E.J Newman [1], the complex networks can be grouped into four categories:

2.1. Social Networks

A social network is a social structure made up of a set of social actors (such as individuals or organizations) and a set of the dyadic ties between these actors. it is from these networks that modeling the real world was introduced empirically through the Milgram experiment. Most studies on these networks suffer from problems of inaccuracy, subjectivity, and samples with small size.

2.2. Information Networks

A information network may be reported to the classic example of a network of citations between scientific papers. The structure of the information being stored in the nodes, this is why the term information network are used. The World Wide Web with its web pages and hyper-links is also an information network.

2.3. Technological Network

Technological network is a network created primarily by human for the distribution of a service or energy. Power grids and computers networks, are included.

2.4. Biological Network

A biological network is a network of elements affecting the living. An example of biological network may be a network of interactions between proteins.

3. Modeling of Complex Networks

In real networks, while local interactions are generally well known the overall result of all interactions is still poorly understood, for example the emergence property. Understanding these global properties yet touches on issues essential: the dynamics of interactions in a social network or a computer network is for example linked to the issue of the spread of viruses (computer or biological), for power grids, the problem of robustness of a large network.

The recent increase in capacity and collection of a large number of statistical data on such networks has allowed the development of studies of these objects. In particular, it has been observed experimentally that such networks share common macroscopic properties[3].

To model complex systems, generally we construct models themselves potentially complex. Morality, trying to simplified the complex system, we will worsen the complexity by mutilation without solving the problem considered. Thus, analytical modeling through solving models of complex problem that mathematics, statistics of the decision and other propose to treat, involving closure of systems. These models often lead to methods in search of a problem that suits them, while the modeling of contemporary complex systems is looking for methods that suit them. That is why, there are several types of modeling complex networks or in general for complex systems. In this case:

1. The Milgram Experiment: This theory said that each of us on the planet can be connected to another person by following a chain of knowledge containing no more than six intermediate [2],
2. Random graphs: Such a model consists of a set of n nodes connected by edges which are placed randomly uniformly between pairs of nodes. Erdös and Rényi gave several versions of their model.

The most commonly studied is the one called where each edge between two nodes is present independently with a probability *p* and absent with a probability (1-*p*) [3, 4],

3. Small-World model of Watts & Strogatz: this is a variant of the random graphs [5],
4. Small-World model navigable of Kleinberg: In 2000, Kleinberg proposed the first model of small world having the property of navigability, that is to say the first graph model whose diameter is Polylogarithmic in the number of nodes and whose polylogarithmic paths can be discovered by a decentralized algorithm between any pair of vertices [6],
5. Small-World model of Lebhar based on Kleinberg model: In 2005, Lebhar's research helped develop a very efficient routing algorithm and nearly optimal [7].
6. ,etc.

3.1. Graph Theory

Graph theory is the study of graphs, which are mathematical structures used to model pairwise relations between objects. A "graph" in this context is made up of "vertices" or "nodes" (V set of vertices, $|V|=N$)and lines called edges that connect them (E set of edges). A graph may be undirected, meaning that there is no distinction between the two vertices associated with each edge, or its edges may be directed from one vertex to another.

For all $i\in V$, we call degree for i the number of vertex j such as $(i,j)\in E$, in other word, the number of neighborhood of i in the graph.

A first statistical characterization is distribution of those degrees by definition where is the number of *vertices* having the same degree k.

We distinguish schematically:

- Graphs or homogeneous networks where decreases very rapidly (exponentially or faster) when *k* is away from the average value,
- Graph or heterogeneous networks where extends over many orders of magnitude and decreases slowly at large *k*.

The parameter with other parameters often intervenes to determine the behavior of dynamic phenomena on the network.

4. News Characteristics

4.1. Clustering Coefficient

The clustering coefficient *C* is the probability that two neighbors for a given vertex are neighbors between them [8]. This corresponds to the local density of a vertex.

$$C(i) = 2 * \frac{|\text{ the number of links between the neighbors of } i \,|}{d_i(d_i - 1)}$$

d_i is the degree of node *i*.

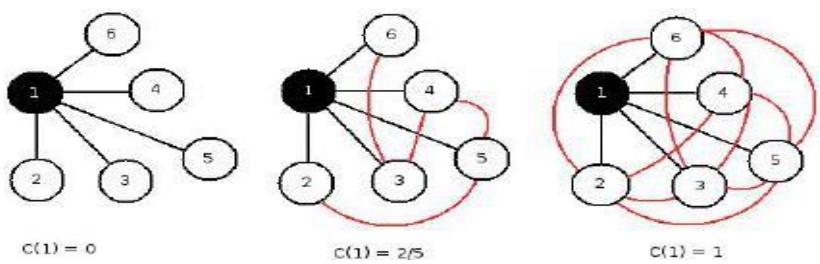


Figure 1. Examples of Clustering Coefficients

4.2. Degree of Correlation

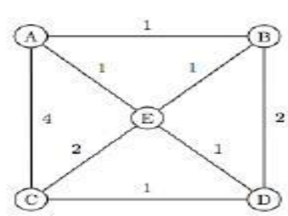
Let d_i : degree of the node i,

d_{ij} : cost of the edge (i, j),

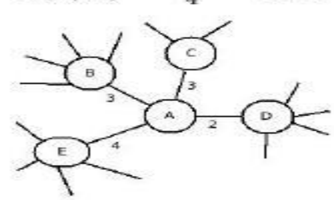
$\Gamma_i = \{ j \in V, (i,j) \in E \}$
V : the set of vertices,
E : the set of edges,
Γ_i : the neighborhood of the node i.

The degree of correlation is defined as the average degree of neighbors of a vertex i (with weighted edges) and noted $d_C(i)$:

$$d_C(i) = \frac{\sum_{j \in \Gamma_i} d_{ij} * d_j}{d_i}$$



$d_C(E) = \frac{15}{4} = 3.75$



$d_C(A) = \frac{21}{2} = 10.5$

4.3. Assortativity

$$d_A(k) = \frac{\sum_{i/k_i=k} d_C(i)}{N_k}$$

with the number of nodes whose degree is equal to k.

The behavior of this function of k defines two classes of networks[9]:

1. Assortativity when increases with k. In social networks, this index corresponds to the fact that high degree vertices are preferentially connected to other vertices of high degree.
2. Disassortativity when decreases with k. In hierarchical network hubs are connected to many vertices of low degree.

4.4. Small Diameter

Formally, the diameter of a network is the longest more shortest paths between two nodes in the network via its connections [10].

In many networks of interactions, the average distance observed is the order of the logarithm of the total number of nodes, while the number links remains much lower than the square of the number of nodes.

5. Existing Models

A model in this sense is to build node per node a network, linking each new node to the existing nodes preferentially according to criteria:

1. Highest degree;
2. Small diameter;
3. Intensive clustering;
4. Distribution of degrees following a probability law.

These criteria allow construction of the network subsequently to measure the diffusion within the network.

5.1. Erdös & Rémi

The model is in the form of random graph $G(n,p)$: a random uniform graph in n nodes. There is an edge between two top with a constant probability p. In this graph, the degree distribution follows a Poisson distribution. This model has several downside, the two important downside are:

1. with experimental observations (impossible in the erdös time) proves that it is rather a power law for the majority of real networks.
2. this graph has a low coefficient of clustering

5.2. Albert & Burabassi

This model is a dynamic model to obtain a degree distribution following the power law. The target of this model is to build node by node a network, linking each new node with existing nodes with preferentially higher degrees.

5.3. Watts & Strogatz

This is a model that has both a small diameter and a high clustering coefficient. From a regular ring of *n* vertices and *k* edges, evenly distributed in relation to their origin, we independently redirect each end edge with a constant probability p to a edge of the ring chosen uniformly randomly.

Downside: The model does not have the dynamic property of navigability.

5.4. Kleinberg

The Kleinberg model of a network is effective at demonstrating the effectiveness of greedy small world routing. The model uses an *n* x *n* grid of nodes to represent a network, where each node is connected with an undirected edge to its neighbors. To give it the "small world" effect, a number of long range edges are added to the network that tend to favor nodes closer in distance rather than farther. When adding edges, the probability of connecting some random vertex *x* to another random vertex *y* is proportional to, where *q* is the clustering coefficient.

Kleinberg proposes a decentralized routing algorithm that computes paths which expectancy length is between any pair of nodes.

This is a greedy algorithm that transmits, in each node, the message at closest neighbor of the target of routing.

6. Axes Research in Complex Networks

Currently, there are several research specifically for the complex system and particularly for complex networks. Following two areas that I consider very important:

- build news decentralized algorithm efficient and dedicated for large networks,
- build new architectures of large networks of small diameter whose shortest paths can be computed in a decentralized manner.

7. Conclusion

The property of small world is sufficiently understood for we can leverage its reproduction on networks constructed, as computer networks, using existing models and decentralized algorithms. However, the emergence of this property in real networks is still largely unknown. The question of the existence of a universal process that would transform every graph into a small navigable world is a fundamental open question for understanding the phenomenon.

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