International Journal of Future Generation Communication and Networking
Vol.7, No.3 (2014), pp.155-166
http://dx.doi.org/10.14257/ijfgcn.2014.7.3.14

Strong Tracking Unscented Kalman Filtering Algorithm Based-on Satellite Attitude Determination System

Min Li[1], Bao-long Zhu[2], Song-yan Wang[3], Ying-chun Zhang[4] and Hua-yi Li[5]

[1,2,3,4,5]*College of astronautics, Harbin Institute of Technology*
Harbin, China, 150001
[1]*liminhit@126.com*, [2]*baolong_zhu@126.com*, [3]*sywang@hit.edu.cn*,
[4]*zhang@hit.edu.cn*, [5]*lihuayi@hit.edu.cn*

Abstract

Combined with strong tracking filter (STF) theory, the Strong Tracking Square-Root Unscented Kalman Filter (UKF)-based satellite attitude determination algorithm is proposed in this paper. QR decomposition and Cholesyk decomposition are introduced in this paper, which improves the stability of filter. In addition, by introduced adaptive fading factor, the prediction error covariance matrix can be adjusted, thus it can guarantee the strong tracking performance of the proposed algorithm. At last, simulation results show that strong tracking square-root UKF has better stability, robustness and mutation status tracking capability than Square-Root UKF and Strong Tracking UKF.

Keywords: *Quaternion; Cholesky decomposition; QR decomposition; satellite attitude determination system; Strong Tracking Square-Root Unscented Kalman Filter (STSRUKF)*

1. Introduction

Satellite attitude determination system is an important subsystem of the satellite attitude control system (ACS). Its accuracy directly affects the attitude control precision of the satellite. As the satellite attitude determination system is a nonlinear model, to derive excellent results, the nonlinear filter methods are needed. Among many nonlinear filters, Extended Kalman filter (EKF) is simple and easy to implement. So it is widely used in dealing with non-linear estimation problem. But EKF is a rough approach for approximating nonlinear system for linear ones, and the higher order terms are neglected. Therefore, it fails to deal with highly complicated nonlinear system. In addition, Jacobian calculations are cumbersome and time-consuming [1-3].

To solve these problems, Julier and J. K first proposed the Unscented Kalman Filter (UKF). The UKF is based on the fact that it is easier to approximate a probability distribution than it is to approximate an arbitrary nonlinear function or transformation, and it uses a deterministic sampling approach to capture the mean and covariance estimates with a minimal set of sample points. Due to better covariance proper ties and simplicity of filter design, the UKF is preferred over the EKF [4-5]. However, the rounding errors of numerical calculation for UKF may destroy the non-negative and asymmetry of covariance matrix, therefore, the convergence rate of algorithm is slowed and the stability of system may be also destroyed. To address these advantages, Square-Root UKF filter is proposed in [6-8]. Compared with the standard UKF, it guarantees the error covariance matrix positive semi-definiteness and improves the computational efficiency and numerical stability by introducing QR decomposition and Cholesyk decomposition to update the error covariance matrix, and it is successfully applied to the satellite attitude estimation in [9].

ISSN: 2233-7857 IJFGCN

EKF, UKF and SR-UKF are applied for accurate model with good precision, but they are not fit for the uncertainty systems. Solving the above difficult problems has attracted much attention of many scholars, Zhou Dong-hua [10-11] proposes the Strong Tracking Filter (STF), which has good precision and robustness for uncertainty model and mutational status tracking capability by introducing adaptive fading factor. Combined with the STF theoretical ideas and UKF, Strong Tracking Unscented Kalman Filter (ST-UKF) is proposed in [12], and it successfully applied for astronomical autonomous navigation and improved system reliability, however, both STF and STUKF are similar to EKF. There is still a rough approach for approximating nonlinear system for linear ones and Jacobian matrix is also need to be calculated, which limits the application of these methods. In addition, STUKF cannot solve problems caused by rounding errors of numerical calculation. Solving the above difficult problems motivates us to purse this investigation.

In this work, the Strong Tracking Square-Root UKF is proposed for satellite attitude determination system. The proposed filter uses the equivalence description between UKF and KF to calculate adaptive fading factor, which solves the disadvantage of Jacobian calculation. And SRUKF instead of UKF is adopted in this algorithm, and it guarantees positive semi-definiteness, which ensures stability and avoids inaccuracy caused by filtering divergence. At last, simulation results demonstrate that, compared with STUKF and SRUKF, the STSRUKF has better stability, robustness for uncertainty model and mutation status tracking capability.

The rest of this article is organized as follows. In Section II, mathematical model and measurement model of satellite attitude determination system are established. In Section III, the STUKF and STSRUKF algorithm theory are constructed for satellite attitude determination system. In Section IV, simulation results on SRUKF, STUKF and STSRUKF, and the comparison of their performance are given. In Section V, conclusive remarks are given.

2. Satellite Attitude Determination System

2.1 Satellite Attitude Description

The satellite attitude can be represented by the quaternion defined as [13-14].

$$q = [q_0 \quad q_1 \quad q_2 \quad q_3]^T \tag{1}$$

where $q_{13} = \hat{n}\sin(\theta/2)$ and $q_0 = \cos(\theta/2)$.

The quaternion satisfies the following constraint:

$$q_0^2 + q_1^2 + q_2^2 + q_3^2 = 1 \tag{2}$$

The quaternion equation of satellite motion is given by:

$$\frac{dq(t)}{dt} = \frac{1}{2}\Omega(\omega(t))q(t) \tag{3}$$

where ω is the angular velocity vector and

$$\Omega(\omega(t)) = \begin{bmatrix} 0 & -\omega_x(t) & -\omega_y(t) & -\omega_z(t) \\ \omega_x(t) & 0 & \omega_z(t) & -\omega_y(t) \\ \omega_y(t) & -\omega_z(t) & 0 & \omega_x(t) \\ \omega_z(t) & \omega_y(t) & -\omega_x(t) & 0 \end{bmatrix}$$

In the absence of angular velocity measured by gyro, the satellite attitude dynamics equation is given by:

$$\frac{d\omega(t)}{dt} = J^{-1}[N(t) - \omega(t)\times(J\omega(t))] \tag{4}$$

where J is the inertia matrix consists of principal moments of inertia, and $N(t)$ is the external control moment.

From equation (3) and (4), the nonlinear state equation can be built as:

$$\frac{dx(t)}{dt} = \begin{bmatrix} f_1(q(t), \omega(t)) \\ f_2(\omega(t)) \end{bmatrix} + w(t) \tag{5}$$

where $f_1(q(t), \omega(t)) = \frac{d\omega(t)}{dt}$, $f_2(\omega(t)) = \frac{d\omega(t)}{dt}$, $w(t)$ is the white Gaussian noise.

2.2 Measurement System Model

The bi-Vector method is to use the projection of gravity vector measured by the accelerometer and magnetic field vector measured by the magneto resistive sensors which is fixed on the body frame to calculate the satellite attitude. As derived in [15], the Euler angles can be calculated as:

$$\begin{cases} \phi = \arctan(\frac{g_{by}}{g_{bz}}) \\ \theta = -\arcsin(\frac{g_{bx}}{g}) \\ \psi = \arctan(\frac{-H_{by}\cos\varphi + H_{bz}\sin\theta}{H_{bx}\cos\theta + H_{by}\sin\varphi\sin\theta + H_{bz}\cos\varphi\sin\theta}) \end{cases} \tag{6}$$

where g denotes the local gravity acceleration, $g_b = [g_x \quad g_y \quad g_z]^T$ denotes the gravity vector obtained by accelerometer in body frame and $H_b = [H_{bx} \quad H_{by} \quad H_{bz}]^T$ denotes the magnetic field vector obtained by the magneto resistive sensors which is fixed on the body frame.

As Euler angles are selected as measurement variables, then the measurement vector can be defined as $z = [\phi \quad \theta \quad \psi]^T$. According to the mapping between the Euler angles and quaternion, the measurement equation can be derived as:

$$z(k) = \begin{bmatrix} \phi(k) \\ \theta(k) \\ \psi(k) \end{bmatrix} = \begin{bmatrix} \arctan(\frac{2(q_2(k)q_3(k) + q_1(k)q_0(k))}{1 - 2(q_1(k)^2 + q_2(k)^2)}) \\ \arcsin(-2(q_1(k)q_3(k) - q_2(k)q_0(k))) \\ \arctan(\frac{2(q_2(k)q_1(k) + q_3(k)q_0(k))}{1 - 2(q_3(k)^2 + q_2(k)^2)}) \end{bmatrix} + v(k) \tag{7}$$

where $q = [q_0 \quad q_1 \quad q_2 \quad q_3]^T$ has been normalized, and v is the white Gaussian noise.

3. STSRUKF-Based Satellite Attitude Determination Algorithm

3.1. UKF Algorithm

The satellite attitude determination system consists of formula (5) and (7) is discredited by fourth-order Ronge-Kutta method, and the discrete-time system can be described as:

$$\begin{cases} x_{k+1} = f(x_k, u_k) + w_k \\ z_k = h(x_k) + v_k \end{cases} \tag{8}$$

where $x_k = [q_{0,k} \quad q_{1,k} \quad q_{2,k} \quad q_{3,k} \quad \omega_{x,k} \quad \omega_{y,k} \quad \omega_{z,k}]^T$ represents a seven dimensional state Vector, u_k is a input Vector, $z_k = [\varphi_k \quad \theta_k \quad \psi_k]^T$ represents the observed signal, w_k is a process Gaussian noise with the variance matrix defined as Q_k, and v_k is a measurement Gaussian noise vector with the variance matrix defined as R_k.

The UKF based satellite attitude determination algorithm is described as following [16-18], and in the process of calculation, the scalar q_{03} should be normalized by equation (9) at any step.

$$q_{03} = \frac{q_{03}}{\sqrt{q_0^2 + q_1^2 + q_2^2 + q_3^2}} \tag{9}$$

1) Setting the process noise covariance Q_k and measurement noise covariance R_k. Initializing the initial state x0 and P_0 :

$$\hat{x}_0 = E(x_0) \tag{10}$$

$$P_0 = E[(x_0 - \hat{x}_0)(x_0 - \hat{x}_0)^T] \tag{11}$$

2) Calculating the sigma points

$$\chi_k = [\ \hat{x}_k \quad \hat{x}_k + \sqrt{(n+\lambda)(P_k)}_i \quad \hat{x}_k - \sqrt{(n+\lambda)(P_k)i}\ ] \tag{12}$$

3) Time updating

$$\chi_{k+1|k} = f(X_{k+1|k}, u_k) \tag{13}$$

$$\hat{x}_{k+1|k} = \sum_{i=0}^{2n} W_i^m \chi_{k+1|k,i} \tag{14}$$

$$p_{k+1|k} = \sum_{i=0}^{2n+1} W_i^c (\hat{\chi}_{i,k+1|k} - \hat{x}_{k+1|k})(\hat{\chi}_{i,k+1|k} - \hat{x}_{k+1|k})^T + Q_{k+1} \tag{15}$$

$$\xi_{i,k+1|k} = h(\chi_{i,k+1|k}) \tag{16}$$

$$\hat{z}_{k+1|k} = \sum_{i=0}^{2n} W_i^m \xi_{i,k+1|k} \tag{17}$$

4) Measurement updating

$$P_{xz,k+1} = \sum_{i-0}^{2n+1} W_i^c (\hat{\chi}_{i,k+!|k} - \hat{x}_{k+1|k})(z_{i,k+1|k} - \hat{z}_{k+1|k})^T \tag{18}$$

$$P_{z,k+1} = \sum_{i-0}^{2n+1} W_i^c (z_{i,k+!|k} - \hat{z}_{k+1|k})(z_{i,k+1|k} - \hat{z}_{k+1|k})^T + R_{k+1} \tag{19}$$

$$K_k = P_{xz,k+1} P_{z,k+1}^{-1} \tag{20}$$

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K(z_{k+1} - \hat{z}_{k+1|k}) \tag{21}$$

$$P_{k+1} = P_{k+1|k} - K_{k+1} P_{z,k+1} K_{k+1}^T \tag{22}$$

The scalar weights of sigma points are selected as:

$$W_0^m = \frac{\lambda}{n+\lambda} \tag{23}$$

$$W_0^c = W_0^m + (1 - \alpha^2 + \beta) \tag{24}$$

$$W_i^c = W_i^m = \frac{1}{2(n+\lambda)} \tag{25}$$

where n is the state dimension, and α, β, λ are the adjustable parameters of filter, of which α and β are sufficient small scalars.

3.2 STUKF Algorithm

If the considered system model is accurate, excellent results can be obtained using UKF. But the environment of satellite operation is complex, which leads to the uncertainty of satellite model, and the noise statistics are also time-varying in satellite ACS. So the estimation accuracy using UKF decreases and even it fails [19]. However, good estimation accuracy, robustness and mutation status tracking ability can be obtained by STUKF even if the model is uncertain and the system is disturbance-driven.

The difference between STUKF and UKF is that the error covariance matrix can be adjusted by introduced adaptive fading factor μ_k .As formula (26)

$$P_{k+1|k} = \mu_{k+1} \sum_{i=0}^{2n+1} W_i^c (\hat{\chi}_{i,k+1|k} - \hat{x}_{k+1|k})(\hat{\chi}_{i,k+1|k} - \hat{x}_{k+1|k})^T + Q_{k+1} \tag{26}$$

The adaptive fading factor calculation is given as follow:

$$u_{k+1,i} = \begin{cases} u_{k+1,i}, u_{k+1,i} \geq 1 \\ 1, u_{k+1,i} < 1 \end{cases} \tag{27}$$

$$u_{i,k+1} = trace(N_{k+1}) / \sum_{i=1}^{n} M_{ii,k+1} \tag{28}$$

$$N_{k+1} = V_{k+1} - H_{k+1} Q_{k+1} H_{k+1}^T - l R_{k+1} \tag{29}$$

$$M_{k+1} = F_{k+1|k} P_{k|k} F_{k+1|k}^T H_{k+1} H_{k+1}^T \tag{30}$$

$$V_k = \begin{cases} (z_k - \hat{z}_{k+1|k})(z_k - \hat{z}_{k+1|k})^T, k = 1 \\ \dfrac{\rho V_{k-1} + (z_k - \hat{z}_{k+1|k})(z_k - \hat{z}_{k+1|k})^T}{1+\rho}, k \geq 2 \end{cases} \tag{31}$$

$$F_{k+1|k} = \frac{\partial f(x_k)}{\partial x_k} | x_k = \hat{x}_{k|k} \tag{32}$$

$$H_{k+1} = \frac{\partial h(x_k)}{\partial x_k} | x_k = \hat{x}_{k+1|k} \tag{33}$$

where l is a softening factor, ρ is a forgetting factor, $\mu_{k+1} = diag(\mu_{1,k+1}, \cdots, \mu_{n,k+1})$ is a multiple adaptive fading factor. Multiple adaptive fading factors are more effective than a single adaptive fading factor [20].

According to the above description, Jacobian matrix needs to be calculated in the process of adaptive fading factor calculation, while Jacobian matrix calculation is very complicated and difficult. According to studying equivalence description of STF, a method of adaptive fading factor calculation without calculating Jacobian matrix is given in this paper.

It is assumed that, before the introduction of adaptive assumed fading factor, the state prediction error covariance matrix $P^l_{k+1|k}$, the cross-covariance matrix $P^l_{xz,k+1}$, and outputs the predicted prediction covariance matrix $P^l_{z,k+1}$ are:

$$\begin{aligned} &P^l_{k+1|k} = \\ &E[(x_{k+1} - \hat{x}_{k+1|k})(x_{k+1} - \hat{x}_{k+1|k})^T] = \\ &F_{k+1|k} P_{k|k} F_{k+1|k} + Q_{k+1} \end{aligned} \tag{34}$$

$$\begin{aligned} &P^{l}_{xz,k+1} = \\ &E[(x_{k+1}-\hat{x}_{k+1|k})(z_{k+1}-\hat{z}_{k+1|k})^{T}]= \\ &E[(x_{k+1}-\hat{x}_{k+1|k})(H_{k+1}(x_{k+1}-\hat{x}_{k+1|k}))^{T}]+ \\ &E[(x_{k+1}-\hat{x}_{k+1|k})v^{T}_{k+1}]= \\ &P^{l}_{k+1|k}H^{T}_{k+1}+E[(x_{k+1}-\hat{x}_{k+1|k})v^{T}_{k+1}] \end{aligned} \tag{35}$$

$$\begin{aligned} &P^{l}_{z,k+1} = \\ &E[(z_{k+1}-z_{k+1|k})(z_{k+1}-z_{k+1|k})^{T}]= \\ &H_{k+1}P^{l}_{k+|k}H_{k+1}+R_{k+1} \end{aligned} \tag{36}$$

Because of $(x_{k+1}-\hat{x}_{k+1|k})$ and v_{k+1} are orthogonal, the formula (35) can be written as follow:

$$P^{l}_{xz,k+1} = P^{l}_{k+1|k}H^{T}_{k+1} \tag{37}$$

The formula (34) can be rewritten as follow:

$$F_{k+1|k}P_{k|k}F_{k+1|k} = P^{l}_{k+1|k} - Q_{k+1} \tag{38}$$

If Q_k is a positive definite matrix, the $P^{l}_{k+1|k}$ and $P_{k+1|k}$ are reversible, the formula (36) can be described as follow:

$$H_{k+1} = P^{l\ T}_{xz,k+1}(P^{l\ T}_{k+1|k})^{-1} \tag{39}$$

According to the formula (29) and (39), the N_{k+1} can be described as follow:

$$N_{k+1} = V_{k+1} - (P^{l\ T}_{k+1|k})^{-1}P^{l\ T}_{xz,k+1}Q_{k+1}(P^{l\ T}_{k+1|k})^{-1}P^{l\ T}_{xz,k+1} - lR_{k+1} \tag{40}$$

According to the formula (30), (38) and (39), the M_{k+1} can be described as follow:

$$M_{k+1} = [P^{l}_{k+1|k} - Q_{k+1}](P^{l\ T}_{k+1|k})^{-1}P^{l\ T}_{xz,k+1}P^{l}_{xz,k+1}P^{l\ -1}_{k+1|k} \tag{41}$$

It is can be seen that the advantage of adaptive fading factor calculation through formula (27), (28), (31), (40) and (41) is that it is not required to calculate the Jacobian matrix. The $P_{k+1|k}$ can be gotten after the adaptive fading factor is calculated. And $P_{k+1|k}$ instead of $P^{l}_{k+1|k}$ is used to calculate $P_{xz,k+1|k}$ and $P_{z,k+1|k}$.

3.3 STSRUKF algorithm

Though STUKF can get good estimation accuracy, robustness and mutation status tracking capability even if the system is uncertain and disturbed the disadvantage of the non-negative and asymmetry of covariance matrix caused by rounding errors of numerical calculation for UKF is not overcame. However, the problem can be solved by SRUKF instead of UKF. In the SRUKF, the square-root of state error covariance matrix and the square-root of output error covariance matrix are calculated using QR decomposition and Cholesky decomposition. In view of this, the STSRUKF algorithm is concluded as follow, and in the process of calculation, the parameter q_{03} should be normalized by equation (9) at any step.

1) Initializing the initial state x_0 and S_0

$$\hat{x}_0 = E(x_0) \tag{42}$$

$$S_0 = chol(E(x_0-\hat{x}_0)(x_0-\hat{x}_0)^{T}) \tag{43}$$

2) Calculating the sigma points

$$X_{k+1|k} = [x_k, x_k+\gamma S_k, x_k-\gamma S_k] \tag{44}$$

3) Calculating $\hat{x}_{k+1|k}$ and $S^{l}_{k+1|k}$

$$\chi_{k+1|k} = f(X_{k+1|k}, u_k) \tag{45}$$

$$\hat{x}_{k+1|k} = \sum_{i=0}^{2n} W_i^m \chi_{k+1|k,i} \tag{46}$$

$$S^l_{k+1|k} = qr\{[\sqrt{W_1^c}(\chi_{1:2n,k+1|k} - \hat{x}_{k+1|k}), \quad \sqrt{Q_{k+1}}]'\} \tag{47}$$

$$S^l_{k+1|k} = cholupdate\{S^l_{k+1|k}, \chi_{0,k+1|k} - \hat{x}_{k+1|k}, W_0^c\} \tag{48}$$

4) Recalculating Sigma points using $\hat{x}_{k+1|k}$ and $S^l_{k+1|k}$

$$X^l_{k+1|k} = [\hat{x}_{k+1|k}, \hat{x}_{k+1|k} + \gamma S^l_{k+1|k}, \hat{x}_{k+1|k} - \gamma S^l_{k+1|k}] \tag{49}$$

5) Calculating $\hat{z}_{k+1|k}$、 $P^l_{xz,k+1}$ and H_{k+1}

$$\xi^l_{i,k+1|k} = h(X^l_{i,k+1|k}) \tag{50}$$

$$\hat{z}_{k+1|k} = \sum_{i=0}^{2n} W_i^m \xi^l_{i,k+1|k} \tag{51}$$

$$P^l_{xz,k+1} = \sum_{i=0}^{2n} W_i^c (X^l_{i,k+1|k} - \hat{x}_{k+1|k})(\xi^l_{i,k+1|k} - \hat{z}_{k+1|k})^T \tag{52}$$

$$H_{k+1} = P^{l\ T}_{xz,k+1} (S^{l\ T}_{k+1|k} S^l_{k+1|k})^{-1} \tag{53}$$

6) Calculating adaptive fading factor μ_{k+1}

$$V_k = \begin{cases} (z_k - \hat{z}_{k+1|k})(z_k - \hat{z}_{k+1|k})^T, k = 1 \\ \dfrac{\rho V_{k-1} + (z_k - \hat{z}_{k+1|k})(z_k - \hat{z}_{k+1|k})^T}{1+\rho}, k \ge 2 \end{cases} \tag{54}$$

$$N_{k+1} = V_{k+1} - (S^l_{k+1|k}{}^T S^l_{k+1|k})^{-1} P^l_{xz,k+1}{}^T Q_{k+1} (S^l_{k+1|k} S^l_{k+1|k}{}^T)^{-1} P^l_{xz,k+1}{}^T - lR_{k+1} \tag{55}$$

$$M_{k+1} = [S^l_{k+1|k} S^l_{k+1|k}{}^T - Q_{k+1}] H_{k+1} H^T_{k+1} \tag{56}$$

$$u_{i,k+1} = trace(N_{k+1}) / \sum_{i=1}^{n} M_{ii,k+1} \tag{57}$$

$$u_{k+1,i} = \begin{cases} u_{k+1,i}, u_{k+1,i} \ge 1 \\ 1, u_{k+1,i} < 1 \end{cases} \tag{58}$$

where l is a softening factor and ρ is a forgetting factor.

7) Calculating $S_{k+1|k}$

$$S_{k+1|k} = qr\{[diag(\mu_{k+1})\sqrt{W_1^c}(\chi_{1:2n,k+1|k} - \hat{x}_{k+1|k}), \quad \sqrt{Q_{k+1}}]^T\} \tag{59}$$

$$S_{k+1|k} = cholupdate\{S_{k+1|k}, diag(\mu_{k+1})(\chi_{0,k+1|k} - \hat{x}_{k+1|k}), W_0^c\} \tag{60}$$

8) Recalculating Sigma points using $\hat{x}_{k+1|k}$ and $S_{k+1|k}$

$$X_{k+1|k} = [\hat{x}_{k+1|k}, \hat{x}_{k+1|k} + \gamma S_{k+1|k}, \hat{x}_{k+1|k} - \gamma S_{k+1|k}] \tag{61}$$

9) Calculating $P_{xz,k+1}$ and $S_{z,k+1|k}$

$$\xi_{i,k+1} = h(X_{i,k+1|k}) \tag{62}$$

$$S_{z,k+1} = qr\{[\sqrt{W_1^c}(\xi_{1:2n,k+1|k} - \hat{z}_{k+1|k}), \quad \sqrt{R_{k+1}}]^T\} \tag{63}$$

$$S_{z,k+1} = cholupdate\{S_{z,k+1}, \xi_{0,k+1} - \hat{z}_{k+1|k}, W_0^c\} \tag{64}$$

$$P_{xz,k+1} = \sum_{i=0}^{2n} W_i^c (X_{i,k+1|k} - \hat{x}_{k+1|k})(\xi_{i,k+1|k} - \hat{z}_{k+1|k})^T \tag{65}$$

10) Measurement updating

$$K_{k+1} = (P_{xz,k+1} / S^T_{z,k+1}) / S_{z,k+1} \tag{66}$$

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_{k+1}(z_{k+1} - \hat{z}_{k+1|k}) \quad (67)$$

$$U = K_{k+1} S_{z,k+1} \quad (68)$$

$$S_{k+1} = cholupdate\{S_{k+1|k}, U, -1\} \quad (69)$$

4. Simulation Results and Analysis

In order to verify the feasibility and efficient of the satellite attitude determination algorithms proposed in this paper, the SRUKF, STUKF and STSRUKF are applied for satellite ACS. Simulation with the same parameters is implemented on the mat lab R2010a software. The parameters are given as follow:

Original state estimate:

$$\hat{x}_0 = [1 \quad 0 \quad 0 \quad 0 \quad 0.03 \quad 0.04 \quad 0.03]^T$$

Original error covariance matrix:

$$P = 20\mathrm{diag}([10^{-5} \quad 10^{-5} \quad 10^{-5} \quad 10^{-5} \quad 10^{-5} \quad 10^{-5} \quad 10^{-5}])$$

Process noise covariance matrix:

$$Q = 20diag([10^{-8} \quad 10^{-8} \quad 10^{-8} \quad 10^{-8} \quad 10^{-8} \quad 10^{-8} \quad 10^{-8}])$$

Measurement noise covariance matrix:

$$R = 20diag([10^{-10} \quad 10^{-10} \quad 10^{-10}])$$

Inertia matrix:

$$J = \begin{bmatrix} 49.96 & 2.68 & 0.24 \\ 4.7 & 55.40 & 0.24 \\ 0.24 & 0.24 & 63.00 \end{bmatrix}$$

In order to compare robustness, stability and mutation status tracking capability of three algorithms, in $t = 80s \sim 150s$, the process noise is magnified 10 times, measurement noise is magnified 20 times, and a constant interference $x = [0 \quad 0 \quad 0 \quad 0 \quad 0.005 \quad 0.005 \quad 0.005]^T$ is also introduced. Figures 1~6 are absolute estimation error of angular Velocity in the x axis, absolute estimation error of angular Velocity in the y axis, absolute estimation error of angular Velocity in the z axis, absolute estimation error of roll angle, absolute estimation error of pitch angle and absolute estimation error of yaw angle.

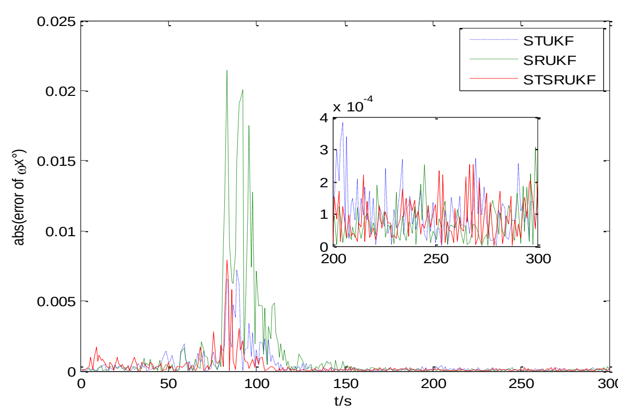


Figure 1. Absolute Estimation Error of Angular Velocity about X Axis

From Figures 1-6, it can be seen that STSRUKF and SRUKF can get better estimation accuracy than STUKF when system is stable. However, if the system is disturbance-driven and the noise statistics changes, SRUKF is more easily to be affected than STUKF and STSRUKF, and the poor estimation precision of attitude and angular velocity is obtained by it. Compared with SRUKF, STUKF can get better estimation precision of attitude and angular velocities, and it is relatively accurate and stable and quickly converge despite that the estimation precision of attitude and angular velocity are poor at the beginning of that system is disturbed and the noise statistics changes. The estimation precision of angular velocities obtained by STSRUKF are also affected at the beginning of that system is disturbed and the noise statistics changes, but they are best relative to the estimation precision obtained by SRUKF and STUKF, and compared with SRUKF and STUKF, the convergence of STSRUKF is the fastest. The estimation precision of attitude obtained by STSRUKF is also best and keeps invariant in the entire process of that system is disturbed and the noise statistics changes. It has the best performance among the three algorithms.

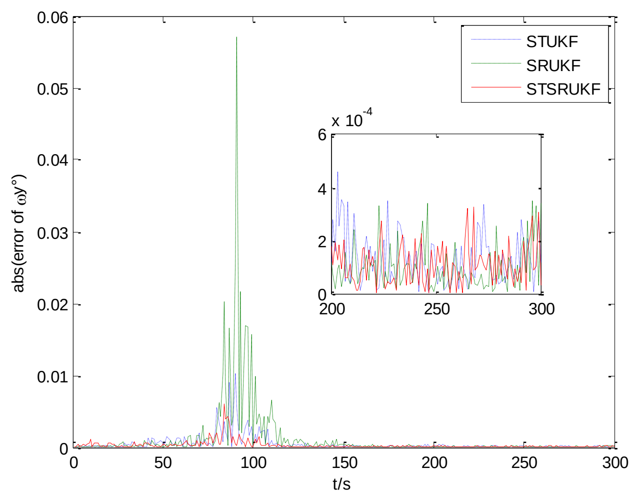


Figure 2. Absolute Estimation Error of Angular Velocity About Y Axis

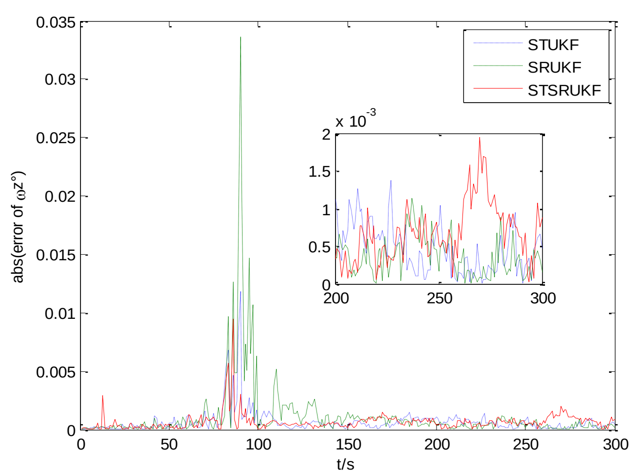


Figure 3. Absolute Estimation Error of Angular Velocity about z Axis

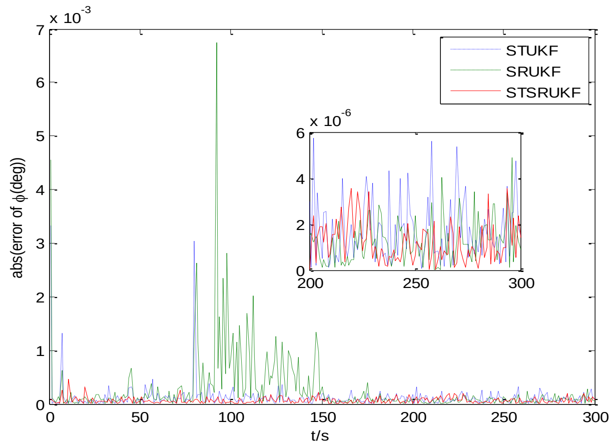


Figure 4. Absolute Estimation Error of Roll Angle

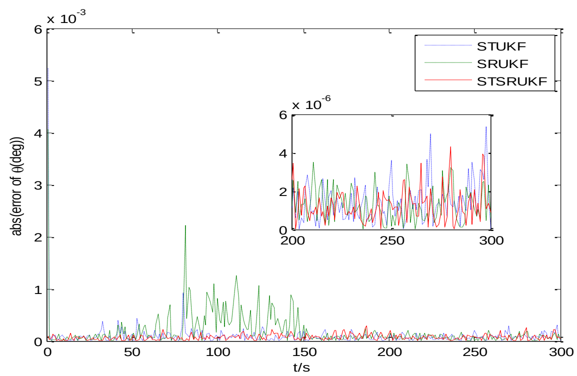


Figure 5. Absolute Estimation Error of Pitch Angle

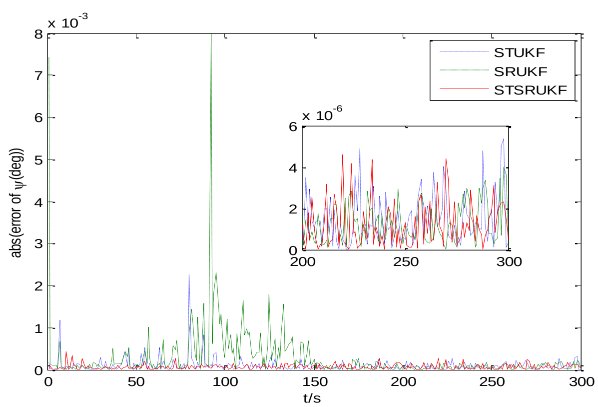


Figure 6. Absolute Estimation Error of Yaw Angle

5. Conclusions

To address the problem of non-gyro satellite attitude and angular velocity estimation, combined with SRUKF and STF theoretical ideas, an STSRUKF-based is proposed in this paper. The algorithm uses the equivalence description between UKF and KF to calculate adaptive fading factor, which avoid calculating Jacobian matrix, and SRUKF instead of UKF is applied, which guarantees positive semi definiteness and improves stability and accuracy. Simulation results demonstrate that, compared with STUKF and SRUKF, the STSRUKF has best stability, robustness and mutation status tracking capability.

Acknowledgment

This work is partially supported by National Natural Science Foundation of China (Grant No. 61304237) and Open Funding for National Defense Key Subject Laboratory of Micro and Small Spacecraft Technology (Grant No. 20090450126).

References

[1] M. L. Psiaki, "Backward smoothing extended Kalman Filter", Journal of Guidance, Control, and Dynamics, vol. 28, no. 5, **(2005)**, pp. 885-894.
[2] J. Joseph, L. Viola Jr, "A Comparison of Unscented and Extended Kalman Filtering for Estimating Quaternion Motion", Proceedings of the 2003 American Control Conference, IEEE Press, **(2003)**, pp:2435-2440.
[3] S. J. Julier, J. K. Uhlamnn and H. F. Durrant-Whyte, "A new method for the nolinear transformation of means and covariances in fiters and estimators", IEEE Transactions on Automatic Control,vol.45, **(2000)**, pp:477-482.
[4] H. L. Shou, C. T. Lin and C. L. Chang, "Micro-Satellite Attitude Angle Rate Estimation Unscened Kalman Fiter Approach",Proceeding SCIE Annual conference 2010, Taipei, Tai-wan, **(2010)**, pp:1278-1290.
[5] P. L. Wu and J. J. Kong, "Underwater Bearing-only Target Tracking Based on Square-rootUKF", Journal of Nanjing University of Science and Technology, vol. 33, no. 6, **(2009)**, pp. 751-755
[6] B. Akin, U. Orguner and A. Ersak, "State estimation of induc-tion motor using unscented Kalman filter", Proc. Of IEEE Conference on Control Applications, CCA, vol. 2, **(2003)**, pp. 915-919.
[7] R. Van der Merwe, E. A. Wan, "The square-root Unscented Kalman filter for stateand parameter-estimation", proceedings of the International Conference on Acoustics, Speech, and Signal Processing, New York: Inst of Electrical and Electronics Engineers, **(2001)**, pp. 3461-3464.
[8] S. A. Holmes, G. Klein and D. W. Murray, "Square Root Unscented Kalman Filters for State Estimation of Induction Motor Drives", IEEE Transactions on Pattern Analysis and Machine Intelligence , vol. 31, no. 7, **(2009)**, pp. 92-99.
[9] X. Y. Jiang and G. F. Ma, "Gyroless Satellite Attitude Estimation for Square Root Unscented Kalman Filter", Journal of Nanjing University of Science and Technology, vol. 29, no. 4, **(2005)**, pp. 399-402.
[10] D. H. Zhou, Y. G. Xi and Z. J. Zhang, "Suboptimal fading extended Kalman filtering for nonliear systems", Control and Decision, vol. 5, no. 5, **(1999)**, pp. 1-6.
[11] D. H. Zhou, Y. G. Xi, Z. H. J. Zhang, "A suboptimal multi. pie fading extened Kalman filter", Acta Automatiea Sinica, vol. 16, no. 7, **(1991)**, pp. 689-695.
[12] W. B. Yang and S. Y. Li, "Autonomous navigation filtering algorithm for spacecraft based on strong tracking UKF", Systems Engineering and Electronics, vol. 33, no. 11, **(2011)**, pp. 2485-2491.
[13] Y. Shi and C. Z. Han, "Adaptive UKF Method with Applications to Target Tracking", Journal of Acta Automatica Sinica, vol, 37, **(2011)**, pp:755-759.
[14] D.-J. Jwo and F. C. Chung, "Fuzzy Adaptive Unscented Kalman Filter for Ultra-Tight GPS/INS Integration", Proceding of 2010 International Symposium on Computational Intelligence and Design, Hang Zhou , China, **(2010)**, pp:229-235.
[15] R. W. Zhang, "Satellite Orbit&attitude dynamics and control", Beihang University press, China, **(2005)**.
[16] W. D. Zhou, X. W. Qiao and Y. R. Ji, "SINS/GPS tightly integrated navigation system based on quaternion square root unscented Kalman filter", vol. 32, no. 12, **(2010)**, pp. 2643-2647
[17] R. Wan, R. van der Merwe, "The unscented Kalman filter for non linear estimation", IEEE Proceedings of Symposium 2000 on Adaptive Systems for Signal Processing, Communica tion and Control, Lake Louise, Alberta, Canada, **(2000)**, pp. 153-158.
[18] N. Sunderhauf, S. Lange and P. Protzel, "Using the unscented Kalman filter in mono-SLAM with inverse depth parame- terization for autonomous airship control", IEEE International Workshop on Safety, Security and Rescue Robotics, Piscat away, NJ, USA IEEE, **(2007)**, pp. 1-6.
[19] J. J. Li, Y. C. Zhang and H. Y. Li, "Application of ASUKF in autonomous spacecraft navigation", Journal of Harbin Engineering University, vol. 32, no. 5, **(2011)**, pp. 575-580.

[20] M. H. Qian, W. Huang and L. Sun, "Attitude estimation of strong tracking UKF based on multiplie fading factors", Systems Engineering and Electronics, vol. 35, no. 3, **(2013)**, pp. 580-585.

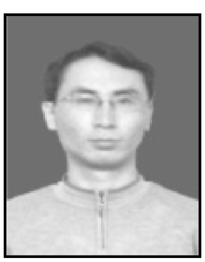
Authors

Min Li, was born in China, on December 7, 1989. He is a student of the University of Harbin Institute of Technology, Control Science and engineering, in 2013. His research interests concern: Satellite attitude estimation and control.

Bao-long Zhu, was born in China, on September 9, 1990. He is a student of the University of Harbin Institute of Technology, Control Science and engineering, in 2013. His research interests concern: Engineering applications of control theory, Optimal estimation.

Song-yan Wang, was born in China, on October 6, 1976. She graduated the University of Harbin Institute of Technology, Control Science and engineering, in 2000. She received the PhD degree in Control Science and engineering from the University of Harbin Institute of Technology, in 2005. She is an Associate Professor at the University of Harbin Institute of Technology, in China. Her research interests concern: Spacecraft attitude estimation and control.

Ying-chun Zhang, was born in China, on March 8, 1961. He graduated the University of Harbin Institute of Technology, Control Science and engineering, in 1987. He received the Master degree in Control Science and engineering from the University of Harbin Institute of Technology, in1987. He is a Professor at the University of Harbin Institute of Technology, in China. His research interests concern: Design of satellite control system, fault diagnosis, Attitude estimation



Huayi Li, was born in China, on June 6, 1978. He graduated the University of Harbin Institute of Technology, astronautics science and technology, in 2008. He received the phD degree in astronautics science and technology from the University of Harbin Institute of Technology, in 2008. He is a Lecturer at the University of Harbin Institute of Technology, in China. His research interests concern: Satellite Formation.

