BER Analysis of Semi-blind Channel Estimation in MIMO Systems

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Abstract

The huge increase of the MIMO (multiple input multiple output) capacity required perfect channel knowledge at the receiver. In this paper, we analyze the effect of imperfect channel estimation error on MIMO systems' BER performance. By utilizing whitening-rotation (WR) algorithm for recovering the detected signal at the receiver and calculating channel coefficients, it can be proved that semi-blind channel estimation obtains better results in comparison with blind ones. In addition, we also analyze symbol error rate (SER) of MIMO systems in such estimation schemes.

Keywords: *MIMO; CSI; whitening-rotation channel estimation*

1.Introduction

MIMO systems have been shown to provide dramatic capacity gain. Specifically, this large capacity gain requires perfect knowledge of the instanneous channel fading at the receiver which rarely happens in true MIMO systems. Therefore, the impact of imperfect channel channel knowledge and channel estimation error on capacity is difficult issue to investigate. While the performance of coherent MIMO communication systems can be assumed that the channel state information (CSI) being known at the receiver. In modern wireless communication standards, the pilot-based channel estimation schemes are used although this technique is associated with power consumption overhead and bandwidth inefficiency. Especially, the transmitters require total power constraints and the channel coefficients can be updated frequently due to fading transmission environment. In such scenarios, non-coherent detection algorithms in which either the CSI is resolved in detectors or can be estimated by the second order statistic of the received data [1, 2].

Besides, in MIMO, we also concern that the finite antenna spacing in array antennas introduces spatial correlation. Thanks to finite spacing properties, we can calculate mutual coupling parameter which affects signal transmission and reception because of the resulting antenna impedance mismatch. The mutual coupling effect is especially pronounced in tightly spaced arrays. In fact, demand for compact size mobile station, the effect of mutual coupling cannot be neglected and thus has to be taken into account while assessing the MIMO link performance. It has been shown that the mutual coupling may improve the channel capacity when the spacing of antenna elements in the transmit and receive array antennas is between 0.2 to 0.4 λ (carrier wavelength). The reason is that in this case the mutual coupling decreases the spatial correlation and increases the channel effective degree of freedom. However, the equalization at the receiver of MIMO based on the assumption that the perfect channel state information (CSI). In practical cases, perfect CSI is not achievable due to channel estimation errors which is analyzed in following section of this paper. In order to solve this difficulty, the MIMO system has to estimate properties fading channel before decoding the received data [3, 4].

Furthermore, in a multipath-rich wireless channel, utilizing the MIMO antennas achieves high data rate, without increasing the total transmission power or bandwidth. The capacity has been proved that grow linearly with the number of antennas while the wireless channel is known a priori. System performance depends on the equality of channel estimation and the number of pilot symbols. It is desirable to limit the number of training or pilot sequences because these pilots reduce the spectral efficiency.

It is true that the system performance will become better thanks to perfect estimated channel coefficients. The conventional training based algorithm (*e.g.*, Least square channel estimation or MMSE scheme) require longer training sequences which can be replaced by utilizing the superimposed training based schemes. However, such methods lead to increase the total transmit power. On the other hand, blind channel estimation schemes are computationally complex and ill-condition with phase ambiguities. In this paper, we apply the semi blind channel estimation based on WR scheme which utilize the analysis of the covariance matrix of received signal in oder to enhance the accuracy of the estimated channel [5]. Moreover, with the multimedia terminals being designed into smart phones which will demand very high data rate in the uplink, low transmit power requirements at the mobile devices to maintain a acceptable battery, and the potential for utilizing many antennas at the base stations, it is more desirable to launch MIMO system having optimal channel schemes. Motivated by the above analysis, we proved that WR-based channel estimation algorithm in the presence of channel estimation errors thanks to effect space time trellis code still satisfy high requirements in modern wireless application [6, 7].

The rest of the paper is organized as follows. In Section II, we present the WR-based channel estimation for flat fading MIMO channel cases. In next section, we analysis impact of channel estimation error on MIMO performance. The numerical results are presented in Section IV and conclusions are drawn in Section V.

Notation: The uppercase letters denote matrices, $(.)^H$ stands for complex conjugate transpose.

2. Channel Estimation based on WR Matrices

In this work, we analysis of MIMO system with N_T transmit and N_R receive antennas. The MIMO flat fading is characterized by a $N_R \times N_T$ matrix. Assuming that the transmit signal vector $\mathbf{x}(n)=[\mathbf{x}_1(n),\ldots,\mathbf{x}_{N_T}(n)]^T$ whose elements are independent identically distributed (i.i.d) Gaussian random variables with zero mean and unit variance $\sigma_x^2=1$, the received signal vector $\mathbf{y}(n)=[\mathbf{y}_1(n),\ldots,\mathbf{y}_{N_R}(n)]^T$ can be described as

$$\mathbf{y}(n)=\mathbf{H}\mathbf{x}(n)+\mathbf{v}(n) \tag{1}$$

In fact, the WR-based semi-blind schemes are suited for high speed wireless communications since they significantly reduce the training overhead, while resolving ambiguities associated with the blind channel estimation algorithms. The main idea of this unique scheme can be presented as analysis of the channel matrix $\mathbf{H}$ in the expression $\mathbf{H}=\mathbf{W}\mathbf{Q}^H$, where $\mathbf{H}$ is the whitening matrix and $\mathbf{Q}$ is the unitary rotation matrix. Since the unitary matrix $\mathbf{Q}$ is constrained as $\mathbf{Q}\mathbf{Q}^H=\mathbf{Q}^H\mathbf{Q}=\mathbf{I}_{N_T}$. The whitening matrix $\mathbf{W}$ can be estimated blindly using the second order statistics of the received signal. The matrix $\mathbf{W}$ can be calculated based on the covariance matrix

$$\mathbf{R}=P_t\mathbf{W}\mathbf{W}^H+\sigma_v^2\mathbf{I} \tag{2}$$

where P_t denotes transmit power. Next, it can be seen that matrix $\mathbf{W}$ calculated by [8]

$$\mathbf{W} = \mathbf{U}_W \mathbf{\Sigma}_W^{\frac{1}{2}} \tag{3}$$

where $\mathbf{U}_W \mathbf{\Sigma}_W \mathbf{V}_W^H = SVD\left[\frac{1}{P_t}\left(\mathbf{R} - \sigma_v^2 \mathbf{I}\right)\right]$

In addition, $\mathbf{Q}$ can be found by training based estimation scheme.

3. MIMO Performance Taking into Account Channel Estimation Errors

In real MIMO system, $\mathbf{H}$ has to be replaced by an estimated channel matrices which carries estimation errors. Utilizing whitening-rotation based channel estimation, the estimation error is affected by the number of antennas and the spatial correlation.

Assuming that the channel estimation error is denoted by e, the estimated channel matrix $\hat{\mathbf{H}}$ can be written as [9]

$$\hat{\mathbf{H}} = \mathbf{H} + \mathbf{e}\Phi \tag{4}$$

where $e\Phi$ is called the estimation error which is uncorrelated with the channel $\mathbf{H}$, e is the measure of how accurate the channel estimation performance approaches [10-14].

The received signal can be rewritten as

$$\mathbf{Y} = \hat{\mathbf{H}}\mathbf{X} + \mathbf{e}\Phi\mathbf{X} + \mathbf{V} \tag{5}$$

Assuming that the correlation coefficient is the same as comparing the real channel gain with the estimated version. We can find out this coefficient as follow

$$\lambda = \frac{E\left[h_{ij}\hat{h}_{ij}^*\right]}{\sqrt{E\left[\left|h_{ij}\right|^2\right].E\left[\left[\left|\hat{h}_{ij}\right|^2\right]\right]}} = \frac{1}{\sqrt{1+e^2}} \tag{6}$$

where $h_{ij}, \hat{h}_{ij}$ are the $(i,j)^{th}$ element of $\mathbf{H}$ and $\hat{\mathbf{H}}$ respectively. We also have the normalized mean square error (NMSE) of the channel estimation as

$$NMSE = \frac{E\left[\left|h_{ij} - \hat{h}_{ij}\right|^2\right]}{E\left[\left|h_{ij}\right|^2\right]} \tag{7}$$

And this NMSE parameter is related to e by the following expression

$$NMSE = e^2 \tag{8}$$

In the WR-base channel estimation, BER performance can be affected by error of estimation of the matrix $\mathbf{Q}$ and $\mathbf{W}$. Using long pilot sequence, we can decrease estimated error while waste of bandwidth. Furthermore, longer data block in blind channel estimation of $\mathbf{W}$ leads to accelerate computation time.

4. Simulation Results

In these simulations, we assume a MIMO system with 2 transmits antennas and 2 receive antennas, in which the QPSK modulation scheme is applied. In order to obtain optimal diversity gain, these simulations utilize space time trellis code for transmitted signal. In addition, we consider a Rayleigh channel whose elements are i.i.d. complex Gaussian variables with zero mean and unit variance. In order to enhance the simulation results, the simulation program run 20000 iterations which are so-called the Monte Carlo iterations.

For these simulations, we use $\mathbf{X}_p = 50$ symbols for training sequence compared to the total pure data length of $\mathbf{X} = 500$ symbols. Assuming the MIMO fading channel is static for the packet duration (or as the term of block fading channel as most of literature mentioned). In real environment, the MIMO channel can be time-variant during the period of a packet.

The first experiment (Figure 1) shows the relationship between channel estimation MSE versus Eb/No. This simulation result illustrates comparison of the estimations using WR-based approach and perfect channel estimation one. It is can be seen that the MSE of both methods linear decrease as SNR increases.

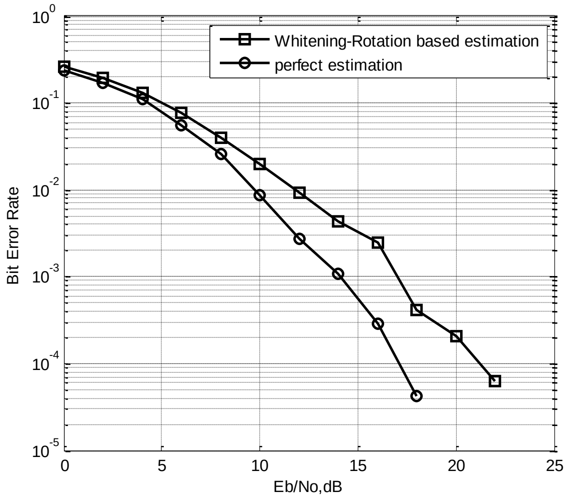

Figure 1. The MSE Performance of WR Algorithm and Perfect Channel Case

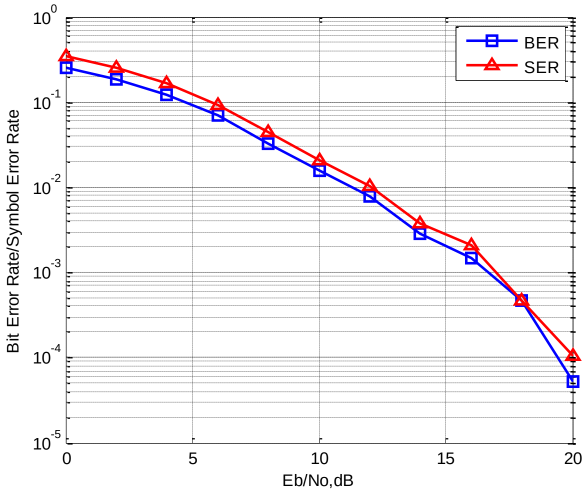

Figure 2. The BER/SER Performance of MIMO using WR-based Channel Estimation

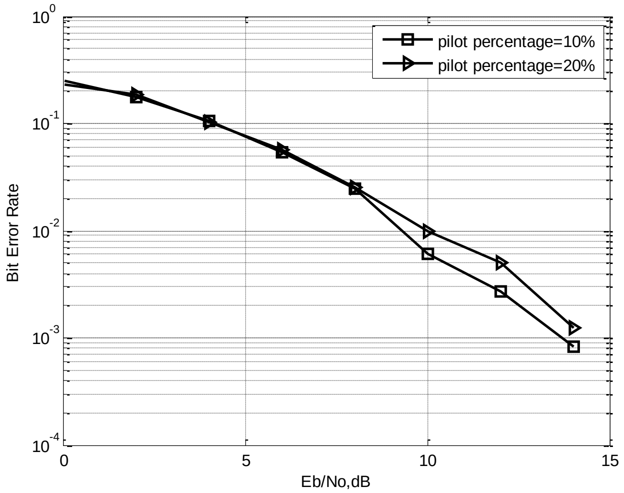

Figure 3. The BER Performance versus Eb/No with Different Training Sequence (pilot) Percentage

In next experiment, the paper illustrates BER performance and SER as well. These graph lines have the similar shape because we utilize QPSK modulation scheme.

The second experiment (in Figure 3) depicts the MSE performance with different percentage of training sequence in data frame of MIMO systems. It can be seen that the stable quality in WR-based channel estimation.

5. Conclusion

In this paper, we briefly studied the effect of CSI on BER/SER performance of MIMO systems utilizing space time trellis code. It is true that the perfect CSI leads to quality of detection algorithm at the receiver. We analytically proposed WR-based estimation algorithm which is considers as hybrid solution between blind scheme and pure one in presence of channel estimation error.

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Author

Dinh-Thuan Do obtained his PhD in Communications Engineering from the School of Electronics and Communications, Vietnam National University (VNU-HCMC) in 2012. In 2009, he was visiting scholar with Communications Engineering Institute, National Tsing Hua University, Taiwan. In additions, he was senior engineer in VinaPhone Company which is the biggest mobile network in Vietnam from 2003 to 2009. Since 2010, he is serving as a full time faculty in the School of Communications Engineering, Ton Duc Thang University. His research interest includes wireless communication network, cooperative communications and digital signal processing.