

# The Minimum Mean Square Precoding Method for Eliminating Asynchronous Interferences in Cognitive Radio System

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## **Abstract**

*If there is a certain distance between the primary users and the second, there are asynchronous interferences because of existing nonsynchronous signals. And the interferences do not influence only the primary user system, but they could also affect second user system and increase interference. To solve this problem and to eliminate interference to primary and secondary asynchronous users among which there are influences in wireless communication system, a novel kind of precoding method based on the minimum mean square error is put forward in this paper. In this method, asynchronous interferences are eliminated in the precoding process and this process also satisfies users' server quality. Analysis and simulation show that the proposed scheme effectively, under the condition of the limitation of interferences in main user system, enhance the second user system capacity and improve the reliability of the transmission system of second users system.*

**Keywords:** *Cognitive Radio, Spectrum Share, MIMO, Asynchronous Interferences*

## **1. Introduction**

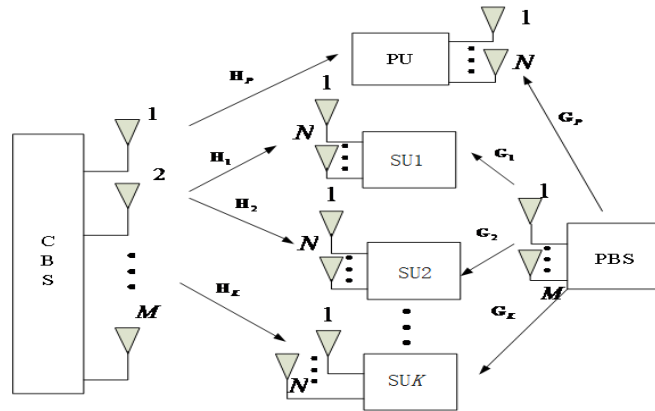
When the primary user (PU) system is being on communication, other second user (SU) system need to work using different communication resource in order to avoid bring about interferences to PU. In study many different communication resources are used efficiently, such as space resource, code resource [1-3]. Researchers pay much attention on multi-antenna technology which makes full use of space resource, such as Multiple Input Multiple Output (MIMO), smart antenna and distributed antenna.

In [1-3], there is an assumption that the interference coming from SU is being in exact synchronization with PU and that the interference, coming from the process that SU receives signals sent by many CBS, is also in exact synchronization. These assumptions are easy to eliminate the interferences coming from many SU systems and interferences of PU [4-5]. However, PU and SU systems could not be in exact synchronization in practice, and the process that CBS signals reach multi-users could not also be exactly synchronized. Prior studies did not consider asynchronous interferences, and the asynchronous interferences could not be eliminated. The result is that the transmission rate of PU and SU system is decreased and bit error rate is increased. A minimum mean square precoding method for eliminating asynchronous interferences in PU and SU system is put forward with asynchronous interference model in [6]. This method effectively improves PU and SU system capacity and the bit error rate of SU system through simulation and analysis.

## **2. System Model**

Similarly to [4], the proposed system model is shown in Figure 1, and PU system which has a Primary Base Station (PBS) will communicate with a primary user. There is one Cognitive Base Station (CBS) which serves  $K$  sub-users in SU cognitive network. PBS and CBS both have  $M$  antennas, and primary user and  $K$  sub-users are allocated with  $N$  receiving antennas. Signal received by primary user on the same frequency will be made interference by signals sent by CBS. At the same time slot, information symbols

coming from CBS are  $\mathbf{s} = [s_1, s_2, \dots, s_K]^T$ , and in these symbols  $s_k$  is sent to sub-user  $k$  ( $k = 1, \dots, K$ ). These is an assumption that  $\mathbf{s}$  is a unit-energy vector and it meets equation  $E[\mathbf{s}\mathbf{s}^H] = \mathbf{I}_K$ .



**Figure 1. Cognitive Model of Multi-users**

The signals coming from CBS are described as following equation.

$$\mathbf{x} = \mathbf{T}\mathbf{s} \quad (1)$$

In the equation (1),  $\mathbf{T} = [t_1, t_2, \dots, t_K]$  is a precoding matrix when CBS sends signals, and  $t_k$  ( $k = 1, \dots, K$ ) is a  $M \times 1$  vector. Then sub-user  $k$  will receive signals as follows:

$$\mathbf{y}_k = \sqrt{\beta_k} \mathbf{H}_k \mathbf{x} + \sqrt{P_p} \mathbf{G}_k \mathbf{s}_p + \mathbf{n}_k \quad (2)$$

In the above equation, the parameters are as follows:

$\mathbf{s}_p \in \mathbb{C}^{M \times 1}$  are signals coming from PBS;  $\beta_k$  is path loss from CBS to sub-user  $k$ ;

$P_p$  is sending power of PBS;  $\mathbf{H}_k \in \mathbb{C}^{N \times M}$  is the channel from CBS to sub-user  $k$ ,  $1 \leq k \leq K$ ;

$\mathbf{G}_k \in \mathbb{C}^{N \times M}$  is the channel from PBS to sub-user  $k$ ,  $1 \leq k \leq K$ ;

$\mathbf{G}_p \in \mathbb{C}^{N \times M}$  is the channel from PBS to primary user;

The elements in  $\mathbf{H}_k$ ,  $\mathbf{G}_k$  and  $\mathbf{G}_p$  are independent and identically distributed random variables of compound gaussian, and mean value is zero and variance is 1;

$\mathbf{n}_k \in \mathbb{C}^{N \times 1}$  is noise vector which mean value is zero and covariance matrix is

$$\mathbf{R}_{\mathbf{n}_k \mathbf{n}_k} = E[\mathbf{n}_k \mathbf{n}_k^H] = \mathbf{I}_N;$$

The signals received by primary user is as follows:

$$\mathbf{y}_p = \sqrt{P_p} \mathbf{G}_p \mathbf{s}_p + \mathbf{H}_p \mathbf{x} + \mathbf{n}_p \quad (3)$$

In equation (3),  $\mathbf{H}_p \in \mathbb{C}^{N \times M}$  is the channel from CBS to primary user;  $\mathbf{n}_p \in \mathbb{C}^{N \times 1}$  is noise vector which mean value is zero and covariance matrix is  $\mathbf{R}_{n_p n_p} = E[\mathbf{n}_p \mathbf{n}_p^H] = \mathbf{I}_N$ .

The asynchronous interference model is similar to [6]. The standard transmission delay parameter  $\tau_k^{(s)}$  and  $\tau_k^{(p)}$  are set, and  $\tau_k^{(s)}$  is a delay parameter between CBS to sub-user  $k$ , and  $\tau_k^{(p)}$  a delay parameter between PBS to sub-user  $k$ . The delay time when CBS system sends information is  $\Delta \tau_k^{(s)} = \tau_k^{(s)} - \tau_k^{(p)}$  (when  $\Delta \tau_k^{(s)}$  is more than zero, CBS send information ahead;  $\Delta \tau_k^{(s)}$  is less than zero, CBS send information after). In SU system, the difference between delay adjusting value of users  $k$  and  $j$  is  $\tau_{jk}^{(s)} = (\tau_k^{(s)} - \Delta \tau_j^{(s)}) - \tau_k^{(p)} = \Delta \tau_k^{(s)} - \Delta \tau_j^{(s)}$ .  $\tau_{jk}^{(s)}$  is the time D-value between the time when signals coming from user  $j$  at PBS station arrive at user  $k$  and the time when PBS interferes signals of user  $k$ .

The model of asynchronous interference is described in Figure 2. Asynchronous interferences are taken place between signals in a row that are sent from CBS to user  $j$ , asynchronous interferences between signal  $m_{jk}^{(b)}$  and  $m_{jk}^{(b)} + 1$ . Asynchronous interference for user  $k$ , which is made by signals coming from user and sent by CBS, is described as  $\mathbf{i}_{jk}^{(b)}$ , and it is also described as following equation.

$$\mathbf{i}_{jk}^{(s)} = \rho(\delta_{jk}^{(b)} - T_s) s_j(m_{jk}^{(b)}) + \rho(\delta_{jk}^{(b)}) s_j(m_{jk}^{(b)} + 1) \quad (4)$$

In (4) equation,  $\rho(\tau) = \int_0^{T_s} g(t) g(t - \tau) dt$ , and  $\delta_{jk}^{(b)} = \tau_{jk}^{(b)} \bmod T_s$ , and  $T_s$  is symbol period. According to description in [6], there is a limit  $E[\mathbf{i}_{jk}^{(s)}(m) \mathbf{i}_{jk}^{(p)}(m)^H] = \beta_{jk}^{(s,p)} \mathbf{I}_{L_j}$ , and  $L_j$  is amount of data received by user.

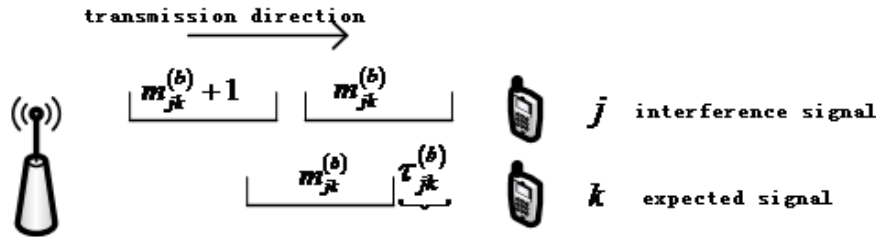


Figure 2. The Model of Asynchronous Interferences in Downlink

Statistic characteristics of  $\beta_{jk}^{(b_1, b_2)}$  are enumerated as follows:

If  $j \neq k$ ,

$$\beta_{jk}^{(s,p)} = 0, \quad \text{if } |m_{jk}^s - m_{jk}^p| > 1 \quad (5)$$

$$\beta_{jk}^{(s,p)} = \rho \left( \delta_{jk}^{(s)} \right) \rho \left( \delta_{jk}^{(p)} - T_s \right), \quad \text{if } m_{jk}^s = m_{jk}^p + 1 \quad (6)$$

$$\beta_{jk}^{(s,p)} = \rho \left( \delta_{jk}^{(s)} \right) \rho \left( \delta_{jk}^{(p)} \right) + \rho \left( \delta_{jk}^{(s)} - T_s \right) \rho \left( \delta_{jk}^{(p)} - T_s \right), \quad \text{if } m_{jk}^s = m_{jk}^p \quad (7)$$

$$\beta_{jk}^{(s,p)} = \rho \left( \delta_{jk}^{(s)} \right) \rho \left( \delta_{jk}^{(p)} - T_s \right), \quad \text{if } m_{jk}^s = m_{jk}^p - 1 \quad (8)$$

If  $j = k$ ,

$$\beta_{kk}^{(s,p)} = 1 \quad (9)$$

### 3. The Precoding Project based on MMSE

According to the above model, optimization function is defined as follow:

$$\begin{aligned} & \min_{\{\mathbf{t}_1, \dots, \mathbf{t}_k\}} E_{\mathbf{H}_k, \mathbf{G}_k} [\varepsilon_k(\mathbf{t}_k)] \\ & \text{s.t.} \begin{cases} E[\sum_{k=1}^K \text{Tr}(\mathbf{t}_k \mathbf{t}_k^H)] \leq P \\ \mathbf{H}_p^H \mathbf{T}^H \mathbf{T} \mathbf{H}_p \leq P_{th} \end{cases} \end{aligned} \quad (10)$$

In the above equation,  $\varepsilon_k(\mathbf{t}_k)$  is the definition of mean square error, and its equation is  $\varepsilon_k(\mathbf{t}_k) = \left\| \mathbf{s}_k - \sqrt{\beta_k} \mathbf{H}_k \mathbf{x} - \sqrt{P_p} \mathbf{G}_k \mathbf{s}_p - \mathbf{n}_k \right\|^2$ , and  $P$  is the restriction of power sent by CBS and  $P_{th}$  is the threshold of interference power to primary user.

The restriction  $E[\sum_{k=1}^K \text{Tr}(\mathbf{t}_k \mathbf{t}_k^H)] \leq P$  is illustrated that the sending power of second user station is restricted and the maximum power is no more than  $P$ , and the second restriction is that the most interference power for primary is no more than  $P_{th}$ . These restrictions could ensure the quality of service of the physical layer in the primary user system.

For the optimal question, CSI is gained only through restricted feedback system, because CBS is frequency-division duplex (FDD) system, and the CSI is channel state information after quantization.  $\hat{\mathbf{H}}_k$ ,  $\hat{\mathbf{G}}_k$  and  $\hat{\mathbf{G}}_p$  are known in CBS system. For good analysis  $\varepsilon_k(\mathbf{t}_k)$  is simplified as follow:

$$\varepsilon_k(\mathbf{t}_k) = \left[ \left\| \mathbf{s}_k - \mathbf{y}_k \right\|_F^2 \right] = \left[ \left\| \mathbf{s}_k - \sum_{b=1}^B \mathbf{H}_k^{(b)} \mathbf{T}_k^{(b)} \mathbf{s}_k - \sum_{b=1}^B \sum_{\substack{j=1 \\ j \neq k}}^K \mathbf{H}_k^{(b)} \mathbf{T}_j^{(b)} \mathbf{i}_{jk}^{(b)} - \mathbf{n}_k \right\|_F^2 \right] \quad (11)$$

Formula (5)-(9) are inserted into equation (11). But there is assumption that  $\mathbf{J}_k$  is  $\mathbf{J}_k = \sum_{b=1}^B \sum_{\substack{j=1 \\ j \neq k}}^K \mathbf{H}_k^{(b)} \mathbf{T}_j^{(b)} \mathbf{i}_{jk}^{(s)}$  and it is interferences which could influence the judgment of receiving signals. For this reason it will be eliminated. Equation (11) is simplified as following simple form.

$$\varepsilon_k(\mathbf{t}_k) = E \left\{ \left\| s_k - \mathbf{H}_k \mathbf{T}_k s_k - \mathbf{J}_k - \mathbf{n}_k \right\|_F^2 \right\} \quad (12)$$

The optimization of minimum mean square error is a question of Semi-Infinite Programming (SIP). Lagrangian function is constructed with the optimization principle of Lagrange multiplication. And the Lagrangian function is described as following equation.

$$L(\mathbf{t}_k, \lambda, \eta) = E_{\mathbf{H}_k \mathbf{G}_k} [\varepsilon_k(\mathbf{t}_k)] + \lambda \left( \mathbf{t}_k^H \mathbf{t}_k - \frac{P}{K} \right) + \eta (\mathbf{H}_p^H \mathbf{t}_k \mathbf{H}_p - P_{th}) \quad (13)$$

In the equation of Lagrangian function,  $\mathbf{t}_k$  is the single variable, and  $\lambda$  and  $\eta$  are the Lagrangian coefficients. Make partial derivatives of the three parameters respectively, and the result is expressed as follows:

$$\begin{aligned} \frac{\partial L}{\partial \mathbf{t}_k} &= \beta_k \mathbf{t}_k^H \mathbf{t}_k \mathbf{H}_k \mathbf{H}_k^H + P_p \mathbf{t}_k^H \mathbf{t}_k \mathbf{G}_k^H \mathbf{G}_k - \beta_k P_p \mathbf{t}_k^H \mathbf{t}_k \mathbf{G}_k^H \mathbf{G}_k \mathbf{H}_k \mathbf{H}_k^H \\ &\quad - \mathbf{t}_k^H \mathbf{H}_k \mathbf{G}_k \mathbf{G}_k^H + 2\lambda \mathbf{t}_k^H + 2\eta \mathbf{t}_k \mathbf{H}_p = 0 \end{aligned} \quad (14)$$

$$\frac{\partial L}{\partial \lambda} = \mathbf{t}_k^H \mathbf{t}_k - \frac{P}{K} = 0 \quad (15)$$

$$\frac{\partial L}{\partial \eta} = \mathbf{t}_k \mathbf{H}_p = 0 \quad (16)$$

The results of formula (15) and formula (16) are inserted into formula (14), and the formula of precoding vector is calculated through simplifying formula (14).

$$\mathbf{t}_k = \frac{1}{\Omega_k} [\mathbf{D}_k + \lambda_k \mathbf{I}_{N,B}]^{-1} \mathbf{H}_k^H \mathbf{A}_k \quad (17)$$

In the formula (17),

$$\lambda_k \text{ is a Lagrangian parameter; } \mathbf{A}_k = \mathbf{H}_k \mathbf{B}_k; \mathbf{D}_k = \begin{bmatrix} \mathbf{D}_k^{(1,1)} & \mathbf{D}_k^{(1,2)} & \dots & \mathbf{D}_k^{(1,B)} \\ \mathbf{D}_k^{(2,1)} & \mathbf{D}_k^{(2,2)} & \dots & \mathbf{D}_k^{(2,B)} \\ \dots & \dots & \dots & \dots \\ \mathbf{D}_k^{(B,1)} & \mathbf{D}_k^{(B,2)} & \dots & \mathbf{D}_k^{(B,B)} \end{bmatrix},$$

and the factor  $\mathbf{D}_k^{(b_1, b_2)}$  in  $\mathbf{D}_k$  matrix is expressed as follow:

$$\mathbf{D}_k^{(b_1, b_2)} = \sum_{j=1}^K \frac{\beta_{kj}^{(s,p)} \mathbf{H}_j^{(s)H} \mathbf{H}_j^{(p)}}{\Omega_j} \quad (18)$$

The mean power of received signals is  $\Omega_k = E \left[ tr \{ y_k y_k^H \} \right]$ .

The format of the precoding method is similar to the normalized MMSE, and the difference is the increase of element  $\mathbf{D}_k$  in formula (17) and it is caused by asynchronous interferences according to equation (18). At transmitting terminal, if the  $\mathbf{D}_k$  is eliminated through making an invert, other interferences including asynchronous interferences are reduced. In the fourth segment of this paper, the simulation of this method shows that system capacity and BER in SU system is improved with the proposed scheme( the primary system capacity and SU BER is improved comparing with other schemes because the interferences of primary system are restricted).

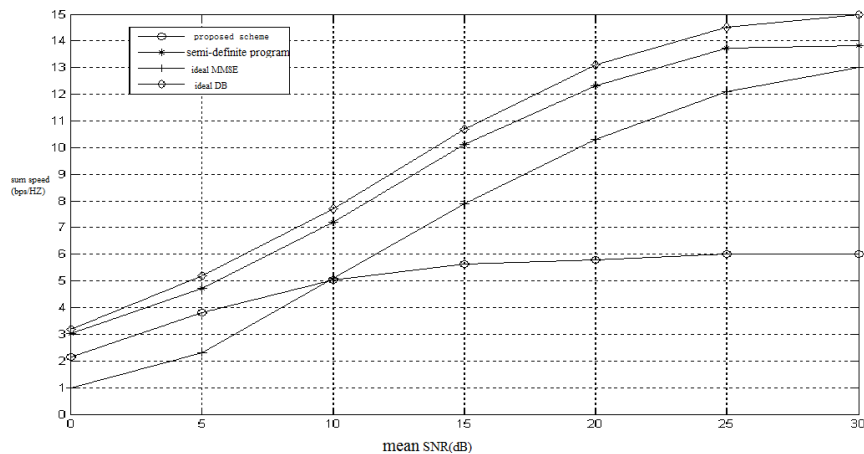
#### 4. Simulation and Analysis

In this part, the performance of maximum mutual information between PU and SU systems and the performance of bit error rate in SU system in proposed scheme are simulated. The maximum mutual information and BER of the below three schemes are compared and the results are showed as follows:

- 1) use the semi-definite programming precoding scheme[4], the “semi-definite program” is labeled in Figures 3,4 and 5;
- 2) use the traditional BD precoding method[7], the “ideal BD” is labeled in Figures 3,4 and 5;
- 3) use the method based on MMSE[8], the “ideal MMSE” is labeled in Figures 3,4 and 5;

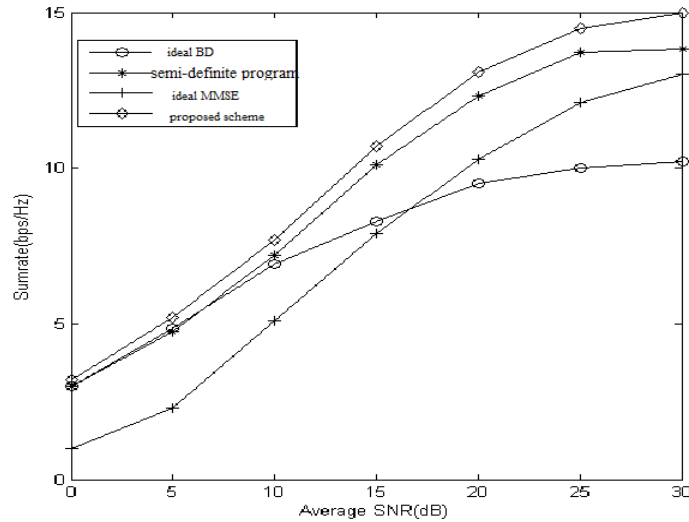
The elimination of asynchronous interferences is not considered respectively in the above three schemes.

In the process of simulation,  $M = 6$ ,  $N = 2$ ,  $K = 3$ ,  $P = P_p = 20dB$ . The performance of maximum mutual information for the PU system is showed in Figure 3. In the Figure 3, with the same sum mean mutual information, the proposed scheme has more than 0.7 dB gain through the comparison with the semi-definite programming scheme. When mean SNR is in the interval 15-30dB, the proposed scheme and the semi-definite programming scheme both have full multiplexing gain, but the proposed scheme is better than the semi-definite programming scheme and a little less than ideal MMSE scheme (1bps/s/Hz).



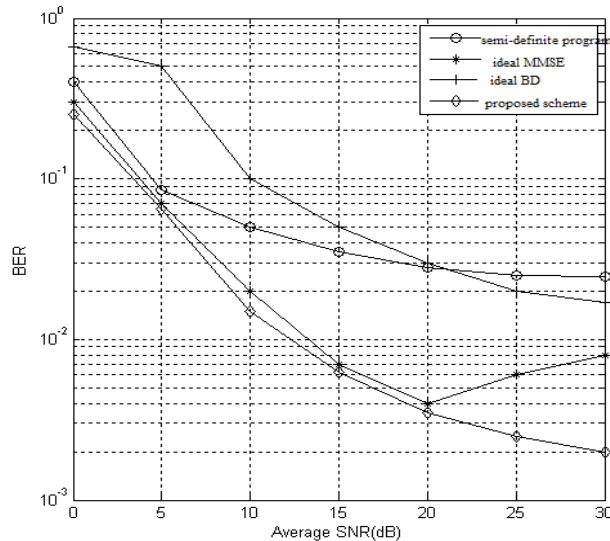
**Figure 3. Maximum Mutual Information in PU System**

The performance of maximum mutual information of the four schemes in SU system is showed in Figure 4. The proposed precoding scheme improves the transmitting performance of SU system and is much better than other three schemes that do not consider asynchronous interferences.



**Figure 4. Maximum Mutual Information in SU System**

Figure 5 shows the changing curve of bit error rate of the four schemes. With the increase of SNR, the semi-definite programming scheme and ideal BD scheme have ceiling effect and the proposed scheme effectively overcome this effect and has lower bit error rate.



**Figure 5. Bit Error Rate of SU System**

## 5. Conclusion

A precoding method in cognitive radio system of MIMO is proposed and asynchronous interference is considered in this method. This scheme does not only eliminate the multi-user interference and interference for the primary user, but also considers the asynchronous multi-user interferences and interferences coming from primary user for sub-user. This scheme does not only improve the maximum mutual information between SU system and PU system but also improves the transmitting reliability of the SU system.

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