Missing Observations and Evolutionary Spectrum for Random Fields

Rachid Sabre

AgroSup/Laboratoire Le2i, UMR CNRS 6306, Université de Bourgogne,
26, bd Docteur Petitjean 21000 Dijon
r.sabre@agrosupdijon.fr

Abstract

There are innumerable situations where the data observed from a non-stationary random field are collected with missing values. In this work a consistent estimate of the evolutionary spectral density is given where some observations are randomly missing

Keywords: *Spectral density, non-stationary processes, periodogram, smoothing estimate, oscillatory process*

1. Introduction

Spectral analysis for stationary processes has been extensively studied in recent years. However, in many applications the signals must be modeled as non-stationary processes. This has motivated several authors to study non-stationary processes assuming that they are locally stationary. Priestley [11, 19] established the theory of the evolutionary spectrum generalizing spectral analysis developed for stationary processes. The evolutionary spectrum is time-dependent and describes the local power-frequency distribution at each time-instant. Other studies based on the Wold-Cramér decomposition have contributed to the development of the evolutionary spectrum [8, 13, 12, 14]. The applications of the evolutionary spectrum cover various scientific fields: signal and image processing [3, 1], seismic [16], oceanography, music [21]. The estimation of the evolutionary spectral density is studied in [19, 8, 6, 15, 7].

Moreover, Jones [4] was the first to consider the problems of missing data problems in spectral analysis. More precisely he studied the case where a block of observations was periodically unobtainable. In parallel, the theory of amplitude-modulated stationary processes was developed by Parzen [9], he applied this theory to solve the problem of periodic missing data problems. Bloomfield [2] has considered stationary processes with randomly missing data. He has given an asymptotically unbiased estimator of the spectral density and shown under suitable conditions that its variance converges to zero. We cite in this paper a few works that have contributed to finding solutions to the problems of missing observations: [17, 10, 5].

The aim of the present paper is to consider the problem of the randomly missing data for the class of non-stationary oscillatory random fields. Using the same techniques introduced by Bloomfield [2] for stationary processes, we give a consistent estimate of the evolutionary spectral density. The paper is organized as follows. In Section 2, we give some notations, assumptions and the amplitude modulating function Y_{t_1,t_2} . In Section 3, we construct a periodogram and we show that it is an asymptotically unbiased estimator. Since, we smooth the periodogram in the neighborhood of the time-instant t via a weight function and we show that it is a consistent estimate of the (weighted) average value of $h_{t_1,t_2}(\omega_{01},\omega_{02})$ in the neighborhood of the time-instant (t_1, t_2). Section 4 is devoted to proving theorems. In Section5, we study numerical results and simulation. The concluding comments are given in Section 6.

2. The Amplitude Modulating Function, Y_{t_1,t_2}

As in Priestley [11, 19], we consider a non-stationary centred oscillatory random field $\{X_{t_1,t_2}, t_1, t_2 \in Z\}$ i.e.

$$X_{t_1,t_2} = \int_{-\pi}^{+\pi}\int_{-\pi}^{+\pi} e^{i(t_1\omega_1, t_2\omega_2)} A_{t_1,t_2}(\omega_1,\omega_2) dZ_1(\omega_1,\omega_2); \quad t_1, t_2 \in Z, \tag{1}$$

where the function $A_{t_1,t_2}(\omega_1,\omega_2)$ is given by

$$A_{t_1,t_2}(\omega_1,\omega_2) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{i(\theta_1 t_1 + \theta_2 t_2)} dF_{\omega_1,\omega_2}(\theta_1,\theta_2),$$

$t_1, t_2 \in Z$ *and* $\omega_1, \omega_2 \in [-\pi, \pi]$,

where F_{ω_1,ω_2} is a measure satisfying: $\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \left| dF_{\omega_1,\omega_2}(\theta_1,\theta_2) \right| = 1$ and Z_1 is a process with orthogonal increments defined on the interval $[-\pi, +\pi]^2$ and $E\left| dZ_1(\omega_1,\omega_2) \right|^2 = d\mu_1(\omega_1,\omega_2)$ where μ_1 is a positive measure. The evolutionary spectral measure is defined by Priestley [11, 19] at each (t_1, t_2) by

$$dH_{t_1,t_2}(\omega_1,\omega_2) = \left| A_{t_1,t_2}(\omega_1,\omega_2) \right|^2 d\mu_1(\omega_1,\omega_2). \tag{2}$$

Our choice of oscillatory random field is motivated by the fact that it has a physical interpretation and the variance of the process is interpreted as a measure of the total power of the process at time t, because $Var(X(t_1,t_2)) = \int_{-\infty}^{+\infty} dH_{t_1,t_2}(\omega_1,\omega_2)$ .The evolutionary spectral density of the process $\{X(t_1,t_2)\}$ is given by $h_{t_1,t_2}(\omega_1,\omega_2)$ and defined as follows:

$$h_{t_1,t_2}(\omega_1,\omega_2) = \frac{dH_{t_1,t_2}(\omega_1,\omega_2)}{d\omega_1 d\omega_2}, \qquad \omega_1, \omega_2 \in R. \tag{3}$$

Assume that the process $\{X_{t_1,t_2}\}$ is observed with randomly missing observations. As Bloomfield [2], we consider the process L_{t_1,t_2} defined as the product of the process $\{X_{t_1,t_2}\}$ and another process $\{Y_{t_1,t_2}\}$ defined as follows:

$$L_{t_1,t_2} = X_{t_1,t_2} Y_{t_1,t_2} \qquad \textit{where} \qquad Y_{t_1,t_2} = \begin{cases} 1 & \textit{if } X_{t_1,t_2} \textit{ is observed} \\ 0 & \textit{if not} \end{cases}$$

The process L_{t_1,t_2} is equal to a modified version of the original process $\{X_{t_1,t_2}\}$ by replacing the missing observations by $E(X_{t_1,t_2})$ with zero as mean value, since $\{X_{t_1,t_2}\}$ is centred.

To simplify matters, we suppose, as Bloomfield [2], that $\{Y_{t_1,t_2}\}$ is stationary, independent of X_{t_1,t_2} and satisfying:

$$P\{Y_{t_1,t_2}=1\}=p>\frac{1}{2},$$
$$P\{Y_{t_1,t_2}=0\}=1-p,$$

The assumption of stationarity means that the statistical properties of the process Y are not time-depend. This case is often encountered in practice especially when collecting data provided by devices that are partially defective. Set

$$\xi_{r_1,r_2}=\frac{1}{p}E\{Y_{t_1,t_2}Y_{t_1+r_1,t_2+r_2}\} \tag{4}$$

$$\nu_{q,r,s}=\frac{1}{p^2}E\{Y_{t_1,t_2}Y_{t_1+q_1,t_2+q_2}Y_{t_1+r_1,t_2+r_2}Y_{t_1+s_1,t_2+s_2}\}; \quad q_i, r_i, s_i \in Z \tag{5}$$

Since $E(Y_{t_1,t_2})=p$, we obtain

$$Cov\{Y_{t_1,t_2},Y_{t_1+r_1,t_2+r_2}\}=E\{Y_{t_1,t_2}Y_{t_1+r_1,t_2+r_2}\}-E\{Y_{t_1+r_1,t_2+r_2}\}E\{Y_{t_1,t_2}\}$$
$$=p\xi_{r_1,r_2}-p^2=p(\xi_{r_1,r_2}-p)$$

This implies that ξ_{r_1,r_2} is symmetrical in (r_1,r_2). In the remainder of this paper, we assume the following hypotheses:

H_1)

There exists a real number $V>0$ such that

$$\sum_{q=-\infty}^{\infty}\left|\nu_{r,q,q+s}-\xi_{r_1,r_2}\xi_{s_1,s_2}\right|\le V(\|(r_1,r_2)\|+\|(s_1,s_2)\|+1)<\infty, \tag{6}$$

H_2)

$$\xi_{r_1,r_2}>0 \text{ and } p\xi_{r_1,r_2}\ge 2p-1>0 \qquad r_1,r_2\in Z \tag{7}$$

Remark 2.1

- The first hypothesis H_1) means that the sum,

$$\sum_{q=-\infty}^{\infty}Cov(Y_{t_1,t_2}Y_{t_1+r_1,t_2+r_2},Y_{t_1+q_1,t_2+q_2}Y_{t_1+q_1+s_1,t_2+q_2+s_2})$$

is bounded by a function proportional to $p^2(\|(r_1,r_2)\|+\|(s_1,s_2)\|+1)$.

- The second hypothesis H_2) implies for each (t_1,t_2), the probability that X_{t_1,t_2} is observed (i.e. not missing) is greater than $\frac{1}{2}$.

3. Estimation of the Evolutionary Spectral Density

We begin by giving some definitions introduced by Priestley [11, 19]. Let F the family of oscillatory functions $\left\{A_{t_1,t_2}(\omega_1,\omega_2)e^{i(t_1\omega_1+t_2\omega_2)}\right\}$. For each family F, we define the function $B_F(\omega_1,\omega_2)=\int \|(\theta_1,\theta_2)\|\, |dF_{\omega_1,\omega_2}(\theta_1,\theta_2)|$. Let C be in the class of families F such that $B_F(\omega_1,\omega_2)$ is bounded for all (ω_1,ω_2). For each family F we define the following constant B_F termed the characteristic width of F:

$$B_F=\left[\sup_{(\omega_1,\omega_2)} B_F(\omega_1,\omega_2).\right]^{-1}$$

The characteristic width of the process X_{t_1,t_2} is defined by $B_X=\sup_{F\in C} B_F$. For more details about the definitions see Priestley [11, 19].

In this section, we propose a periodogram constructed as follows:

$$I_{t,T}(\omega_{01},\omega_{02})=\left|\sum_{u_2=t_1-T_1}^{t_1+T_1}\sum_{u_2=t_2-T_2}^{t_2+T_2} g_{u_1,u_2}\frac{L_{t_1-u_1,t_2-u_2}}{S}e^{-i(\omega_{01}(t_1-u_1)+(t_2-u_2)\omega_{02})}\right|^2, \tag{8}$$

where $S=\left(2\pi\sum_{u_1,u_2} p\xi_{0,0}\left|g_{u_1,u_2}\right|^2\right)^{\frac{1}{2}}$, and $\left\{g_{u_1,u_2}\right\}$, is a filter satisfying the following conditions:

$C_1:\ g_{u_1,u_2}\geq 0\ ;\ g_{u_1,u_2}=g_{-u_1,-u_2}$,

$C_2:\ \sum_{u_1,u_2,v_1,v_2} p\xi_{u_1-v_1,u_2-v_2}\, g_{u_1,u_2}\, g^*_{v_1,v_2}<\infty$, where ξ is defined in (4)

C_3:) g_{u_1,u_2} has a finite ``width" defined by:

$$B_g;\ \sum_{u_1,u_2,v_1,v_2=-\infty}^{+\infty} p\left|\xi_{u_1-v_1,u_2-v_2}\right| \|(u_1,u_2)\| \left|g_{u_1,u_2}\right|\left|g^*_{v_1,v_2}\right|<\infty, \tag{9}$$

$C_4:\ B_g \ll B_F$,

C_5: For any real numbers k_1,k_2, we have

$$\left|\int_{-\infty}^{\infty}\Gamma(s_1,s_2)h_{t_1,t_2}(s_1+k_1,s_2+k_2)ds_1ds_2-h_{t_1,t_2}(k_1,k_2)\int_{-\infty}^{\infty}\Gamma(s_1,s_2)ds_1ds_2\right|<\frac{B_g}{B_F},$$

where the function Γ is defined by:

$$\Gamma(s,s')=\sum_{u_1,u_2,v_1,v_2} p\xi_{u_1-v_1,u_2-v_2}\, g_{u_1,u_2}\, g^*_{v_1,v_2}\, e^{-i\left(u_1s_1-v_1s_1'+u_2s_2-v_2s_2'\right)}.$$

The function Γ is highly concentred with relation to the function h_{t_1,t_2}.

When this condition is satisfed, we say as in Priestley ([19] page 829) that the function Γ is δ-function with respect to h_{t_1,t_2} in order $\left(\frac{B_g}{B_F}\right)$.

C_6 : $g_{u_1,u_2} = O\left(e^{-\|(u_1,u_2)\|}\right)$

The following theorem shows that the periodogram $I_{t,T}(\omega_{01},\omega_{02})$ is an asymptotically unbiased estimator of the evolutionary spectral density $h_{t_1,t_2}(\omega_{01},\omega_{02})$.

Theorem 3.1 *Let t_1, t_2 be an integer numbers; ω_{01},ω_{02} are real numbers and suppose that $\frac{B_g}{B_X} < \varepsilon$, then*

$$E\left[I_{t,T}(\omega_{01},\omega_{02})\right] = h_{t_1,t_2}(\omega_{01},\omega_{02}) + O(\varepsilon).$$

To prove the theorem 1, we have need the two following lemmas

Lemma 3.1 *For any $t_1, t_2, t_1^{'}, t_2^{'}, \lambda_1, \lambda_2$ real numbers, we have*

$$\left|\int e^{i(t_1 s_1 + t_2 s_2)} e^{-i(t_1^{'} s_1^{'} + t_2^{'} s_2^{'})} \Gamma\left(s + k, s^{'} + k^{'}\right) dF_{\lambda_1,\lambda_2}(s_1,s_2) dF_{\lambda_1,\lambda_2}(s_1^{'},s_2^{'}) - \Gamma\left(k,k^{'}\right)\int e^{i(t_1 s_1 + t_2 s_2)} e^{-i(t_1^{'} s_1^{'} + t_2^{'} s_2^{'})} dF_{\lambda_1,\lambda_2}(s_1,s_2) dF_{\lambda_1,\lambda_2}(s_1^{'},s_2^{'})\right| < 2\frac{B_g}{B_F}$$

Lemma 3.2 *Let θ_1,θ_2, λ_1,λ_2, t_1,t_2 and $t_1^{'},t_2^{'}$ be real numbers, we have*

$$\left|A_{t_1,t_2}(\lambda_1,\lambda_2) A^{*}_{t_1^{'},t_2^{'}}(\lambda_1,\lambda_2)\right| \left|\Gamma_{t_1,t_2,t_1^{'},t_2^{'},\lambda_1,\lambda_2}(\theta_1,\theta_2) - \Gamma(\theta,\theta)\right| \le 2\frac{B_g}{B_F}, \text{ where}$$

$$\Gamma_{t_1,t_2,s_1,s_2,\lambda_1,\lambda_2}(\theta_1,\theta_2) = \sum_{u_1,u_2,v_1,v_2} p\,\xi_{u_1-v_1,u_2-v_2}\, g_{u_1,u_2}\, g^{*}_{v_1,v_2}\, \beta(u,v,\theta) \tag{10}$$

where

$$\beta(u,v,\theta) = \frac{A_{t_1-u_1,t_2-u_2}(\lambda_1,\lambda_2) A^{*}_{s_1-v_1,s_2-v_2}(\lambda_1,\lambda_2)}{A_{t_1,t_2}(\lambda_1,\lambda_2) A^{*}_{s_1,s_2}(\lambda_1,\lambda_2)} e^{-i[(u_1-v_1)\theta_1 + (u_2-v_2)\theta_2]}.$$

In order to obtain a consistent estimate of $\left\{h_{t_1,t_2}(\omega_{01},\omega_{02})\right\}$, we smooth the periodogram in the neighborhood of the time-instant (t_1,t_2) via a weight function:

$$\hat{h}_{t_1,t_2}(\omega_{01},\omega_{02}) = \sum_{v_1,v_2 \in M} w_{T_1^{'},T_2^{'},v_1,v_2} I_{t_1-v_1,t_2-v_2}(\omega_{01},\omega_{02}). \tag{11}$$

where $w_{T_1^{'},T_2^{'},v_1,v_2}$ is a weight-function depending on the parameters $T_1^{'},T_2^{'}$ and satisfying

a) $w_{T_1^{'},T_2^{'},v_1,v_2} \ge 0$, for all v_1,v_2, $T_1^{'},T_2^{'}$

b) $w_{T_1',T_2',v_1,v_2} = 0$, $v_1, v_2 \notin M$, where M is a set of integers surrounding zero.

c) $w_{T_1',T_2',v_1,v_2} = w_{T_1',T_2',-v_1,-v_2}$,

d) $\sum_{v_1,v_2 \in M} w_{T_1',T_2',v_1,v_2} = 1$,

e) $\sum_{v_1,v_2 \in M} w^2_{T_1',T_2',v_1,v_2} < \infty$.

f) We assume that there exists a constant C such that

$$\lim_{T_1',T_2' \to \infty} T_1', T_2' \sum_{u_1,u_2 \in M} \left| W_{T_1',T_2',u_1,u_2} \right|^2 = C, \; where$$

$$W_{T_1',T_2',u_1,u_2} = \sum_{v_1,v_2 \in M} e^{-i(u_1 v_1 + u_2 v_2)} w_{T_1',T_2',v_1,v_2}.$$

The following theorem shows that the estimator $\hat{h}_{t_1,t_2}(\omega_{01}, \omega_{02})$ is an asymptotically unbiased of the (weighted) average value of $h_{t_1,t_2}(\omega_{01}, \omega_{02})$ in the neighbourhood of (t_1, t_2).

Theorem 3.2 *Let* $(\omega_{01}, \omega_{02})$ *be an element of* $[-\pi, \pi]^2$ *and suppose that* $\frac{B_g}{B_X} < \varepsilon$ *, then*

$$E\left[\hat{h}_{t_1,t_2}(\omega_{01}, \omega_{02})\right] = \bar{h}_{t_1,t_2}(\omega_{01}, \omega_{02}) + O(\varepsilon)$$

where $\bar{h}_{t_1,t_2}(\omega_{01}, \omega_{02}) = \sum_{v_1,v_2 \in M} w_{T_1',T_2',v_1,v_2} h_{t_1-v_1,t_2-v2}(\omega_{01}, \omega_{02})$.

To show that the variance converges to zero, as in Priestley ([11]) and Mélard [8], we assume that the process L_{t_1,t_2} is Gaussian.

Theorem 3.3 *Let* $(\omega_{01}, \omega_{02})$ *be an element of* $[-\pi, \pi]^2$ *and suppose that the process* L_{t_1,t_2} *is Gaussian, then we have*

$$Var\left[\hat{h}_{t_1,t_2}(\omega_{01}, \omega_{02})\right] = O\left(\frac{1}{T_1', T_2'}\right).$$

4. Numerical Studies

As in Bloomfield [2], we suppose that our process $\{X_{t,s}\}_{t,s \in Z}$ is observed at the successive instants $(t_1, s_1), (t_2, s_2), ..., (t_n, s_n)$ where $\tau_i = | t_{i+1} - t_i |$ $\tau_i' = | s_{i+1} - s_i |$ are independent random variables, each with the probability distribution $\{f_{r_1,r_2} = P[(\tau, \tau') = (r_1, r_2)]\}$, and the finite mean p^{-1}. As in Feller ([18], pp. 282-283), we define a process $\{Y_{t,s}'\}$ which coincides with $\{Y_{t,s}\}$ except at origin $Y_{0,0}' = 1$.The event "$Y' = 1$" is termed persistent and recurrent event. Using (6) we obtain

$$\xi_{r_1,r_2} = p^{-1}E\left\{Y_{t_1,t_2}Y_{t_1+r_1,t_2+r_2}\right\} = P\left\{Y_{t_1+r_1,t_2+r_2} = 1/Y_{t_1,t_2} = 1\right\}$$
$$= P\left\{Y'_{r_1,r_2} = 1\right\}$$

Feller ([18], pp. 282-283) has shown that

$$\xi_{r_1,r_2} = \sum_{s_1,s_2=1}^{r_1,r_2} f_{s_1,s_2}\xi_{r_1-s_1,r_2-s_2},\ r_i, s_i = 1,2...$$

The process L_{t_1,t_2} was obtained from the process X by omitting certain observations with the renewal-type mechanism defined above with $f_{1,1} = \frac{8}{9},\ f_{2,2} = \frac{1}{9},\ f_{r_1,r_2} = 0$ otherwise.

The simulation of the process X :

Using the same method as in [20] for the simulation of Markov Gauss random field, we simulate the Gaussian random field $Y = (Y_{n_1,n_2})_{n_1,n_2 \in Z}$ such that the covariance function is given by $C_Y(n_1,n_2) = e^{-\sqrt{(n_1+n_2)}}$, and its spectral density is $f_Y(\lambda_1,\lambda_2) = \frac{1}{\pi(1+\lambda_1^2+\lambda_2^2)}$.

The random field $X_{t,s}, t, s \in Z$ is given by the following model

$$X_{t,s} = c_{t,s}Y_{t,s}, \qquad t,s \in Z$$

where $c_{t,s} = \exp\left(-\frac{(t+s-500)^2}{2*200^2}\right)$ and $A_{t,s}(\omega_1,\omega_2) = c_{t,s}$ is independent of (ω_1,ω_2). With respect to the family $F = \left\{c_{t,s}e^{i(\omega_1 t+\omega_2 s)}\right\}$; $X_{t,s}$ has an evolutionary spectral density function $h_{t_1,t_2}(\omega_1,\omega_2) = c^2_{t_1,t_2} f_Y(\omega_1,\omega_2)$. The curve of the estimator with 5000 observations (Figure 2) and that of the spectral density (Figure 1) are very similar. So the estimator is quite satisfactory. If we take more observations (around 10000), the estimator becomes much smoother and the curve much approaches the density.

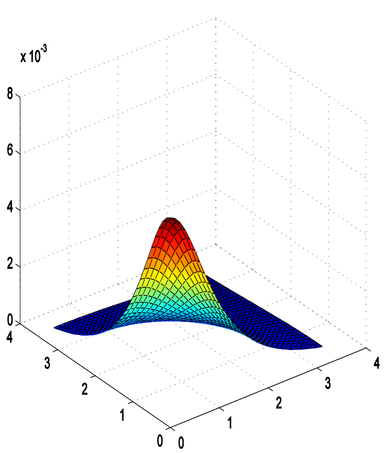

Figure 1. Density $h_{100,12}$

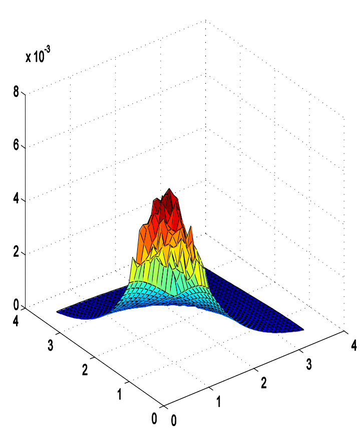

Figure 2. Estimator $\hat{h}_{100,12}$

5. Conclusion

We have proposed in this paper some results about the estimation of the evolutionary spectral density for non-stationary random fields where the data observed are collected with missing values. The approach is based on the technique used by Bloomfield [2] for stationary processes combining the estimates of evolutionary spectrum introduced by Priestley [11]. This work could be applied to several cases when the process is non-stationary as for example in:

- the segmentation of a sequence of images of a dynamic scene and the detection weeds in a farm field.
- the study of geostatistical mapping of certain chemical factors in agricultural soil.

This work could be supplemented by the study of optimal smoothing parameters using cross validation methods that have been proven in the field. It will also be extended to non-Gaussian process by assuming some hypotheses as for example when the cumulates are finite.

Acknowledgements

I would like to thank the anonymous referees for their interest in this paper and their valuable comments and suggestions.

References

[1] A. I. Al-Shoshan and L. F. Chaparro, "Identification of non-minimum phase systems using evolutionary spectral theory", Signal Processing, vol. 55, no. 1, **(1996)**, pp. 79-92.
[2] P. Bloomfield, "Spectral analysis with randomly missing observations", J. Roy. Statistical Society, vol. 32, no. 3, **(1970)**, pp. 369-380.
[3] S. D. Cabrera, B. C. Flores, G. Thomas and J. Vega-Pineda, "Evolutionary spectral estimation based on adaptive use of weighted norms", Advanced Signal Processing Algorithms, Architectures, and Implementations IV, Franklin T. Luk; SPIE, vol. 2027, **(1993)**, pp. 168-179.
[4] R. H. Jones, "Spectrum estimation with missing observations", Annals of the Institute of Statistical Mathematics, vol. 23, no. 1, **(1971)**, pp. 387-398.
[5] J. -C. Hung, "A genetic algorithm approach to the spectral estimation of time series with noise and missed observations", Information Sciences, vol. 178, no. 24, **(2008)**, pp. 4632-4643.
[6] A. S. Kayhan, A. El-Jaroudi and L. F. Chaparro, "Data-Adaptive Evolutionary Spectral Estimation", IEEE Transactions on signal processing, vol. 43, no. 1, **(1995)**, pp. 204-213.
[7] A. S. Kayhan and M. G. Amine, "Spatial evolutionary spectrum for DOA estimation and blind signal separation", IEEE transactions on signal processing, vol. 48, no. 3, **(2000)**, pp. 791-798.
[8] G. Mélard and A. Herteleer-de Schutter, "Contributions to evolutionary spectral theory", Journal of time series analysis, vol. 10, no. 1, **(1988)**, pp. 41-63.
[9] E. Parzen, "On spectral analysis with missing observations and amplitude modulation", Sankhya, Series A, no. 25, **(1963)**, pp. 383-392.
[10] P. M. T. Broersen, "Automatic spectral analysis with missing data", Digital Signal Processing, vol. 16, no. 6, **(2006)**, pp. 754-766.
[11] Priestley, M. B., Evolutionary spectra and non-stationary processes, J.Roy.Statist. Soc. Ser,B, 27: 204-237 **(1965)**
[12] M. Rachdi and R. Sabre, "Mixed-spectra analysis for stationary random fields", Statistical Methods and Applications, no. 18, **(2009)**, pp. 333-358.
[13] R. Sabre, "Spectral density estimation for stationary stable random fields", Journal Applications Mathematicae, vol. 23, no. 2, **(1995)**,pp. 107-133.
[14] R. Sabre, "Discrete estimation of spectral density for symmetric stable process", Statistica, vol. 2, **(2000)**, pp. 1-26.

[15] S. I. Shah, L. F. Chaparro, A. El-Jaroudi and J. M. Furman, "Evolutionary Maximum Entropy Spectral Estimation and HeartRate Variability Analysis", Multidimensional Systems and Signal Processing, vol. 9, no. 4, **(1998)**, pp. 453- 458.
[16] J. Tezcan, "Evolutionary Power Spectrum Estimation Using Harmonic Wavelets", Seismic Design and Analysis of Structures, (**2003)**, pp. 37-41.
[17] Y. Wang, P. Stoica, J. Li and T. L. Marzetta, "Nonparametric spectral analysis with missing data via the EM algorithm", Digital Signal Processing, vol. 15, Issue 2, **(2005)**.
[18] W. Feller, "An Introduction to Probability Theory and its Applications", vol. 1, 2nd Ed. New York: Wiley, **(1957)**.
[19] M. B. Priestley, "Spectral analysis and time series Probability and Mathematical Statistics", Academic Press, **(1988)**.
[20] F. Messaci, "Estimation de la densité spectrale d'un processus en temps continu par échantillonnage poissonnien", Université de Rouen, (Thèse de 3 ème cycle), **(1986)**.
[21] M. Caetano and X. Rodet, "Evolutionary spectral envelope morphing by spectral shape descriptors", In Proceeding of International Computer Music Conferences (ICMC), **(2009)** 16-21 August; Montreal, Canada.

Authors

Rachid Sabre

Responsible of MAIS, AgroSup, Member of the Laboratory Le2i, UMR CNRS 6306, University of Bourgogne, France.