

Analysis of Semi-Blind Channel Estimation based on Second Order Statistics in MIMO

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Abstract

Although MIMO (multiple input multiple output) represent a solution to increase the capacity and the reliability of wireless channels, it is particularly challenging from a channel estimation perspective. There are many algorithms that can be applied in MIMO channel estimator. Motivated by analysis of the autocorrelation matrix at the receiver, which is considered as a important step in computation of channel parameter based on whitening-rotation (WR) channel estimation algorithm, this paper modify transmit signal frame by embedded its second order statistic data. This idea is described as solution resolved ill-condition of WR-based channel estimation approach. We show that considerable improvement in the performance of channel estimator can be obtained by using this modified algorithm, especially for larger number of transmit/receive antennas.

Keywords-*MIMO; LS estimation; whitening-rotation channel estimation*

1. Introduction

MIMO has been the focus of a wide array of research in communications because of its high transmission capability and its robustness to multipath delay [1]. In a MIMO system, the number of channel parameters is much larger than the conventional SISO one, making the channel estimation scheme particularly critical. Therefore, this increase in the number of estimated channel parameters lead to a smaller estimation accuracy which is counteracted by using a longer training pilot. Disadvantage issue of training based channel estimation less attractive. In this paper, we evaluate the blind channel estimation in MIMO. This method, by exploiting the observations associated to the unknown symbols perform the channel estimation, potentially leads to an enhancement in the estimation accuracy in comparison with the typical training based approach [2-4].

In addition to these, MIMO system itself provide the more advantage of increased data communication rates for the same SNR by using orthogonal space time block code for transmit data. In particular, the system's ability to approach the MIMO capacity heavily relies on the channel status information (CSI). The fact that radio channels are frequency selective and times varying for wideband mobile communication systems need exact CSI for detection of the original signals. In this paper, we present the WR-based channel estimation approach. In fact, these various MIMO channel estimation can be classified into these categories: training-based methods, blind and semi-blind (SB) ones. Whereas blind methods not only impose high com-

plexity and slow convergence but also suffer from unavoidable decision ambiguities. In general, the family of blind methods for joint channel estimation and data detection does not require training symbols and hence does not reduce the achievable system throughput.

The popular algorithm in channel estimation is based on Maximum-LikeLihood (ML) algorithm. However, the complexity of ML scheme of the MIMO with larger number of transmit and receive antennas too high as the big drawbacks of this technique. The blind scheme used in this paper so-called whitening-rotation algorithm. This blind channel estimation has received considerable attraction in wireless communications. In these systems, digital signals are transmitted via multipath that often cause signal distortion extremely. To obtain the exact input data, channel impulse responses can be estimated using high order statistic (HOS) property of the received output data. Without training sequence, the blind channel estimation approach is usually required. One of the most effective methods is solution based on the second order statistics of the received signal. Applying the eigenvalue decomposition algorithm and orthogonal property of subspaces, channel value can be calculated in approximation schemes. It is observed that the autocorrelation matrix of the received signal is important parameter evaluating the performance of the subspace based channel estimation [5-8].

In previous works, the WR-based channel estimation has been proposed for MIMO system with ill-condition at high SNR [9, 10]. Generally, this approach consists of two steps: 1) estimation of a unitary rotation matrix using training sequence; and 2) estimation of a whitening matrix utilizing autocorrelation matrix of the receive signal. It is worth nothing that the number of receive antenna, using in WR-based channel estimation algorithm, is greater than or equal to the number of transmit antennas. However, the WR-based scheme is only efficient in the specific case of signal-to-noise ratios (SNR) because of the signal perturbation error caused by the blind part of the WR method.

More specifically, the noise part in receive signal is considered as main reason leading to ill-condition of WR-based channel estimation in high SNR. In this paper, we use the new transmit frame with embedded SOS data in order to reduce noise term at the receiver.

The rest of the paper is organized as follows. In Section 2, we introduce the WR-based channel estimation in flat fading MIMO channel. In next section, we analysis of SOS-based transmit frame aiming to reduce the affect by the perturbation signal. The numerical results are presented in Section 4 and conclusions are drawn in Section 5.

Notation: The uppercase letters denote matrices, $(\cdot)^H$ stands for complex conjugate transpose; $\|\cdot\|^2$ denotes Frobenius norm.

2. MIMO Channel Estimation based on Whitening-Rotation Matrices

In this work, we analysis of MIMO system with N_T transmit and N_R receive antennas. The MIMO flat fading is characterized by a $N_R \times N_T$ matrix. Suppose that the transmit signal vector $\mathbf{x}(n) = [\mathbf{x}_1(n), \dots, \mathbf{x}_{N_T}(n)]^T$ whose elements are independent identically distributed (i.i.d) Gaussian random variables with zero mean and unit variance $\sigma_x^2 = 1$, the received signal vector $\mathbf{y}(n) = [\mathbf{y}_1(n), \dots, \mathbf{y}_{N_R}(n)]^T$ can be described as

$$\mathbf{y}(n) = \mathbf{H}\mathbf{x}(n) + \mathbf{v}(n) \quad (1)$$

In this paper, the whitening-rotation based channel estimation algorithm using SVD of \mathbf{H} matrix

$$SVD(\mathbf{H}) = \mathbf{P}\mathbf{S}\mathbf{Q}^H \quad (2)$$

Both \mathbf{P} and \mathbf{Q} must be satisfied the following properties: $\mathbf{P}\mathbf{P}^H = \mathbf{P}^H\mathbf{P} = \mathbf{I}$ and the same one for the matrix \mathbf{Q} . From these decompositions, we get the whitening matrix \mathbf{W} is given by $\mathbf{W} = \mathbf{P}\mathbf{S}$ which can be estimated blindly and especially only based on the received data. The next step can be done by using the training sequence for estimation of the matrix \mathbf{Q} with assumption of \mathbf{W} known perfectly at the receiver. The optimal issue now is presented follow Maximum-Likelihood (ML) estimate scheme for matrix \mathbf{Q} . The ML estimate of \mathbf{Q} is given as a solution of the cost function

$$\begin{aligned} \hat{\mathbf{Q}} &= \arg \min \|\mathbf{Y}_p^H - \mathbf{X}_p^H \mathbf{Q}\mathbf{S}\|^2 \\ &\text{subject to } \mathbf{Q}\mathbf{Q}^H = \mathbf{I} \end{aligned} \quad (3)$$

As a result, the constrained estimate $\hat{\mathbf{Q}}$ will be formulated as

$$\hat{\mathbf{Q}} = \mathbf{U}_p \mathbf{R}_p^H \quad (4)$$

It is worth nothing that \mathbf{R}_p^H can be derived from

$$\mathbf{U}_p \mathbf{S}_p \mathbf{R}_p^H = SVD(\mathbf{X}_p \mathbf{Y}_p^H \mathbf{S}) \quad (5)$$

Combining the calculation of both of matrix \mathbf{W} and $\hat{\mathbf{Q}}$, we obtain the estimated channel $\hat{\mathbf{H}}$

$$\hat{\mathbf{H}} = \mathbf{W}\hat{\mathbf{Q}} \quad (6)$$

with the assumption of the matrix \mathbf{W} known perfectly at the receiver.

3. The Transmit Frame Embedded Second Order Statistic Data

It is known that, the WR-based channel estimation scheme is especially related to the property of the receive signal. The autocorrelation matrix of the receive signal can be written as

$$\mathbf{R}_Y = \frac{1}{N} \sum_{n=1}^N \mathbf{y}(n)\mathbf{y}^H(n) - \sigma_v^2 \mathbf{I} \quad (7)$$

Note that N is the number of slots in the transmit signal frame. Then, we expand above expression in new function

$$\begin{aligned} \mathbf{R}_Y &= \frac{1}{N} \sum_{n=1}^N [\mathbf{H}\mathbf{x}(n) + \mathbf{v}(n)][\mathbf{H}\mathbf{x}(n) + \mathbf{v}(n)]^H - \sigma_v^2 \mathbf{I} \\ &= \mathbf{H} \left[\frac{1}{N} \sum_{n=1}^N \mathbf{x}(n)\mathbf{x}^H(n) \right] \mathbf{H}^H + \mathbf{H} \left[\frac{1}{N} \sum_{n=1}^N \mathbf{x}(n)\mathbf{v}^H(n) \right] \\ &\quad + \left[\frac{1}{N} \sum_{n=1}^N \mathbf{x}^H(n)\mathbf{v}(n) \right] \mathbf{H}^H + \frac{1}{N} \sum_{n=1}^N \mathbf{v}(n)\mathbf{v}^H(n) - \sigma_v^2 \mathbf{I} \end{aligned} \quad (8)$$

Next, we have the minor change in the formulation of \mathbf{R}_y to following one

$$\begin{aligned} \mathbf{R}_y = & \mathbf{H} \left[\frac{1}{N} \sum_{n=1}^N \mathbf{x}(n) \mathbf{x}^H(n) \right] \mathbf{H}^H + \mathbf{H} \left[\frac{1}{N} \sum_{n=1}^N \mathbf{x}(n) \mathbf{v}^H(n) \right] \\ & + \left[\frac{1}{N} \sum_{n=1}^N \mathbf{x}(n) \mathbf{v}^H(n) \right]^H \mathbf{H}^H + \frac{1}{N} \sum_{n=1}^N \mathbf{v}(n) \mathbf{v}^H(n) - \sigma_v^2 \mathbf{I} \end{aligned} \quad (9)$$

Similarly, we also determine the autocorrelation one of transmit signal and then denotes this matrix as follow

$$\Delta \mathbf{R}_x = \frac{1}{N} \sum_{n=1}^N \mathbf{x}(n) \mathbf{x}^H(n) - \sigma_x^2 \mathbf{I} \quad (10)$$

We also rewrite (9) in noise-free case [11-14]

$$\hat{\mathbf{R}}_y = \mathbf{H}(\mathbf{I} + \Delta \mathbf{R}_x) \mathbf{H}^H \quad (11)$$

Whereas, the full receive data as $\hat{\mathbf{R}}_y = \mathbf{H}(\mathbf{I} + \Delta \mathbf{R}_x) \mathbf{H}^H + \Delta \mathbf{R}_v$

In order to simplicity, the paper redefined as the other version of this autocorrelation matrix as

$$\Delta \mathbf{R}_{x1} = \sum_{n=1}^N \mathbf{x}(n) \mathbf{x}^H(n) - N \sigma_x^2 \mathbf{I} \quad (12)$$

Then analysing the autocorrelation transmit signal, we have

$$\Delta \mathbf{R}_{x1} = \lambda \mathbf{X}_{x1} \mathbf{X}_{x1}^H \quad (13)$$

Finally, the total transmit frame consists of the pure data (\mathbf{X}), the training sequence \mathbf{X}_p , and the added information (\mathbf{X}_{x1}). In simulation, the paper tries some controlled coefficients of λ .

4. Simulation Results

In simulations, we assume a MIMO system with 4 transmits antennas and 6 receive antennas, in which the QPSK modulation is applied. In addition, we consider a Rayleigh channel whose elements are i.i.d. complex Gaussian variables with zero mean and unit variance. In order to enhance the simulation results, the paper run the program with 20000 frames which are so-called the Monte Carlo iterations.

For these simulations, we use $\mathbf{X}_p = 100$ symbols for training sequence compared to the total pure data length of $\mathbf{X} = 1000$ symbols and added 5 symbols follow SOS of transmit signal. We also have the remark that assuming the MIMO fading channel is static for the packet duration (or as the term of block fading channel as most of literature mentioned). In real environment, the MIMO channel can be time-variant during the period of a packet.

Because of difficulty in find out *lambda* coefficient, in this paper, we try some simulation run in order to get accepted MSE value in real MIMO applications as Table 1.

Table 1. Some simulation runs with different number of lambda

No.	λ	MSE (dB) with SNR=25dB
1	2	-35
2	4	-33
3	6	-35
4	8	-36
5	10	-33
6	12	-35
7	14	-36
8	16	-34
9	18	-35
10	20	-32

The first experiment (Figure 1) shows the relationship between channel estimation MSE versus Eb/No and comparison of the estimations with LS-based approach and transmit scheme embedded SOS data. It is clearly that the MSE of both methods decrease as SNR increases.

Figure 2, in the second experiment, depicts the MSE performance with many choices of model of MIMO systems. It can be seen that the stable quality in WR-based channel estimation.

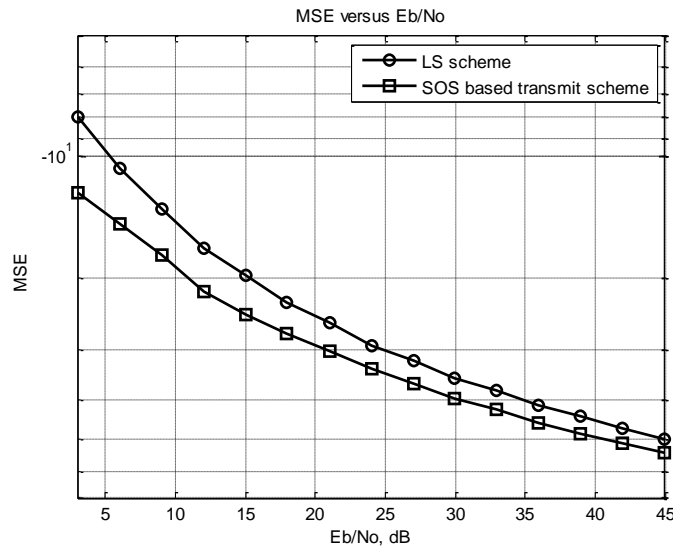


Figure 1. The MSE performance of the modified transmit frame based on WR algorithm

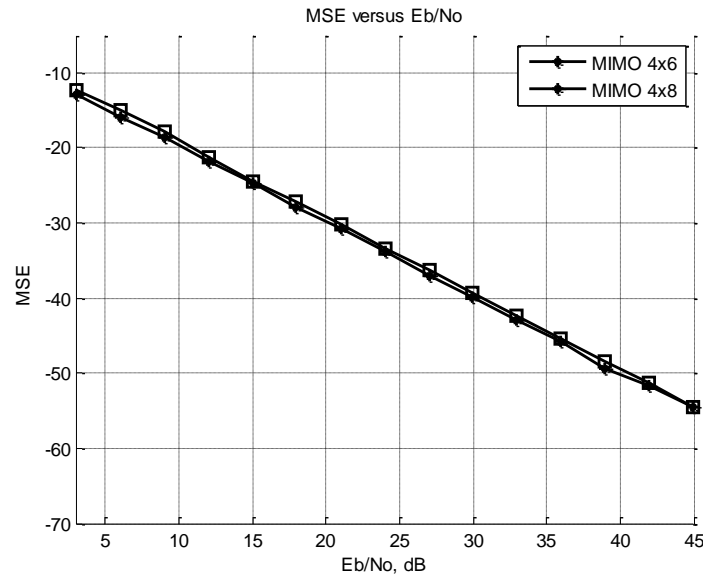


Figure 2. The MSE performance versus Eb/No with different MIMO models

5. Conclusion

We have presented the algorithm using transmit frame combining the information of itself autocorrelation matrix. This scheme makes the performance of the WR-based channel estimation reaching to a significant improvement especially in high SNR domain. This paper also tries various parameters of λ then determines wanted MSE value.

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