

The Optimal Coverage and its Features of the Communication Nodes in Ad Hoc Network

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Abstract

In Ad Hoc network, the randomness of the node's location makes the network topology highly dynamic change, so it brings great challenges to design and realize the routing algorithm. In this paper, by using the geometry and optimization method to analyze the coverage formed by two and three intersected communication nodes in the Ad Hoc network, we get some important changing relationships about the factors of the node's distance d and the communication radius r and the coverage area s , and briefly analyze the least number of the communication channels that the other nodes can obtain in the network, and specially discuss the situations of the optimal complete-coverage and its Features. Finally, according to the conclusions, we construct an optimal complete-coverage for a given communication area, and give its logical structure and corresponding formulas calculating the least number of communication nodes. For the researches, it has important significance to design the more efficient routing algorithm and analyze network survivability, and so on.

Keywords: Ad Hoc network, Optimal Coverage, Feature, Geometry Method

1. Introduction

In recent years, the mobile Ad Hoc network becomes a hot research topic in the field of the wireless mobile network and communication technology. To meet some needs, such as battlefield survival and rescue and so on, the mobile infrastructure that can quickly equip and self-organize is the basic element of this network that is different from other commercial cellular systems [1-2]. The Ad Hoc network is a multi-hop and temporary and self-organized wireless mobile communication network that is composed of a group of mobile nodes with wireless transceiver device, it has the Features that do not need the base station and the specific routing and switching nodes, and it can be randomly built and flexibly accessed, and easy to move, and so on. So it can be widely applied to the military wireless communication, and the business mobile meeting, and the home networking, and the emergency services, and the sensor networks, and the personal field network, *etc* [3-4].

For a long time, the research on the Ad Hoc network mainly focus on the routing strategy and algorithm implementation. Therefore, people have put forward many solutions, and the typical routing strategies include the table-driven routing and the on-demand routing, the representative routing protocols have the DSDV, DSR, AODV, *etc*. [5] The table-driven routing strategy need to periodically exchange routing information to take the initiative to find routing, its periodicity is related to the moving speed of node, it need to consume a large amount of network bandwidth, therefore it is suitable for the network that the node mobility is smaller and the number of nodes is large. The On-demand routing strategy does not need to

periodically broadcast routing information, only to discover route when it has not the destination node, but it will cause large delay, so it is suitable for the network that the scale is small and the node mobility is larger. During this period, research on Ad Hoc network is mainly based on planar structure, *i.e.*, all network nodes are in the same plane, each node has the same function and it can use a single routing strategy.

Some experts have put forward the hierarchical structure of the Ad Hoc network, and attempt to hide the details of the network topology through polymerizing layer by layer, and strive to achieve two goals: one is that reduce the routing table size and the communication overhead to exchange routing information required through reducing the number of route nodes; two is that select to generate a more stable sub-network based on the some kind of the clustering strategy, and reduce the impact on the routing protocol that the changing of the topology have brought, and improve the scalability of network, to overcome various problems of the flat routing [6-8].

Being easy to see, whether it is flat routing or hierarchical routing strategy, it fundamentally dependent on the positional relations between the nodes and the Features of the communication coverage area. So we use the geometry and optimization method to analyze the relationships of the coverage formed by two and three intersection nodes in Ad Hoc network, and give the important relationship between the node distance d , the node communication radius r , the coverage area s and other factors, and briefly analyze the least communication channels that the other nodes can obtain in the network, and specially discuss the situations of the optimal coverage. Finally, we use the conclusions of the paper to construct the optimal complete coverage for a given communication area, and give its logic structure figures and corresponding calculation formulas. The conclusions of the paper are conducive to establish and maintain optimal routing table of the nodes, it has important significance to the research works that design the more efficient routing algorithm and analyze network survivability, and so on [9-11].

2. The Coverage and its Features Formed by Two Intersected Nodes

Firstly, we need to discuss the coverage and its Features formed by two intersected nodes in Ad Hoc network, and then extend to more intersected nodes. Suppose there are node A and node B, its communication radius are r , the wireless communication area of the node A is a circular area that the center is point A and the radius is r , denoted by $\odot A$; Similarly, the communication area of the node B is denoted by $\odot B$, the distance between two nodes is the distance between the center A and B, Assume it is d . By the geometric knowledge, two nodes mainly have two kinds of basic geometric relationships in the network: deviation and intersection, which can be seen as a special case of the intersection, so the following we will mainly discuss the situation of two intersected nodes, as shown in Figure 1 (a).

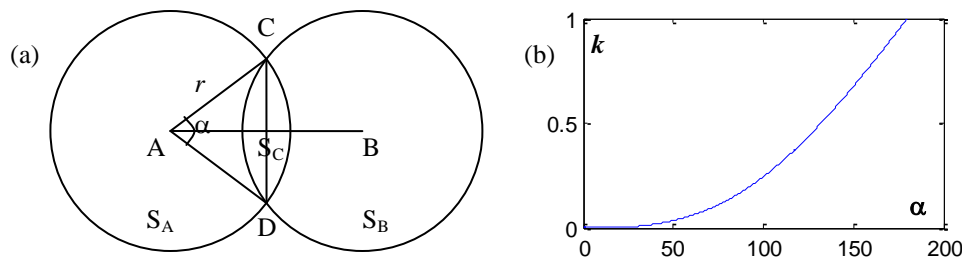


Figure 1. The Coverage and its Features Formed by Two Intersecting Nodes: (a) the Coverage, (b) the Relationships that k Changes with α

2.1. The Coverage Formed by Two Intersected Nodes

In Figure 1(a), line CD is the intersected chord of $\odot A$ and $\odot B$, α is the central angle of the chord CD in $\odot A$, and there have $0^\circ \leq \alpha \leq 180^\circ$. When the $\odot A$ and $\odot B$ are external tangency ($d=2r$), it has $\alpha=0^\circ$, and when the $\odot A$ and $\odot B$ coincide ($d=0$), it has $\alpha=180^\circ$. According to the geometric features of the Figure 1, there have

$$d = |AB| = 2r \cos \frac{\alpha}{2}.$$

Then,

$$\alpha = 2 \arccos \frac{d}{2r} (0 \leq d \leq 2r). \quad (1)$$

According to the Figure 1(a), when two nodes intersect, the whole coverage can be divided into two kinds: the common coverage and the single coverage. Set the area of $\odot A$ is S_A , the area of $\odot B$ is S_B , the area of the common coverage is S_C , the whole coverage is S_{2T} , then there are

$$S_A = S_B = \pi r^2, \quad (2)$$

$$S_C = 2(S_{\text{sectorCAD}} - S_{\triangle CAD}) = 2\left(\frac{\alpha}{360} \cdot \pi r^2 - \frac{1}{2} r^2 \sin \alpha\right) = \pi r^2 \left(\frac{\alpha}{180} - \frac{\sin \alpha}{\pi}\right), \quad (2)$$

$$S_{2T} = S_A + S_B - S_C = 2\pi r^2 - \pi r^2 \left(\frac{\alpha}{180} - \frac{\sin \alpha}{\pi}\right) = \pi r^2 \left[2 - \left(\frac{\alpha}{180} - \frac{\sin \alpha}{\pi}\right)\right]. \quad (3)$$

Assume $k = \frac{\alpha}{180} - \frac{\sin \alpha}{\pi}$, then $S_C = k\pi r^2$, and there have

$$k = \frac{S_C}{\pi r^2} = \frac{S_C}{S_A}. \quad (4)$$

We can see, the proportion coefficient k shows the ratio of the area of the common coverage to the area of one node. Being easy to see, when $0^\circ \leq \alpha \leq 180^\circ$, there are $0 \leq k \leq 1$, and k is monotonically increasing when α is increasing, as shown in the Figure 1(b).

2.2. The Features of the Coverage Formed by Two Intersected Nodes

According to the formula (1) to (4), we can see that central angle α will decrease when the nodes distance d increase and the communication radius r is certain, and the proportion coefficient k will also decrease, and the area S_C of the common coverage will also decrease, but the whole area S_{2T} will increase. Because $S_C = k\pi r^2$ and $S_{2T} = (2-k)\pi r^2$, so the changing relationships between the variables d, α, k are most important. The following we give their changing relationships by listing some typical values, as shown in Table 1.

Table 1. Typical Values about the Variables d, α, k

| α | d | k | α | d | k |
|------------|-----------|--------|-------------|-----------|--------|
| 0° | $2r$ | 0 | 120° | r | 0.3910 |
| 30° | $1.9319r$ | 0.0075 | 135° | $0.7654r$ | 0.5249 |
| 45° | $1.8478r$ | 0.0249 | 150° | $0.5176r$ | 0.6742 |
| 60° | $1.7321r$ | 0.0577 | 180° | 0 | 1 |
| 90° | $1.4142r$ | 0.1817 | | | |

According to the Table 1, for any two given intersected nodes, we can construct the logical structure and ascertain the Features of the formed coverage.

2.3. The Communication Channels Provided by Two Intersected Nodes

According to the Figure 1(a),(b), if the third node appears in the effective communication area formed by two intersected nodes, there have two situations for the number of the communication channels obtained by the third node: a) if the third node is in the common coverage area, it can simultaneously obtain two communication channels and it can communicate with not only node A but also node B; b) if the third node only is in the single coverage area of node A or B, it can only obtain one communication channel and it can only communicate with one of node A or B.

We can see that the proportion coefficient k reflects not only the changes of the area proportion but also the probability that the third node can obtain 1 or 2 communication channels, so if we make the proportion coefficient k to increase, the probability can be increased that the third node obtain the more number of communication channels.

2.4. The Optimal Coverage Formed by Two Intersected Nodes

According to the formula (2),(3), increasing the area of the common coverage S_C certainly makes the whole coverage S_{2T} to decrease, and vice versa. Therefore, there are two special situations:

One is that $k=1$. For this situation, there have $S_C=S_{2T}=\pi r^2$, it means that the area of the common coverage reaches maximum and the area of the whole coverage reaches minimum, and the node A coincides with the node B, so the probability that the third node obtain 2 communication channels is $k=1$.

The other is that $k=0$. For this situation, there have $S_C=0$ and $S_{2T}=2\pi r^2$, it means that the area of the common coverage reaches minimum and the area of the whole coverage reaches maximum, this moment node A and B is circumscribe, and the probability that the third node obtain 2 communication channels is $k=0$.

Are there some optimal coverage between the above two special situations?

In the practical application, we usually consider the situation that the coverage area formed by a certain number of the nodes can or not completely cover a rectangular area. For simplicity, the following we will discuss the situation that the effective rectangular area reaches maximum in the coverage area formed by two intersected nodes.

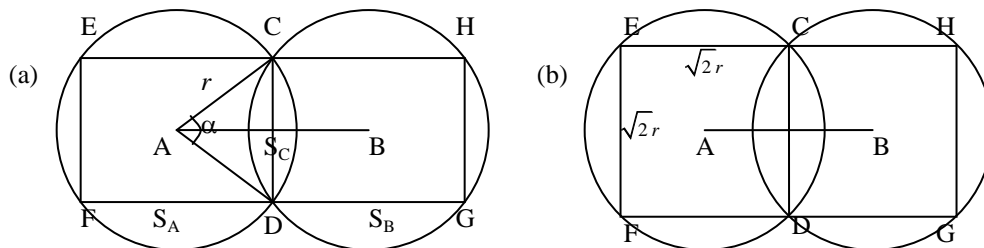


Figure 2. The Rectangular Coverage Formed by Two Intersected Nodes: (a) The General Coverage, (b) the Optimal Coverage

According to the geometrical relationships in Figure 2 (a), there are

$$|CD| = 2r \sin \frac{\alpha}{2}, |CE| = 2r \cos \frac{\alpha}{2}.$$

$$S_{\square EFGH} = 2 |CD| \cdot |CE| = 8r^2 \sin \frac{\alpha}{2} \cos \frac{\alpha}{2} = 4r^2 \sin \alpha .$$

According to the features of sin function, when $\alpha=90^\circ$ the rectangular area $S_{\square EFGH}$ will reaches maximum, and the maximum value is $S_{\square EFGH} = 4r^2$, see the Figure 2 (b).

From the Table 1, the distance between two nodes is $d=1.4142r$, and the proportion coefficient is $k=0.1817$.

3. The Coverage and its Features Formed by Three Intersected Nodes

In Figure 1(a), if the communication area of any third node intersects with the communication area of the node A or B, the distance between the node A and B should be less than or equal to $2r$, then drawing two circles as the center of the node A and B and the radius $2r$, the third node must be in the union area of two circles, as shown in Figure 3.

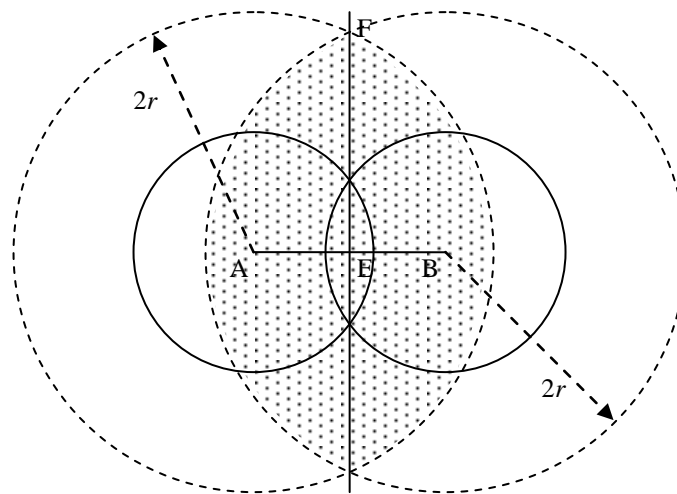


Figure 3. The Area that the Third Node may Appear in

We can see there are three situations that the third node can appear in the area:

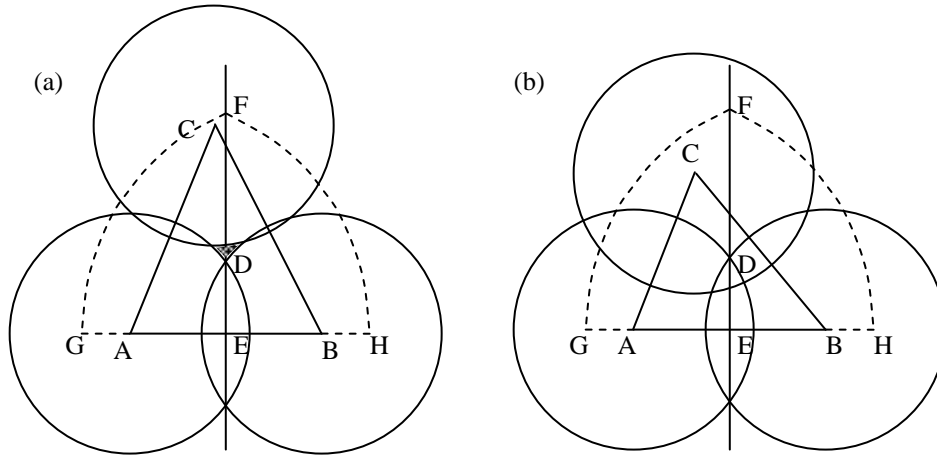
- a) Appearing outside of the two dashed circles. For this situation, the communication area of the third node cannot intersect with the communication area of the node A or B.
- b) Appearing inside of the two dashed circles but outside of the shaded area. For this situation, the third node can only intersects with one of the nodes A and B, and it can be referred to section 2 for the coverage and its Features.
- c) Appearing inside of the shaded area. For this situation, the third node simultaneously intersects with the nodes A and B, and it need to be discussed.

3.1. The Coverage Formed by Three Intersected Nodes

In Figure 3, the shaded area is symmetrical about the line AB and EF, so the following will discuss the situation that the third node appears on the left side of the shaded area above the line GH and EF.

Assume the third node to node C, and its communication radius is r . According to the geometric relationships of the shaded area of the top of the line AB in Figure 3, when the node C appears in the area, the coverage can be divided into the incomplete-coverage and the complete-coverage, as shown in Figure 4 (a), (b).

In Figure 4 (a), the shaded area is not covered by any one of the three communication nodes, so called as the incomplete-coverage. But in Figure 4 (b), the whole area can be covered, so called as the complete-coverage.



**Figure 4. The Coverage Formed by Three Intersected Nodes:
 (a) The Incomplete-Coverage; (b) The Complete-Coverage**

3.2. The Critical Line between the Incomplete-Coverage and the Complete-Coverage

It has great significance to the fast route or optimal route that finds the critical line between the incomplete-coverage and the complete-coverage. From the Figure 4, whether the complete -coverage can be formed, it depends on the distance of the node C to the intersection point D. If $|CD| > r$, then it must be the incomplete-coverage, else complete-coverage. So drawing a circle as the center of the intersection point D and the radius r , denoted by $\odot D$. Suppose that the $\odot D$ intersects with the line EF at the point P, and the $\odot D$ intersects with the arc GF at the point M, the arc GMP is the critical line in a fan-shaped area GEF, as shown in Figure 5.

If the node C appears inside the area GMPE (the shaded area in Figure 5), it must form the complete-coverage, else it will form the incomplete-coverage. The situation of the right side of line EF is symmetrical, and the below of line GH is also symmetrical with the upper part.

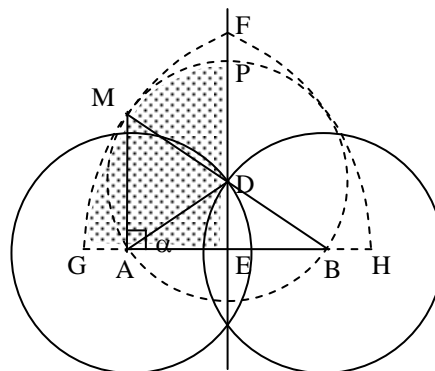


Figure 5. The Critical Line Between the Incomplete-Coverage and the Completer-Coverage

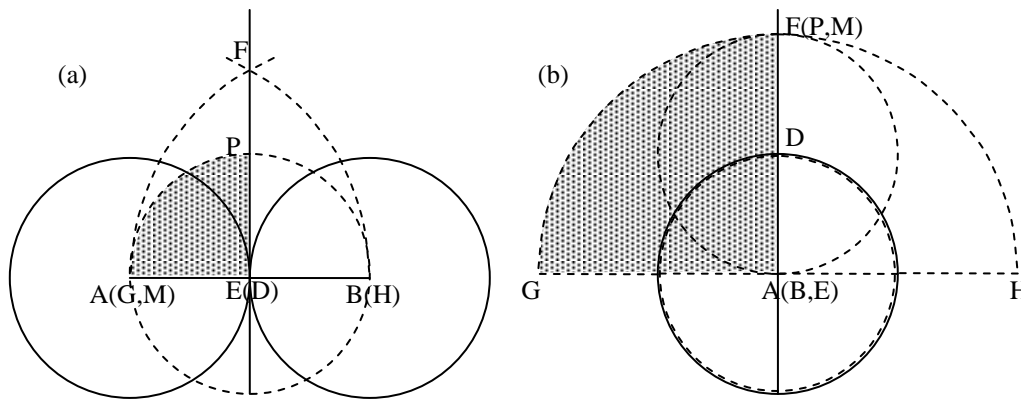
In Figure 5, does the circumferential line of $\odot D$ intersect with arc GF?

According to the Figure 3, arc GF is formed by drawing the circle as the center B and radius $2r$, so the furthest distance from the point D are the point G and F on the arc GF, the nearest distance from point D is the point that the extended line of line BD intersects with arc GF. Because $DM=r$, so the intersected point is the point M that the circumferential line of $\odot D$ intersects with arc GF, as shown in Figure 5.

Because $DA=DB=DM=DP=r$, and BM is the diameter of $\odot D$, and the line EF and AB are perpendicular bisector, so ΔMAB is a right triangle. According to the Figure 1, it has

$$|MA| = 2|DE| = 2r \sin \frac{\alpha}{2} \quad (0^\circ \leq \alpha \leq 180^\circ). \quad (5)$$

So the point that the perpendicular line MA intersects with arc GF is the point M, it has two extreme situations, as shown Figure 6 (a),(b).



**Figure 6. Two Special Situations of the Critical Line:
(a) when $\alpha=0^\circ$, (b) when $\alpha=180^\circ$**

For the figure 6(a), when $\alpha=0^\circ$ and $|MA|=0$, it means that the point M coincides with the point A, so the point D coincides with the point E, the critical arc GMP degenerates to the arc PM. For the figure 6(b), when $\alpha=180^\circ$ and $|MA|=2r$, it means that the point M and F and P are coincident, so the critical arc GMP degenerates to the arc FG.

They are respectively corresponding to the two situations that the node A is circumscribe with the node B and the node A coincides with the node B.

3.3. The Optimal Complete-Coverage Formed by Three Intersected Nodes

When the third node C is closer to the point P in the shaded area, three nodes can cover greater area, as shown Figure 7. Here, three nodes have the following important features: (a) They all intersect, (b) It is a complete-coverage, (c) The coverage area reaches maximum. Therefore, it is called as the optimal complete-coverage.

In Figure 7, denoting the total area covered by three intersecting node as S_{3T} , and assuming the common area that node A intersects with node B is S_{C1} , and the common area that node A intersects with node C is S_{C2} , and the common area that node B intersects with node C is S_{C3} . Because the point C is on the perpendicular bisector EF of line AB, then $S_{C2}=S_{C3}$, so there are

$$S_{3T} = (S_A + S_B + S_C) - (S_{C1} + S_{C2} + S_{C3}) = 3\pi r^2 - (S_{C1} + 2S_{C2}).$$

Suppose the distance of node A and B is d_1 , and the central angle corresponding to the intersecting chord formed by the node A and B is α_1 , and the proportion coefficient of the common area is k_1 . For the situation that node A intersects with node C, assuming that there are d_2, α_2, k_2 . According to the formula (2) and (4), there are

$$S_{C_1} = k_1 \pi r^2, \quad S_{C_2} = k_2 \pi r^2, \quad k_1 = \frac{\alpha_1}{180} - \frac{\sin \alpha_1}{\pi}, \quad k_2 = \frac{\alpha_2}{180} - \frac{\sin \alpha_2}{\pi}, \quad (6)$$

$$S_{3r} = \pi r^2 [3 - (k_1 + 2k_2)]. \quad (7)$$

In the isosceles triangle $\triangle ABC$, there are

$$\begin{aligned} |AE| &= r \cos \frac{\alpha_1}{2}, \quad |CE| = |CD| + |DE| = r + r \sin \frac{\alpha_1}{2}, \\ d_2 &= \sqrt{|AE|^2 + |CE|^2} = \sqrt{\left(r \cos \frac{\alpha_1}{2}\right)^2 + \left(r + r \sin \frac{\alpha_1}{2}\right)^2} = \sqrt{2} r \sqrt{1 + \sin \frac{\alpha_1}{2}}. \\ \alpha_2 &= 2 \arccos \frac{d_2}{2r} = 2 \arccos \sqrt{\frac{1}{2} \left(1 + \sin \frac{\alpha_1}{2}\right)} \end{aligned} \quad (8)$$

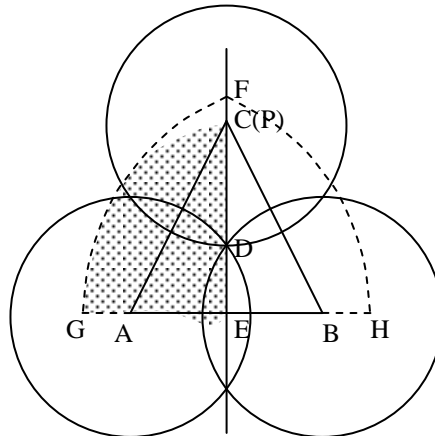


Figure 7. The Optimal Complete-Coverage Formed by Three Intersected Nodes

Generalizing the formula (6)-(8), we can get the formulas to calculate the total covered area formed by three intersected nodes when d_1 is arbitrarily given, as the following:

$$\begin{cases} \alpha_1 = 2 \arccos \frac{d_1}{2r} (0 \leq d_1 \leq 2r) \\ \alpha_2 = 2 \arccos \sqrt{\frac{1}{2} \left(1 + \sin \frac{\alpha_1}{2}\right)} \\ k_1 = \frac{\alpha_1}{180} - \frac{\sin \alpha_1}{\pi}, \quad k_2 = \frac{\alpha_2}{180} - \frac{\sin \alpha_2}{\pi} \\ S_{3r} = \pi r^2 [3 - (k_1 + 2k_2)] \end{cases} \quad (9)$$

Being easy to see, the proportion coefficients k_1, k_2, k_3 are not exactly equal In Figure 7, and there is $k_2=k_3$, but k_1 is not equal to k_2 or k_3 . So there are two questions:

Question 1: What value of d_1 can make $k_1=k_2=k_3$?

Question 2: For an arbitrarily given value of d_1 , if it makes $k_1=k_2=k_3$, how the optimal complete -coverage will change in the Figure 7?

For question 1, if there are $k_1=k_2=k_3$, the triangle ΔABC must be an equilateral triangle, and the line CE is perpendicular bisector of the line AB , then in right-angled triangle ΔACE , there are

$$\begin{cases} |CE| = r + r \sin \frac{\alpha_1}{2} = |AC| \sin 60^\circ = \frac{\sqrt{3}}{2} d_1 \\ |AE| = r \cos \frac{\alpha_1}{2} = \frac{|AB|}{2} = \frac{d_1}{2} \end{cases} \quad (10)$$

There, $0^\circ \leq \alpha_1 \leq 180^\circ$. We can resolve the formula (10) and get $\alpha_1=60^\circ$, $d_1=1.732r$, $k_1=0.0577$.

Further, we can conclude the changing relationships of α_i, d_i, k_i ($i=1,2,3$) when three nodes intersect to form a optimal complete-coverage, seen as Table 2.

Table 2. The Changing Relationships of the Variables α_i, d_i, k_i ($i=1,2,3$)

| The Situations | The Central Angles α_i ($i=1,2,3$) | The Node Distances d_i ($i=1,2,3$) | The Proportion Coefficients k_i ($i=1,2,3$) |
|----------------|--|--|---|
| 1 | $0^\circ \leq \alpha_1 < 60^\circ, \alpha_2 = \alpha_3 > 60^\circ$ | $d_1 > 1.732r, d_2 = d_3 < d_1$ | $k_1 < 0.0577, k_2 = k_3 > k_1$ |
| 2 | $\alpha_1 = \alpha_2 = \alpha_3 = 60^\circ$ | $d_1 = d_2 = d_3 = 1.732r$ | $k_1 = k_2 = k_3 = 0.0577$ |
| 3 | $60^\circ < \alpha_1 \leq 180^\circ, \alpha_2 = \alpha_3 < 60^\circ$ | $d_1 < 1.732r, d_2 = d_3 > d_1$ | $k_1 > 0.0577, k_2 = k_3 < k_1$ |

In Table 2, the second situation is completely symmetrical, and the first situation and the third situation are not completely symmetrical. According to the Table 2, The network topology of the three situations can be drawn, as shown in the Figure 8 (a),(b),(c).

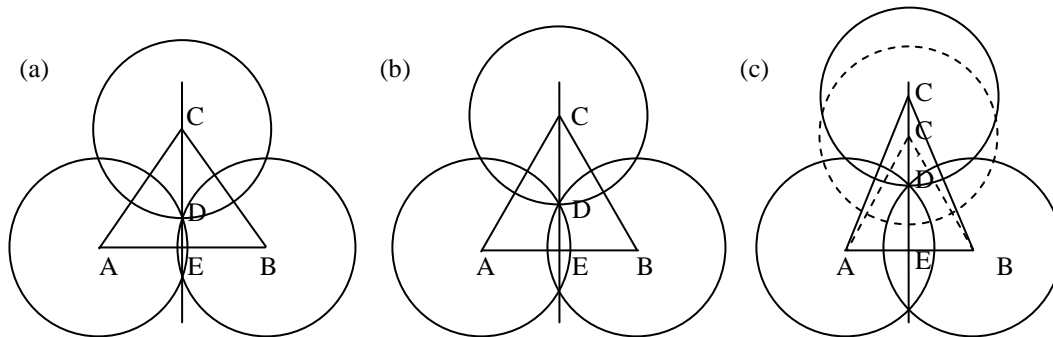


Figure 8. The Three Situations of the Optimal Complete-Coverage Formed by Three intersected Nodes: (a) $k_2=k_3 > k_1$, (b) $k_1=k_2=k_3$, (c) $k_2=k_3 < k_1$

For the question 2, the Figure 8 (b) is satisfied with the requirement. For the Figure 8 (a), obviously it cannot make $k_1=k_2=k_3$. For the Figure 8 (c), if we make the node C to move downward to an appropriate position along the line CE and make the triangle ΔABC to be an equilateral triangle, then we can make $k_1=k_2=k_3$, as shown the dotted parts in Figure 8 (c).

3.4. The Number of the Communication Channels Provided by Three Intersected Nodes

According to the Figure 4, the coverage formed by three intersected nodes can provide two communication channels for other nodes when it is an incomplete-coverage, but the uncovered area will not be able to provide the communication channel. When the coverage formed by three intersected nodes is a complete-coverage, it can provide three communication channels at most.

4. A Simple Application

The conclusions of the section 2 in the paper can be applied to the situations that any two nodes intersect in the Ad Hoc network, and it can be used as basis to analyze more intersected nodes. The conclusions of the section 3 in the paper reflect the impact of the new node added to the existing topology, and which discuss the optimal complete-coverage is propitious to build the cluster-head layer of the optimal topology in the hierarchy Ad Hoc Network.

4.1. The Problem

In the hierarchy Ad Hoc Network, it is very important for the performance of the entire network that properly selects cluster-head and builds cluster-head layer. Next we select a problem that communication area is complete-coverage to illustrate the application of the conclusions in the paper.

Suppose that the Ad Hoc network is a rectangular area, the length of the communication is m and the width of the communication area is n (Unit: m). Now we use the communication nodes that radius is r to completely cover the entire area, please provide a optimal design solution of the Ad Hoc network.

4.2. The Optimal Solution of the Problem

For the purpose, we can select a solution from the Figure 8(a), (b), (c). Because the proportion coefficient k of the common area and the communication channels are not given in the application problem, so we can select the special Figure 8 (b). For Figure 8 (b), its optimal features are " $\alpha_1=\alpha_2=\alpha_3=60^\circ, d_1=d_2=d_3=1.732r, k_1=k_2=k_3=0.0577$ ", it can be expressed as "the third node only intersects with the two adjacent nodes, and make the coverage reach to be fully symmetrical and optimal complete-coverage." Therefore, we can continue to add the new nodes to the Figure 8 (b), until the $m \times n$ communication area is optimal completely covered, as shown in the Figure 9 (a).

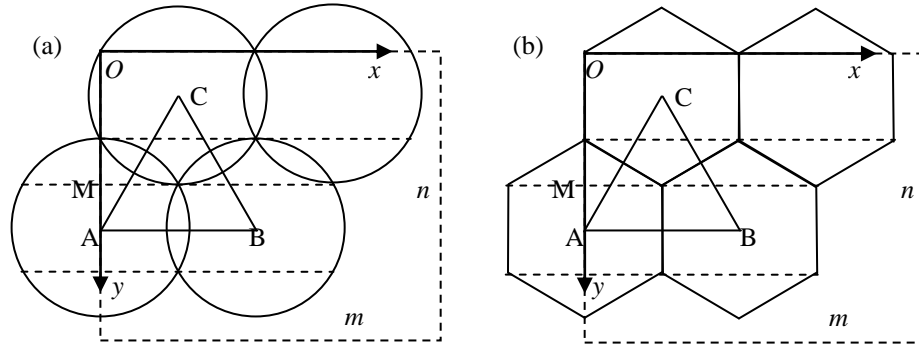


Figure 9. According to the Figure 8(b) Optimally and Completely Covering the $m \times n$ Communication Area

Because each node has the feature of “ $\alpha=60^\circ$ ”, so if we sequentially link the all intersections in the circumference of the same node, the sides will justly form a regular hexagon, and then using the nodes to the optimally and completely cover the $m \times n$ Communication Area is equal to using the regular hexagon that the side length d is $1.732r$ to the optimally and completely cover the area, as shown in the Figure 9 (b). According to the Figure 9 (a),(b), we can get the formula to compute the number of the nodes required.

In the Figure 9 (b), we can get $d=|AB|=1.732r$ as the step length in x direction, and get $h=|OM|=1.5r$ as the step length in y direction. Assume the quotient of the length m of the communication area divided by the step length d in x direction is p_x , and its remainder is q_x , and the quotient of the width n of the communication area divided by the step length h in y direction is p_y , and its remainder is q_y , so there are

$$m \div d = p_x \cdots q_x, n \div h = p_y \cdots q_y . \quad (11)$$

Then we can calculate the number of the nodes required according to the following steps:

(1) For the number of the nodes in odd rows of x direction (denoted as x_1), it can be calculated as the following formula:

$$x_1 = \begin{cases} p_x & , q_x = 0 \\ p_x + 1 & , q_x > 0 \end{cases} . \quad (12)$$

(2) For the number of the nodes in even rows of x direction (denoted as x_2), it can be calculated as the following formula:

$$x_2 = \begin{cases} p_x + 1 & , 0 \leq q_x \leq 0.5d \\ p_x + 2 & , 0.5d < q_x < d \end{cases} . \quad (13)$$

(3) For the number of rows required in y direction (denoted as y), it can be calculated as the following formula:

$$y = \begin{cases} p_y + 1 & , 0 \leq q_y \leq r \\ p_y + 2 & , r < q_y < 1.5r \end{cases} . \quad (14)$$

Further, we can calculate the total rows of the odd rows and the even rows, denoted as y_1 and y_2 , and denoted as $y_0 = \lfloor y/2 \rfloor$ (the symbol $\lfloor \cdot \rfloor$ means that it is rounded down), so there are

$$y_1 = \begin{cases} y_0 & , y \text{ is even} \\ y_0 + 1 & , y \text{ is odd} \end{cases}, y_2 = y_0. \quad (15)$$

(4) For the total number of nodes required, it can be calculated as the following formula:

$$t = y_1 \cdot x_1 + y_2 \cdot x_2 \quad (16)$$

4.3. The Sample Data and the Computing Results

The following we will use a group of data to apply the above computing steps.

Assume the length m of the communication area is 1000m, and the width n is also 1000m, and the communication radius r of each node is 100m. According to the Figure 9, we can firstly get $d=1.732r=173.2$, $h=1.5r=150$. By the formula (11)-(16), there are

$$p_x=5, q_x=134, p_y=6, q_y=100,$$

$$x_1=p_x+1=6, x_2=p_x+2=7; y_1=p_y+1=7, y_0=3, y_1=y_0+1=4, y_2=3.$$

$$t = y_1 \times x_1 + y_2 \times x_2 = 4 \times 6 + 3 \times 7 = 45.$$

Therefore, when using the nodes of the communication radius $r=100m$ to optimally and completely cover the communication area of size $1000m \times 1000m$, we need to use at least 45 nodes. Any other node can get one or two communication channels in the communication area, but the probability that the node gets two communication channels is only $k=0.0577$.

5. The Follow-Up Work

In the paper we discuss the Features of the coverage formed by two intersected nodes and three intersected nodes in the Ad Hoc network, and specially analyze the Features of the optimal complete-coverage and the number of the communication channels provided. This research has a strong foundation and a very strong guiding significance to construct the optimal Ad Hoc network and design more efficient optimal routing algorithm.

In the paper, the optimal complete-coverage (the Figure 8 (a), (b), (c)) has not been more deeply analyzed and compared, and in section 4 the application problem is only one of the sub-problems of designing the optimal Ad Hoc network. For lack of space, the optimal coverage and its Features formed by four or more communication nodes cannot be discussed here, and it need to be complemented.

Acknowledgment

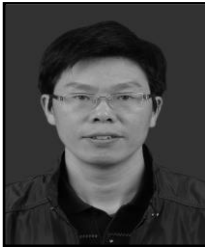
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