On The Equivalence of the PA-SLNR and PA-SINR MU-MIMO Precoding Design Criterien

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Abstract

In this work, we develop on average relationship between the Per-Antenna Signal to Interference plus Noise Ratio (PA-SINR) and the Per-Antenna Signal to Leakage plus Noise Ratio (PA-SLNR) performance metrics. Simulation results confirm the equivalence between the two metrics which opens up the door to use PA-SLNR instead of PA-SINR to construct simple Multi-user Multiple Input Multiple Output (MU-MIMO) precoding algorithm.

Keywords: *Multi-user Multiple Input Multiple Output (MU-MIMO), Per-Antenna Signal to Interference plus Noise Ratio (PA-SINR , Per-Antenna Signal to Leakage plus Noise Ratio (PA-SLNR), Precoding*

1. Introduction

Multi-user Multiple Input Multiple Output Broadcast Channel (MU-MIMO-BC) precoding design has received significant attention recently [1-3]. This consideration is due high predictable performance/capacity gain of both MIMO and multi-user spatial dimensions. In the recent literature; due to the complexity of nonlinear precoding methods [4-7], several linear precoding techniques were proposed. In these methods both joint and independent optimization techniques were used to mitigate the multiuser interference (MUI) [8-12]. Given the availability of channel state information at the transmitter and receiver, various conditions and objective functions were used to study this problem. In [8, 9, 13-15] various closed form solutions based on minimum mean square error (MMSE) design criteria were proposed. The methods in [8, 15] impose dimensionality limitations that the number of base station antennas should be greater than or equal to the total numbers of receive antennas for all users. On the other hand the methods in [9, 14] relax the dimensionality constraint but with some performance loss at high SNR values when the total number of receive antennas at all users exceeds the number of base station antennas. In [10] an iterative sum mean square error (SMSE) criteria method which is also called joint transceiver design was developed to compute both the precoding and decoding matrices. In spite of its high performance, computational complexity and restrictions on the number of antennas were the main demerits of this method. The work by [11] proposing a beam-forming precoding design utilizing Per-user Signal to Leakage and Noise Ratio (PU-SLNR) performance cost function. In this method, the precoding vectors for all users are obtained by solving a series of optimization problems using generalized eigenvalue decomposition (GEVD). In [12] PU-SLNR precoding matrix design based on GEVD for MU-MIMO spatial Multiplexing is proposed. This work shows that the precoding matrix computation using GEVD is simple but it's sensitive to the singularity of the leakage plus noise power matrix, thus there is some performance loss at high SNR values. To solve the singularity problem in matrix computation, Fukunaga-Koontz transforms (FKT) and GSVD based computation were independently utilized in [16] and [17] respectively. The precoding design based on PU-SLNR cost function introduced in [12, 16, 17] were totally neglect to take into account the interference between streams multiplexed to each individual user (intra-user antenna interference). The author in [18] proposes Per-Antenna Signal to Leakage plus Noise Ratio (PA-SLNR) performance criteria with FKT precoding solution. This method takes into account the intera-user interference cancelation, thus it has better BER performance.unlike the Per-Antennas Signal to Interference plus Noise Ratio (PA-SINR) which is well known design performance metric and related to the QoS service factor, the PA-SLNR metric is mysterious. In this work we are going to establish on average relation between the PA-SINR and the PA-SLNR and demonstrate by Monte-Carlo integral simulation that the improvement of the input PA-SLNR is equivalent to the improvement of the output Per-User SINR.

The rest of this work is organized as follows: section 2 review the efficient PA-SLNR precoding method proposed in [18]. On average relation between the PA-SLNR and the PA-SLNR is developed in section 3. Mote-Carlo integral simulation to prove the relation is given in section 4 and section 5 conclude the paper.

2. Precoding Based on Per-antenna Signal to Leakage plus Noise Ratio (PA-SLNR) Maximization

The objective function originally proposed in [11] maximizes the SLNR for each user, thus the precoder designed is to cancel inter-user interference only. The work in [18] however, proposes a new cost function that maximizes the PA-SLNR which would help to minimize the intra-user antenna interference as well as the inter-user interference. Thus, the proposed cost function leads to a better precoder that maximizes the overall SLNR per user more efficiently. This is justified because the per-antenna signal to leakage plus noise ratio as explained in Figure 1, takes into account the inter-user antenna interference cancellation. For each j^{th} receive antenna of the k^{th} user, the PA-SLNR, γ_k^j is defined as the ratio between the j^{th} receive antenna desired signal power to the interference introduced by the j^{th} antenna intended signal power leaked to all other antennas plus the noise power at that receiving antenna front end. So for the j^{th} receive antenna of the k^{th} user, the signal to leakage plus noise ratio is defined by:

$$\gamma_k^j = \frac{\left\| \mathbf{h}_k^j \mathbf{f}_k^j \right\|_F^2}{\sum_{\substack{i=1 \\ i \neq k}}^{B} \left\| \mathbf{H}_i \mathbf{f}_k^j \right\|_F^2 + \sum_{\substack{i=1 \\ i \neq j}}^{M_k} \left\| \mathbf{h}_k^i \mathbf{f}_k^j \right\|_F^2 + \sigma_{n\,k}^{2\,j}} \tag{1}$$

where $\mathbf{h}_k^j \in \mathbf{C}^{1 \times N_T}$ is received signal row of the j^{th} receive antenna at the k^{th} user. If we define an auxiliary matrix $\widetilde{\mathbf{H}}_k^j$ as the matrix contains the k^{th} user received antenna rows except the j^{th} row as follows:

$$\tilde{\mathbf{H}}_k^j = \begin{bmatrix} h_k^{(1,1)} & h_k^{(1,2)} & \cdots & h_k^{(1,N_T)} \\ \vdots & \vdots & \vdots & \vdots \\ h_k^{(j-1,1)} & h_k^{(j-1,2)} & \cdots & h_k^{(j-1,N_T)} \\ h_k^{(j+1,1)} & h_k^{(j+1,2)} & \cdots & h_k^{(j+1,N_T)} \\ \vdots & \vdots & \vdots & \vdots \\ h_k^{(M_k,1)} & h_k^{(M_k,2)} & \cdots & h_k^{(M_k,N_T)} \end{bmatrix} \in \mathbf{C}^{((M_k-1)\times N_T)} \tag{2}$$

and the combined channel matrixes for all other systems receive antennas except the j^{th} receive antenna of the k^{th} user as:

$$\tilde{\mathbf{H}}_k^j = [\tilde{\mathbf{H}}_k^{j\,T}\,\mathbf{H}_1^T \cdots \mathbf{H}_{k-1}^T \mathbf{H}_{k+1}^T \cdots \mathbf{H}_B^T]^T \tag{3}$$

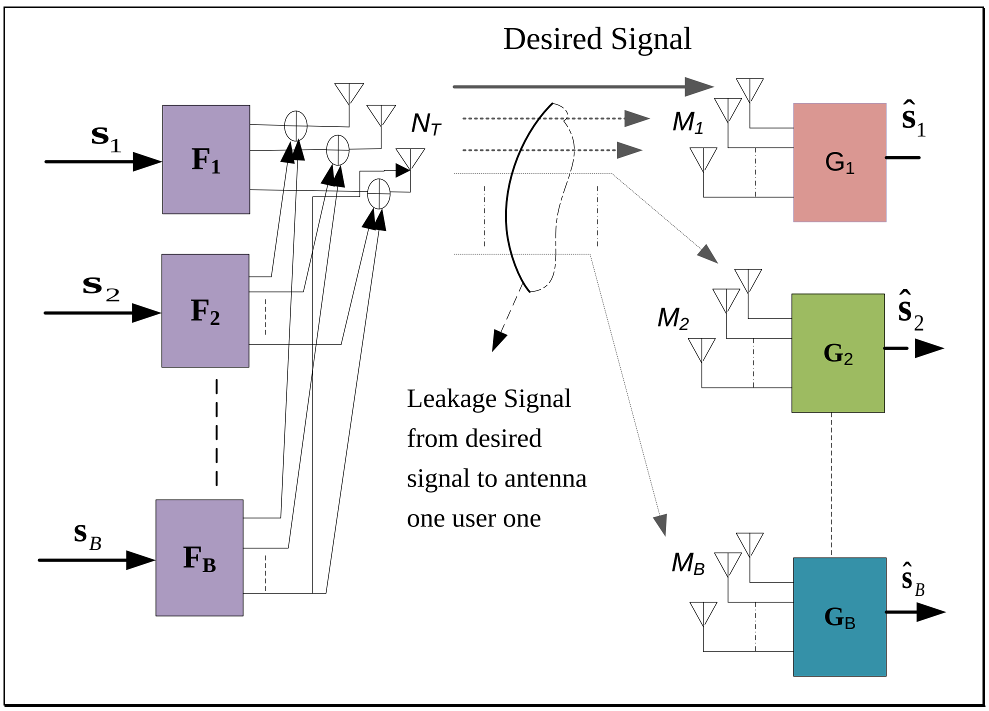

Figure 1. System Model Depicts all Variables

Accordingly, from the equation (2) and equation (3) the expression in (1) can be rewritten as:

$$\gamma_k^j = \frac{\left\| \mathbf{h}_k^j \mathbf{f}_k^j \right\|_F^2}{\left\| \tilde{\mathbf{H}}_k^j \mathbf{f}_k^j \right\|_F^2 + \sigma_{n\,k}^{2\,j}} \tag{4}$$

2.1. Problem Statement:

For any j^{th} receive antenna of the k^{th} user, select the precoding vector $\mathbf{f}_k^j$, such that the Per-Antenna Signal to Leakage Plus Noise Ratio γ_k^j is maximized:

$$\mathbf{f}_k^j = \arg\max_{\mathbf{f}_k^j \in \mathbf{C}^{N_T \times 1}} \frac{\mathbf{f}_k^{j^*} (\mathbf{h}_k^{j^*} \mathbf{h}_k^j) \mathbf{f}_k^j}{\mathbf{f}_k^{j^*} (\tilde{\mathbf{H}}_k^{j^*} \tilde{\mathbf{H}}_k^j + \sigma_{n\,k}^{2\,j} \mathbf{I}_{N_T}) \mathbf{f}_k^j}$$

subject to

$$\text{tr}(\mathbf{F}_\text{k} \mathbf{F}_\text{k}^*) = 1 \quad (5)$$

$$\mathbf{F}_\text{K} = [\mathbf{f}_k^1, \cdots, \mathbf{f}_k^{M_k}]$$

where: $k = 1, \cdots, B$, $j = 1, \cdots, M_k$

The optimization problem in the equation (5) deals with the j^{th} antenna desired signal power in the numerator and a combination of total leaked power from the desired signal to the j^{th} antenna to all other antennas plus noise power at the j^{th} antenna front-end at the denominator. To calculate the precoding matrix for each user we need to calculate the precoding vector for each receive antenna independently. This requires solving the linear fractional optimization problem in the equation (5) $M_k \times B$ times using either GEVD [11] or GSVD [19] which will lead to high computational load at the base stations. The work in [18] propose the FKT transform based solution method to solve such series of linear fractional optimization problems which described as follows:

Algorithm 1: PA-SLNR MU-MIMO precoding based on FKT for multiple B independent MU-MIMO uses.

Input: all B users combined channel matrix and the input noise variance

$\mathbf{H}_\text{com} = [\mathbf{H}_1^T \mathbf{H}_2^T \cdots \mathbf{H}_B^T]^T$, σ_k^2.

Output: multiple B users precoding matrices $\mathbf{F}_k$ such that, $k = 1, \cdots, B$

1 Compute the sum $\mathbf{A} = \mathbf{H}_\text{com}^* \mathbf{H}_\text{com} + \sigma^{2\,j}_{\,k} \mathbf{I}_{N_T}$

2 Compute FKT factor $\mathbf{P} = \mathbf{U}\mathbf{D}^{-\frac{1}{2}}$ from $\text{SVD}(\mathbf{A})$

3 For k = 1 to B

4 For j =1 to M_k

- Transform the j^{th} receive antenna covariance matrix $\mathbf{A}_1$ using the FKT factor $\mathbf{P}$ to the $\tilde{\mathbf{A}}_1$ and select the first eigenvector $\mathbf{v}_k^j$ of $\tilde{\mathbf{A}}_1$

- The precoding vector corresponds to the j^{th} receive antenna at the k^{th} user is: $\mathbf{f}_k^j = \mathbf{P}\ \mathbf{v}_k^j$
- End

5 The k^{th} user precoding matrix is $\mathbf{F}_k = [\mathbf{f}_k^1 \cdots \mathbf{f}_k^{M_k}]$

6 End

The algorithm takes the combined MU-MIMO channel matrix as well as the value of the noise variance as an input and outputs B users precoding matrices. It computes the FKT Factor in step 1 and 2 and iterates B times (step 3-to-6) to calculate the precoding matrices for the B number of users. For each user, there are M_k sub-iteration operations (step 4-to-5) to calculate each individual user precoding matrix in vector by vector basis.

3. A Relation between Average PA-SINR Maximization and Average PA-SLNR Maximization

Let as reconsider the MU-MIMO system configuration which has B users with each k^{th} user $k = 1,2,\cdots,B$ is equipped with M_k receive antenna and there are N_T transmit antennas at the base station/access-point. As depicted in Figure 2, we can formulate the objective function that maximizes the input Per-antenna Signal to Interference plus Noise Ratio (PA-SINR) as follows: For any j^{th} receive antenna of the k^{th} user we can define the PA-SINR, λ_k^j metric as the ratio between the desired signal to the j^{th} received antenna to the total interference caused by all transmitted signals from all other antennas to the j^{th} antenna plus the noise-power at the j^{th} antenna front-end. Thus, λ_k^j is defined as:

$$\lambda_k^j = \frac{\mathbf{f}_k^{j^*}(\mathbf{h}_k^{j^*}\mathbf{h}_k^j)\mathbf{f}_k^j}{(\mathbf{F}_k^{j^*}\mathbf{h}_k^{j^*}\mathbf{h}_k^j\mathbf{F}_k^j + \sigma^2{}_k^j)} \tag{6}$$

where $\mathbf{h}_k^j \in \mathbf{C}^{1\times N_T}$ is the j^{th} received antenna row of the k^{th} user. Consequently, $\mathbf{f}_k^j$ denotes the j^{th} received antenna corresponding precoding vector. If we define the matrix $\tilde{\mathbf{F}}_k^j = \left[\mathbf{f}_k^1 \cdots \mathbf{f}_k^{j-1}\mathbf{f}_k^{j+1} \cdots \mathbf{f}_k^{M_k}\right]$ as the combined precoding matrix which contains all the precoding vectors of the k^{th} user without the j^{th} receive antenna corresponding precoding vector. Furthermore, we can define $\mathbf{F}_k^j = [\tilde{\mathbf{F}}_k^{j^T}\mathbf{F}_1^T \cdots \mathbf{F}_{i-1}^T\mathbf{F}_{i+1}^T \cdots \mathbf{F}_B^T]^T$ as the combined matrix contains other antennas precoding vectors except the j^{th} receive antenna corresponding vector.

Problem Formulation: For any j^{th} receive antenna of the k^{th} user, select the precoding vector $\mathbf{f}_k^j$, where $k = 1,\cdots,B$, $j = 1,\cdots,M_{R_j}$ such that the per-antenna signal to interference plus noise ratio PA-SINR, λ_k^j is maximized.

$$\mathbf{f}_i^j = \underset{\mathbf{f}_i^j \in \mathbf{C}^{N_T \times 1}}{\arg\max} \frac{\mathbf{f}_i^{j^*}(\mathbf{h}_i^{j^*}\mathbf{h}_i^j)\mathbf{f}_i^j}{(\mathbf{F}_i^{j^*}\mathbf{h}_i^{j^*}\mathbf{h}_i^j\mathbf{F}_i^j + \sigma^2{}_i^j)} \tag{7}$$

The optimization in equation (7) is coupled in such way that the computation of the j^{th} receive antenna at the k^{th} user precoding vector depends on other antennas precoding vectors, which are also unknown. One way to solve this optimization is through a long iterative method which is going to be very complex and there is no guarantee of convergence [20]. Given that the PA-SINR quantity has direct relation to the corresponding output SINR and consequently to the BER performance, we will define an on average relation between PA-SINR and PA-SLNR as follows: By defining a distance error value ρ_k^j as the logarithmic ratio between the γ_k^j given in equation (5) and λ_k^j as given in equation (6) as follows:

$$\rho_i^j = \log \frac{\gamma_k^j}{\lambda_k^j} \tag{8}$$

By taking the expectation of both side of the equation (8), we rewrite it as:

$$E(\rho_i^j) = E\left[Log \frac{\gamma_k^j}{\lambda_k^j} \right] \tag{9}$$

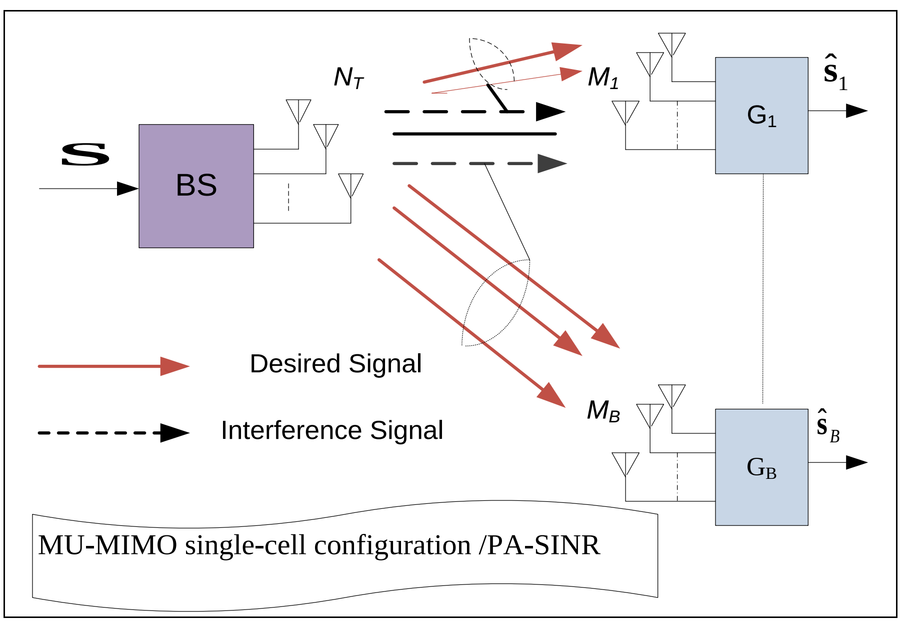

Figure 2. System Model Depicting all Variables for the PA-SINR

Assuming $\mathbf{h}_i^j \mathbf{F}_i^j = \widetilde{\mathbf{H}}_k^j \mathbf{f}_k^j$ (they have the same statistical distribution), we find that $E(\rho_i^j) = 0$ which means that on average, the distance between the PA-SLNR and PA-SINR is zero which means that on average they are equal [21]. Given this asymptotic on average relation and that the output SINR is lower bounded by the input SINR which in turn depends on the individual antennas SINR, we conclude that the output SINR is a good performance metric for system level performance evaluation. In the next section we simulate output SINR versus the input SNR range using the PA-SLNR Precoding to test the hypothesis that the proposed PA-SLNR is equivalent to the input PA-SINR. We will also confirm the hypothesis that the proposed optimization metric has better MUI and inter-user antennas interference cancellation capability over the metrics proposed in [11, 17]

4. System Integral Simulation

To test the proposition that the proposed PA-SLNR is equivalent to the input PA-SINR, Monte-Carlo integral simulation for the MU-MIMO Broadcast channel will be carried out to evaluate the output SINR verses the input SINR for a wide range of input SNR. For each k^{th} user, the received signal at the output can write as follows:

$$\hat{\mathbf{y}}_k = \mathbf{G}_k \mathbf{H}_k \mathbf{F}_k \mathbf{s}_k + \mathbf{G}_k \mathbf{H}_K \sum_{\substack{i=1 \\ i \neq k}}^{B} \mathbf{F}_i \mathbf{s}_i + \mathbf{G}_k \mathbf{n}_k \tag{10}$$

Where $\mathbf{G}_k$ denotes the k^{th} user channel decoder.

In this equation, the first term represents the desired signal to the k^{th} user; the second term is the MUI; and the third term is the received noise at the output. By making use of the equation (10), we can calculate the ratio of the desired signal to the total MUI plus noise for each user in the system.

Figure 3, shows the user average received output SINR and the input SINR of the proposed PA-SLNR-FKT precoding. MU-MIMO system configuration of $B = 3, M_k = 3, N_T = 10$. Input SNR range of 6-22dB is considered in the simulation. It is shown that the output SINR is always better than the input SINR. For example, at 16dB input SNR, there is almost 2 dB gain difference between the input SINR and the received output SINR. This result stress the conclusion that the reduction in PA-SLNR is equivalent to the Reduction in the PA-SINR given that the output SINR is lower bounded by the input SINR. To test the hypothesis that the proposed optimization metric has better MUI and inter-user antennas interference cancellation capability, Monte-Carlo integral simulation for the MU-MIMO Broadcast channel will be carried out to evaluate the output SINR over a wide range of input SNR. The evaluation is carried on for both the proposed method as well as the Per-user SLNR (PU-SLNR) based on the GEVD solution and PU-SLNR based on the GSVD methods. For each system simulation, link configuration equivalent to the system model of Figure 1 is considered.

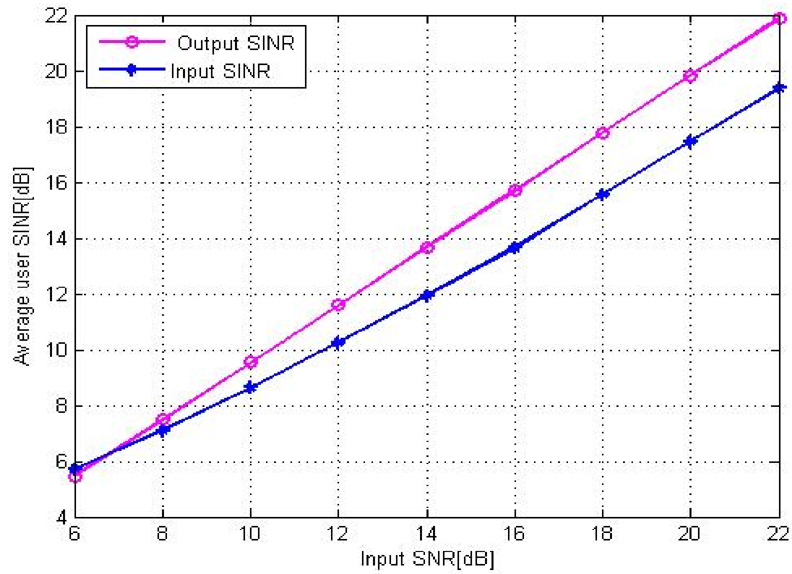

Figure 3. Shows the Average Input/Output SINR for the System Configuration of $B=3, M_k=3, N_T=10$

Figure 4 shows the average user received output SINR of the proposed PA-SLNR-FKT precoding and the related works based on SLNR-GSVD and SLNR-GEVD precoding methods. MU-MIMO system configuration of ($B=3,\ M_k=4,\ N_T=12$) with an input SNR range of 2-22dB is considered in the simulation. The proposed method outperforms both the SLNR-GSVD and SLNR-GSVD methods. For example at 10dB input SNR there is almost 2dB gain difference of receiving output SINR.

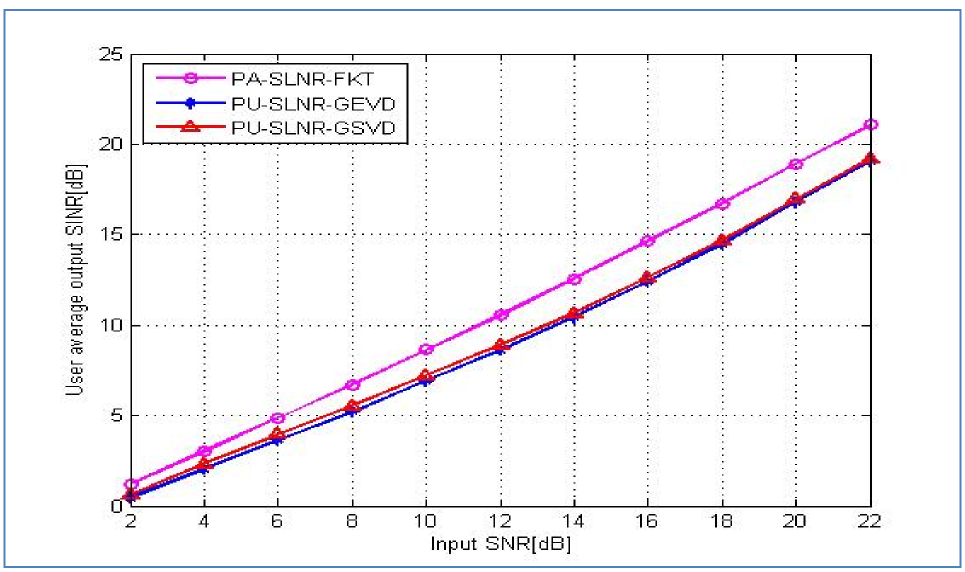

Figure 4. Shows the User Average Output SINR for the System Configuration of $B=3, M_k=4, N_T=12$

5. Conclusion

In this work, we develop on average relation between the PA-SINR and the PA-SLNR MU-MIMO precoding design performance metrics. We test the proportion by making use of Monte-Carlo integral simulation for the MU-MIMO Broadcast channel to evaluate the output SINR verses the input SINR for a wide range of input SNR. We also show that the proposed optimization metric has better MUI and inter-user antennas interference cancellation capability when compared with the conventional PU-SLNR-GEVD and PU-SLNR-GSVD precoding methods.

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