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An Image Filter Arithmetic based on GA, PDE and TV

Zhiyong Fan[1,2], Quansen Sun[1], Zexuan Ji[1], Feng Ruan[2] and Liling Zhao[1,2]

[1]*Nanjing University of Science & Technology, Nanjing 210000, China*
[2]*Nanjing University of Information Science & Technology, Nanjing 210044, China*

Abstract

Rician noise pollutes Magnetic Resonance Imaging (MRI) image and makes later work worse. In allusion to remove noise while lessen the loss of details as low as possible, this paper proposed an filter algorithm which comprehensive utilize Genetic Algorithm (GA), PDE and TV, based on 4th order Partial Differential Equations (PDE) and Total Variation (TV) theory. First, it calculates the Total Variation(TV) and the 4th order PDE of kth image, and gives them weight coefficients. Then, it finds the optimal Standard deviation of image of kth image by adjusting weight coefficients based on GA algorithm. Third, it compares PSNR of kth image and PSNR of k+1th image to find whether the algorithm is over. Experimental results show that our new algorithm presented in this paper is more effective in removing Rician noise and giving better Peak Signal Noise Ratio (PSNR) gains without manual intervention in comparison with other traditional filters.

Keywords: *Image filter, PDE, GA, MRI*

1. Introduction

Magnetic Resonance Imaging (MRI) images are frequently corrupted by Rician noise during image gaining or transmission [1]. This makes noise reduction to be one of the key problems in image processing and the basis of image subsequent processing. Preservation of image details and attenuation of noise are the two important of noise reduction but they are contradictory in nature. So, this research emphasis is on the removal of Rician noise while lessening the loss of details as low as possible.

As the field requires higher levels of reliability and efficiency for the last two decades, mathematical image processing has become an important component. In particular, mathematical frameworks employing recent powerful tools of partial differential equations (PDEs) [2] and functional analysis have been extensively studied to answer fundamental questions in image processing.

Applications of the PDE models can be widely found in a broad range of image restoration tasks such as denoising and enhancement, color image processing and in painting, Image restoration is often necessary as pre-processing for other operations such as segmentation, three-dimensional (3-D) construction, and compression; good denoising methods have strong demands. However, those PDE-based methods can show some drawbacks unless the governing equations are both incorporating appropriate parameters and discredited by suitable numerical schemes.

For the removal of Rician noise in MRI image, we propose a new image filter arithmetic based on GA [3, 4] and VT, 4th order PDE. First, it calculates the Total Variation (TV) and the 4th order PDE of kth image, and gives them weight coefficients. Then, it finds the optimal Standard deviation of image of kth image by adjusting weight coefficients based on GA

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algorithm. Third, it compares the Standard deviation of image of kth image and Standard deviation of image of k+1th image to find whether the algorithm is over or not.

The remaining paper is organized as follows. Section 2 introduces related work. Section 3 presents our algorithm, including workflow. Section 4 gives the experimental results of our proposed algorithm. Finally, Section 5 concludes this paper.

2. Related Works

2.1. Rician Noise

Noised MRI image v can be defined as $v_{(i)} = u_{(i)} + n_{(i)}$, here, $u_{(i)}$ is original image pixels, $n_{(i)}$ is noised pixel. When MR images are computed using the magnitude of a single complex raw data, its distribution can be modeled as a Rician model [15-17].

$$p(m) = \frac{m}{\sigma_n^2} e^{\frac{m^2 + A^2}{2\sigma_n^2}} I_0\left(\frac{Am}{\sigma_n^2}\right) \tag{1}$$

Here, σ^2 is the standard deviation (STD) of Gaussian noise, A is the amplitude of the signal without noise, x is the value in the magnitude image and I_0 is the 0th order modified Bessel function. This model is used by the majority of the noise estimation methods.

When SNR is small enough (i.e. SNR=0), the Rician distribution is considerate as a Rayleigh distribution.

$$p\left(m\right) = \frac{m}{\sigma_n^{\ 2}} e\left(\frac{m^2}{\sigma_n^{\ 2}}\right) \tag{2}$$

When SNR is high (i.e. SNR>3), the Rician distribution is approximated as a Gaussian distribution.

$$p\ \left(m\right) = \frac{1}{2\pi\sigma^2} e\left(-\frac{\left(m^2 - \sqrt{A^2 + \sigma_n^2}\right)^2}{2\sigma_n^2}\right) \tag{3}$$

2.2. Total Variation Minimization (TVM)

Let $\Omega \subset R^2$ be an open subset with Lipschitz boundary. Planar images are often assumed to be in $BV(\Omega)$, the space of function of functions with bounded variation (BV) , $BV(\Omega)$ is the subspace of functions $u \in L^1(\Omega)$ such that the following quantity called BV semi-norm is finite,

$$\int_\Omega |\nabla u| = \sup\left\{\int_\Omega u\left(x\right) div\left(\xi\left(x\right)\right) dx; \xi \in C_0^1\left(\Omega, R^2\right), \|\xi\| \le 1\right\} \tag{4}$$

$BV(\Omega)$ endowed with norm $\|u\|_{BV(\Omega)} = \int\limits_\Omega |\nabla u| + \|u\|_{L^1(\Omega)}$ is a Banach space.

Total variation (TV) minimization (ROF model):

$$u = \arg\min\left\{\int_\Omega |\nabla u|\,dx + \frac{\lambda}{2}\int_\Omega (u-u_0)^2\,dx\right\} \tag{5}$$

It was first proposed by Rudin, Osher and Fatemi [3]. It does an excellent job in preserving edges in image restoration and reconstruction. Mathematically this is reasonable, since images always have jumps or discontinuities; it is natural to study this minimization problem in the space of functions with bounded variation. This phenomenon can also be explained physically, since the corresponding diffusion is orthogonal to the gradient of the image.

2.3. Fourth-order PDEs

High order PDEs is created by:

$$\min_u \int_\Omega F\left(x_1, x_2, \cdots, x_n, du, d^2u, \cdots, d^k u\right) dx_1 dx_2 \cdots dx_n \tag{6}$$

Here, $\Omega \subset R^n, n \geq 2, d^k u$ is the kth-order partial derivative of u.

Lysker, Osher and Tai proposed LOT model:

$$\min_u \left\{T(u) = \int_\Omega \left(\left|u_{xx} + u_{yy}\right|\right) dxdy + \frac{\lambda}{2}\int_\Omega |u-u_0|^2\,dxdy\right\} \tag{7}$$

Chan, Esedoglu, Park proposed CEP model when they used regularization $\int_\Omega |\Delta u|^2 dxdy$.

$$\min_u \left\{T_2(u) = \int_\Omega |\Delta u|^2 dxdy + \frac{\lambda}{2}\int_\Omega |u-u_0|^2 dxdy\right\} \tag{8}$$

3. Proposed Model

3.1. Target Function

TV filter is able to preserve edges and the 4th PDE filter can overcome the staircase effect in smooth regions. So, Wang proposed an algorithm which combines the TV filter and 4th order PDE filter by a weighting function. This provided a good thinking mode and angle for us, but unfortunately, he did not point out how to get those optimal weighting coefficients,

Here, based on it, we proposed a model to do noise filtering work, which synthetically utilizes TV of ROF model, 4th order PDE of CEP model and GA.

Euler-Lagrange of ROF model is

$$\begin{cases} -\nabla \cdot \left(\dfrac{\nabla u}{|\nabla u|}\right) + \lambda(u-u_0) = 0 & u \in \Omega \\ \dfrac{\partial u}{\partial n} = 0 & u \in \partial\Omega \end{cases} \tag{9}$$

Here, $\partial\Omega$ is boundary of Ω; n is normal vector of $\partial\Omega$; $|\nabla u|_\beta = \sqrt{|\nabla u|^2 + \beta^2}, \beta > 0$ is used to avoid trouble of $\nabla u = 0$. λ_1 is control coefficients. And define

$$L(u)v = -\nabla \cdot \left(\frac{\nabla v}{\sqrt{|\nabla u|^2 + \beta^2}} \right) \tag{10}$$

Euler-Lagrange of CEP model is

$$\begin{cases} \Delta^2 u + \lambda_2 (u - u_0) = 0 & u \in \Omega \\ \dfrac{\partial \Delta u}{\partial n} = 0 & u \in \partial\Omega \\ \dfrac{\partial \Delta^2 u}{\partial n} = 0 & u \in \partial\Omega \end{cases} \tag{11}$$

Same as Wang, we define u^* and v^* as solutions of equation (9) and equation(11), they can be calculated by following equation:

$$\begin{cases} u_t^* = -\nabla \cdot \left(\dfrac{\nabla u}{|\nabla u|} \right)_\beta + \lambda_1 (u - u_0) \\ v_t^* = \Delta^2 u + \lambda_2 (u - u_0) \end{cases} \tag{12}$$

$$\begin{cases} u^{*\kappa+1} = u^{*k} - \left[\lambda_1^k \mathrm{I} + L(u) \right]^{-1} \left[L(u) u^{*k} + \lambda_1^k \left(u^{*k} - u_0 \right) \right] \\ v^{*k+1} = v^{*k} - \left[\lambda_2^k \mathrm{I} + \Delta^2 \right]^{-1} \left[\Delta^2 v^{*k} + \lambda_2^k \left(v^{*k} - u_0 \right) \right] \end{cases} \tag{13}$$

TV filter is able to preserve edges and the fourth-order PDE filter can overcome the staircase effect in smooth regions. We build equation (14) to try to preserve the advantages of both filters.

$$w = \tau u^* + (1 - \tau) v^*, \quad \tau \in [0,1] \tag{14}$$

3.2. Workflow

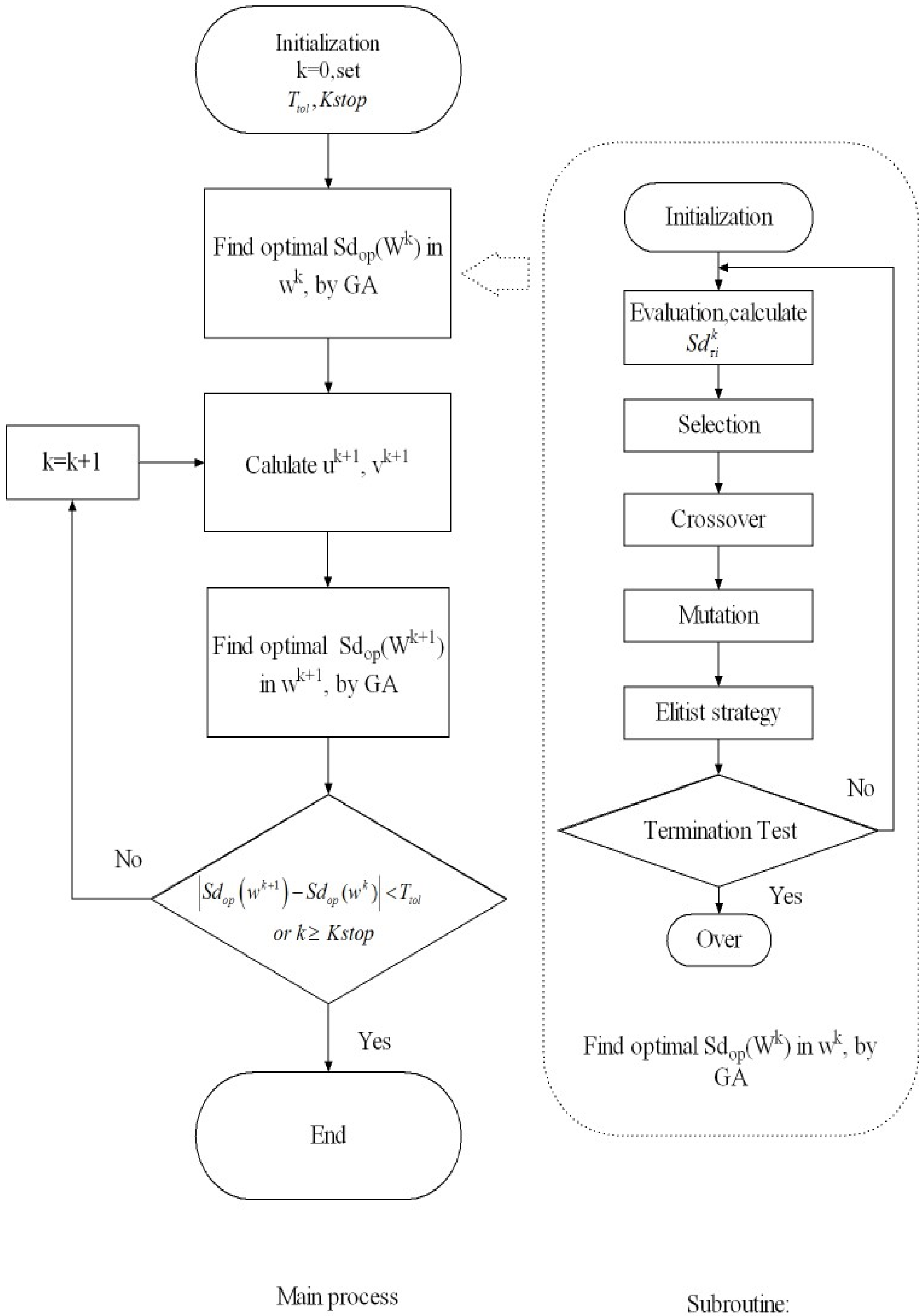


Figure 1. Workflow of our Proposed Algorithm

Our algorithm main process is listed as following:

Parameter:

1. $Sd_{op}\left(w^k\right)$. we get the optimal Standard deviation of image in w^k after we adjust coefficient τ by GA;
2. Ttol: Threshold between $Sd_{op}\left(w^{k+1}\right), Sd_{op}\left(w^k\right)$;
3. *Kstop*: Maximum iterations.

Process:

Step 1. k=0, $Sd_{op}\left(w^0\right)$=0;

Step 2. Calculate u^{*k+1}, v^{*k+1};

Step 3. $w=\tau u^*+(1-\tau)v^*$,adjust coefficient τ by GA, find the optimal PSNR $PSNR_{op}\left(w^k\right)$;

Step 4. If $(Sd_{op}\left(w^{k+1}\right)-Sd_{op}\left(w^k\right)<T_{tol})$ or $(k \geq Kstop)$, go to Step 6; else go to Step 5;

Step 5. $k=k+1$, go to step2;

Step 6. Exit.

Similar to upper following, subroutine of find optimal PSNR in w is listed as following:

Parameter:

1. τ. τ is coefficients we used to adjust $w=\tau u^*+(1-\tau)v^*, \quad \tau \in [0,1]$
2. *Kstop*: Maximum iterations;

Process:

Step 1. (*Initialization*): Generate an initial population containing N_{pop} strings where N_{pop} is the number of strings in each population. These strings contains weight coefficients τ and other parameters in GA we needs;

Step 2. (*Ev*aluation):

[1]. Calculate Standard deviation of image, $Sd\left(w_{\tau i}^k\right)$;

[2]. Update a tentative setoff Pareto optimal solution.

Step 3. (*Selection*): Calculate the fitness value of each string using the random weights in (3).Select a pair of strings from the current population according to the following selection probability.

Step 4. (*Crossover*): For each selected pair, apply a crossover operation to generate two new strings. N_{pop} new strings are generated by the crossover operation.

Step 5. (*Mutation*): For each bit value of the strings generated by the crossover operation, apply a mutation operation with a pre-specified mutation probability.

Step 6. (*Elitist strategy*): Randomly remove N_{elite} strings from the set of N_{pop} strings generated by previous operations, and replace them with N_{elite} strings randomly selected from tentative set of Pareto optimal solutions.

Step 7. (*Termination Test*): If one stopping condition in following is satisfied, go to Step8; if not, return Step2.

- Maximum iterations exceeded;
- The optimal target value is achieved.

Step 8. (*Algorithm termination*): Exit.

4. Experimental Results and Analysis

4.1. Evaluation Index

Peak Signal to Noise Ratio (PSNR) is defined as:

$$PSNR=10\lg\frac{255^2}{\frac{1}{M\times N}\sum_{i=0}^{M}\sum_{j=0}^{N}(A(i,j)\text{-}O(i,j))^2} \tag{15}$$

Here, O is original image with size of M×N pixels，A is filtered image of noised image，(i,j) is coordinates of pixels.

4.2. Experimental Results

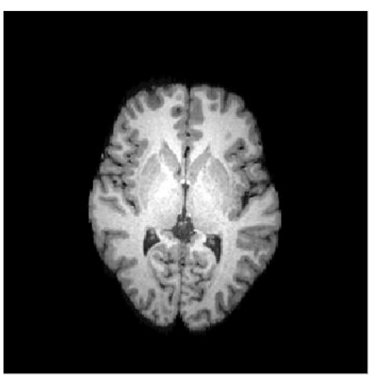
In order to verify the validity of the algorithm, this paper designed two kinds of experimental methods to verify its effectiveness. One is use objective data such as PNSR to objective analyzed its performance; another is make us able to obverse filtering performance directly by naked eyes [19~21].

Experiment 1:

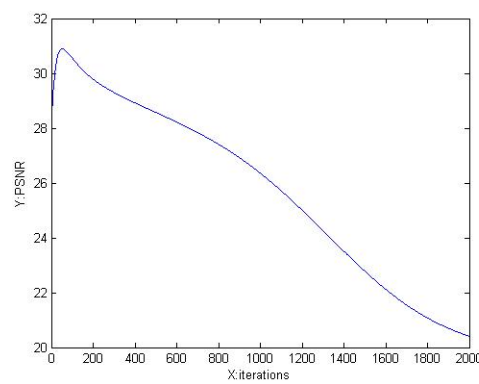
Figure 2(a) is the original MR image we adopted to do experiments. Added Rician noise to it, we did filtering work by classical TV and proposed algorithm in this paper, and show their statics PSNR as Figure. Figure 2(b) and Figure 2(d). Figure 2(c) is Standard deviation of enhanced image which we used to as evaluated function.

From Figure 2(c) and Figure 2(d), we can see that Standard deviation of enhanced image is related to PSNR of proposed algorithm, and they are similar in all iterations, So Standard deviation of enhanced image is a good evaluated function in GA.

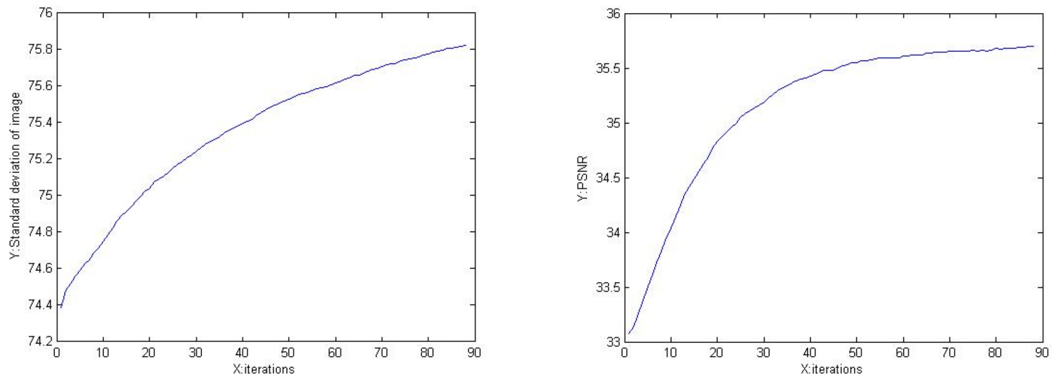
From Figure 2(b) and Figure 2(d), we can see that PSNR of proposed algorithm is better than PSNR of proposed TV, so proposed algorithm has better performance in filtering.



(a).Original image



(b).PSNR of TV



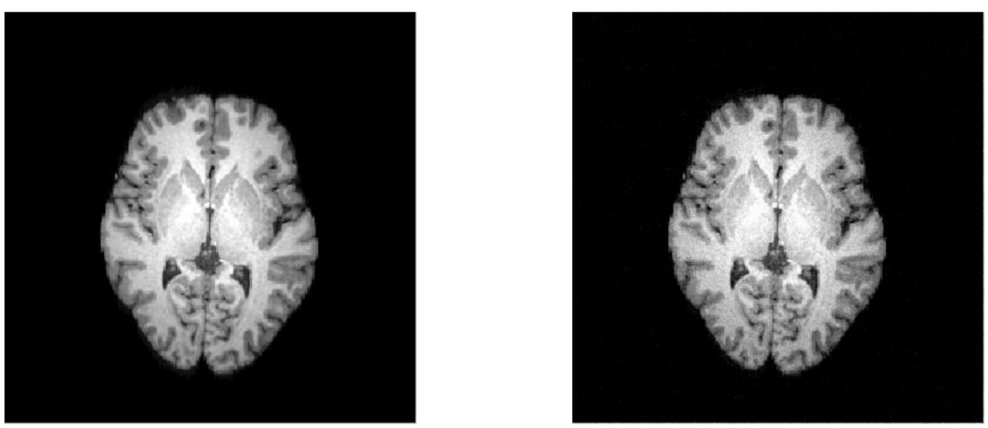
(c). Standard deviation of enhanced image (d). PSNR of proposed algorithm

Figure 2. MSE in Different σ(%) and Different Algorithm

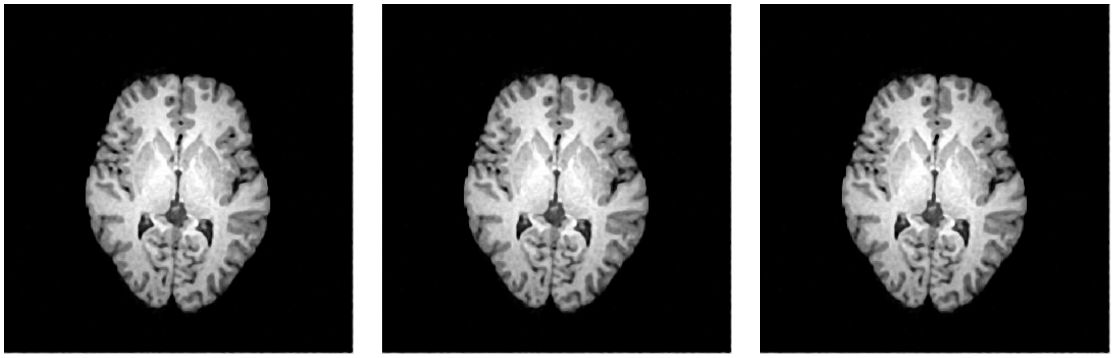
Experiment 2:

At last, we measured our algorithm performance to MRI image. Figure 3(a) is the original MRI. Figure 3(b) is MRI noised by Rician noise. Filtered Figure 3(b) by classical 4th order PDE algorithm, we got Figure 3(c). Filtered Figure 3(b) by classical TV algorithm, we got Figure 3(c). Figure 3(e) is the output of that we did filtering work based on our proposed algorithm.

Through classical 4th order PDE algorithm and TV algorithm and between proposed algorithm's filtering performance comparisons, our proposed algorithm could effectively remove the noise from degraded image of Gaussian noise and protect the image detail better at the same time. The most important is that our proposed algorithm is totally automatically.



(a).original image (b). noised image



(c).output of 4th order PDE (d). output of TV (e). output of proposed

Figure 3. The Algorithm Performance to MRI Image

5. Conclusions

In allusion to removing Rician noise while lessening the loss of details as low as possible, this paper proposed a new image filter arithmetic based on GA, 4th order Partial Differential Equations (PDE) and Total Variation (TV) theory,. It builds evaluation function in GA by Standard deviation of image; and uses GA to find optimal coefficients of value 4th order Partial Differential Equations (PDE) and Total Variation (TV) in different scale and different orientation. Computer simulations and their results are given to verify the efficiency of this algorithm. At last, to MRI image we pay particular attention, experiments data show that our algorithm has excellent performance.

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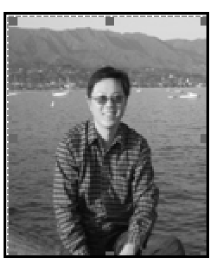
Authors

Zhiyong Fan received MSc from Nanjing University of Information Science and Technology (Nuist) in 2007, China. He is a lecturer in the School of Information and Control Engineering at Nuist, China. Now, he is a PhD student in Nanjing University of Science and Technology, China. His current research interests include medical imaging, image processing and pattern recognition.

Quansen Sun He received his Ph.D. degree in pattern recognition and intelligence system from Nanjing University of Science and Technology (NUST), Nanjing, China, in 2006. He is a professor in the school of computer science and engineering at NUST. He visited the Department of Computer Science and Engineering, The Chinese University of Hong Kong in 2004 and 2005, respectively. His current interests include pattern recognition, image processing, computer vision and data fusion.

Zexuan Ji He received his B.E.in Computer Science and Technology and Ph.D. degree in pattern recognition and intelligence system from Nanjing University of Science and Technology (NUST), Nanjing, China, in 2007 and 2012, respectively. He is a lecturer in the school of computer science and engineering at NUST. His research interests include medical imaging, image processing and pattern recognition.



Feng Ruan Lecturer at Nanjing University of Information Scicence&Technology. He received the B.S. and M.S. degree in the Nanjing University of Information Scicence&Technology.Now he is PHD student in Computer Science Department of Nanjing University of Information Scicence&Technology. His research interests mainly include complex system and algorithm design.

Liling Zhao She is aLecturer at the Information and Control Engineering, and study at Nanjing University of Information Science and Technology. Her research interest covers image processing and application, computer vision.

