Generation of Nakagami Fading Signals with Arbitrary Correlation and Fading Parameters

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Abstract

Mobile radio channel simulators are essential for repeatable system tests in the development, design, or test laboratory. Field tests in a mobile environment are considerably more expensive and may require permission from regulatory authorities. Because of the random, uncontrollable nature of the mobile propagation path, it is difficult to generate repeatable field test results. The Nakagami model can be used to test the performance of radios in a mobile environment in the lab, without the need to perform measurements whilst actually mobile. The mobile fading simulation can also if required be replicated, and the effects can be varied according to the 'velocity' of the mobile receiver. This allows the comparison of the performance of different receivers under standardized conditions that would not normally be possible in actual mobile testing situations. In this paper, two techniques that are capable of generating correlated Nakagami channels with arbitrary fading parameters are proposed. Basically our approach is to generate Nakagami RVs from the square root of directly generated correlated Gamma RVs (not from sum of squares of Gaussian RVs).

Keywords: *gamma and Gaussian random variables, Nakagami random vector, correlations, probability density function.*

1. Introduction

The early study of the error performance of diversity combining systems in a Nakagami environment was concentrated on some simple cases, assuming independent branch signals This implies that the antennas must be separated at least 50 wavelengths so that their correlation is negligible. In the real world, correlation between two antennas used in a base station is typically 0.7 or even higher due to the space limitation. The research focus was therefore gradually shifted to diversity systems with correlated Nakagami distribution. Generation of correlated Nakagami fading channels is therefore an essential issue for a laboratory test of wireless systems or subsystems to operate in a fading environment. Unfortunately, general techniques for this purpose are not available in the literature. The only result available is the technique described in a paper by Ertel and Reed for the generation of two correlated Rayleigh fading envelopes of equal power. This useful technique is further enhanced by introducing some efficient computational methods and by revealing its connection to wireless communications. The idea used is to exploit the fact that the envelope of a complex Gaussian variable follows a Rayleigh distribution.. Once the relation-ship is determined, the Rayleigh envelopes can be generated. In fact, the equal-power constraint imposed on the two Rayleigh signals can be easily removed if one directly invokes the results for the envelope correlation of two complex Gaussian variates.

Generation of correlated Nakagami vector with an arbitrary covariance matrix should follow a different philosophy. A generic technique is derived for arbitrary correlated Nakagami channels. A Nakagami variable is the square root of a gamma variable. One crucial step in our method is to introduce a decomposition principle for representing a gamma vector as a direct sum of independent vectors which, in turn, can be produced from a set of correlated Gaussian sequences. The next step is then to determine the relationship between the given Nakagami covariance structure and its counterpart for the Gaussian vectors.

Diversity combining techniques have been shown to provide an effective means to combat multipath fading and mitigate co-channel interference in mobile wireless communication systems. However, the diversity gain is reduced by the correlation of the multipath signals among the branches. The effects of correlated fading on the performance of a diversity combining receiver has received a great deal of research interest. Previous works used the Rayleigh distribution to model the fading channel statistics, but there has been increased interest in analyzing and evaluating the mobile wireless communication system performance in the Nakagami-m fading channel in order to represent a wider range of realistic fading conditions.[1]

Experimental results have shown that the Nakagami distribution fits experimental data better than Rayleigh, Rice and log-normal distributions. An advantage of the Nakagami distribution is that it can be reduced to the Rayleigh distribution and can model fading conditions more severe or less severe than those in the Rayleigh case. Most previous research investigations in correlated Nakagami fading channels focused on the theoretical analysis of performance (e.g. Bit-Error-Rate and outage probability) for various modulation schemes, with different pre-detection or post-detection diversity combining techniques. Computer-aided modeling of correlated Nakagami fading channels for predicting the performance of a given modulation/coding scheme is essential for the efficient evaluation and validation of system designs.

2. Gamma and Gaussian Random Variables:

Nakagami distribution (also known as m-distribution) is an important probability function (pdf) used in the study of mobile radio communications. A wide variety of fading effect can be modeled as Nakagami fading with different m parameters, including Rayleigh and one-sided Gaussian fading as special cases when m equals to 1 and 1/2, respectively. Furthermore, experimental and theoretical work has shown that the Nakagami distribution is the best m-distribution for data obtained for many urban multipath radio channels. Nakagami distribution is also suitable for modeling the output statistics of diversity combining system that are employed extensively to mitigate multipath faded effect. For maximum diversity gain, it is desirable for all diversity branches in a diversity combining system to be fading independently. However, under practical constraints or operating conditions, the branches may sometimes be correlated. Therefore a flexible algorithm with the ability to generate correlated Nakagami fading branches with arbitrary fading parameters and correlations is handy for the simulation of such systems. In the literature, however, only algorithms generating correlated Nakagami channel with the same m parameter are found. The approach is to use the summation of squared Gaussian random variables (RVs) to obtain Gamma distributed RVs; Nakagami RVs are then obtained from the square root of Gamma RVs. The correlation relationship of the Gaussian vectors is determined by the correlation relationship of the Gamma RVs, which in turn is determined from the correlation relationship of the

Nakagami channels. A limitation of this approach is that it is useful for identical and integer values of m parameter for all the branches only. Although improved the algorithm to non-integer m parameter by introducing a correction factor came into existence, it still cannot deal with branches with different fading parameters.

3. Derivation of Parameters and Correlations for Gamma Random Variables:

For a general I-branch diversity system in Nakagami fading environment, the fading envelope variable xi of the ith ($1 \leq i \leq I$) branch follows the Nakagami-m distribution

$$f(x_i) = \frac{2}{\Gamma(m_i)}\left(\frac{m_i}{P_i}\right)^{m_i} x_i^{2m_i-1} \exp\left(\frac{-m_i}{P_i} x_i^2\right) \tag{3.1}$$

where $\Gamma(.)$ is the Euler Gamma function and

$$P_i = E[x_i^2] \qquad m_i = \frac{E^2[x_i^2]}{E\left[\left(x_i^2 - E(x_i^2)\right)^2\right]} \tag{3.2}$$

The moments and variance for Nakagami RV are given in to be

$$E[x_i^n] = \frac{\Gamma(m_i+0.5n)}{\Gamma(m_i)} \times \left(\frac{P_i}{m_i}\right)^{0.5n} \tag{3.3}$$

$$\text{Var}\,[x_i] = P_i \times \left[1 - \left(\frac{\Gamma(m_i+0.5n)}{\sqrt{m_i} \times \Gamma(m_i)}\right)^2\right] \approx \frac{P_i}{5m_i} \tag{3.4}$$

Representing the fading envelope for all branches in vector form as

$$X = [x_1 \;\; x_2 \;\; \ldots\ldots \;\; x_{I-1} \;\; x_I\,]^T \tag{3.5}$$

The covariance matrix of x is

$$C_X = E\,[(X-E[X])\,(X-E[X])^T\,] \tag{3.6}$$

The square of a Nakagami RV follows the Gamma distribution, i.e.

$$\gamma_i = x_i^2$$

where gamma random variable follows the Gamma distribution

$$f(\gamma_i) = \frac{(m_i/P_i)^{m_i}}{\Gamma(m_i)} \exp\left(-\frac{m_i}{P_i}\,\gamma_i\right) \gamma_i^{m_i-1} \tag{3.7}$$

The moments and variance for a Gamma RV can be obtained in terms of m_i and p_i to be

$$E[\gamma_i^n] = \frac{\Gamma(m_i+n)}{\Gamma(m_i)} \times \left(\frac{P_i}{m_i}\right)^n \tag{3.8}$$

And

$$\mathrm{Var}[\gamma_i] = \frac{P_i^{\,2}}{m_i} \tag{3.9}$$

Similarly, the vector form of γ_i can be represented as

$$\gamma = [\gamma_1 \quad \gamma_2 \quad \cdots \quad \gamma_{I-1} \quad \gamma_I\,]^T \tag{3.10}$$

The covariance matrix of γ is denoted as C_γ, which is a positive definite matrix defined by

$$C_\gamma = \mathrm{E}\,[(\gamma - \mathrm{E}[\gamma])\,(\gamma - \mathrm{E}[\gamma])^{\mathrm{T}}\,] \tag{3.11}$$

Since we are going to generate correlated Nakagami RVs from correlated Gamma RVs, given the specifications for Nakagami RVs, we need to determine the parameters for the corresponding Gamma RVs, i.e.,

$$x_i\,(m_i, P_i)\,, C_X \;\rightarrow\; \gamma_i\,(m_i, P_i)\,, C_\gamma \tag{3.12}$$

From (3.1) and (3.7), m_i and P_i are the same for both Nakagami and Gamma RVs, hence we only need to determine the covariance matrix C_γ of the Gamma RVs based on the knowledge of C_X. First of all, we know that the diagonal elements for the covariance matrix are the variances of γ. Hence, the diagonal elements for C_γ can be directly obtained from m_i and P_i based on (3.9), i.e.,

$$C_\gamma(i, i) = \frac{P_i^2}{m_i} \tag{3.13}$$

Next, the cross covariance between γ_i and γ_j ($i \neq j$) is required. By definition, the normalized covariance (also known as the correlation coefficients) between any two RVs and is

$$\rho = \frac{\mathrm{cov}\,(r_i, r_j)}{\sqrt{\mathrm{var}\,[r_i] \times \mathrm{var}\,[r_j]}} = \frac{\mathrm{E}(r_i, r_j) - \mathrm{E}(r_i)\mathrm{E}(r_j)}{\sqrt{\mathrm{var}\,[r_i] \times \mathrm{var}\,[r_j]}} \tag{3.14}$$

where cov( ) is the covariance. For the relationship between the correlation coefficient ρ_x of Nakagami and ρ_γ of Gamma, there are two cases to be considered.

4.1 Direct Generation of Correlated Gamma RVs

The method used for generating correlated Gamma RVs is the Decomposition method.Our decomposition method is based on Cholesky decomposition of the covariance matrix, and the correlated Gamma RVs are obtained by linear summations of weighted independent Gamma RVs. The weighting coefficients are determined by the decomposed C. [2]

We want to generate an *n* -by-1 correlated Nakagami vector *z* with fading parameter *m* and covariance matrix $\boldsymbol{R}_z$. Before proceeding, let us define some notations.

The symbols

$$X \sim N(0, R_x)$$
$$Y \sim GM(m, R_y)$$
$$Z \sim NK(m, R_z) \tag{4.1}$$

are used to indicate that the vectors ***x*** **,** ***y*** and ***z*** follow a joint Gaussian, gamma, and Nakagami distribution, respectively. The Gaussian process has zero mean, and *m* is the fading parameter for the last two distributions. As a convention in this paper, we use R_x to denote the covariance matrix of the random vector ***x***, and use $x^{\odot r}$ to denote the vector obtained by taking power *r* of each element of ***x***.
Namely

$$[x^r(1) \dots \dots \; x^r(n)]^T \cong X^{\odot r} \tag{4.2}$$

with superscript *T* denoting transposition. Similar symbols can be defined for the same operation on a matrix. We will use *x* (*k*) to denote the *k* th entry of ***x*** and *R*(i, *j*) to denote the (i ,*j*)th element of ***R***. Directly generating a Nakagami sequence is extremely difficult, and no simple methods are available in the literature.

We will take an indirect approach, which follows the philosophy illustrated below:

$$\{X_k\} \rightarrow Y \rightarrow Z \tag{4.3}$$

where $\{\boldsymbol{X}_k\}$ is a set of independent Gaussian vectors, each having covariance matrix ***R****x*. The reason we start from Gaussian vectors ***x*** *k* is that they are easy to generate. Recall that the Nakagami vector ***z*** can be obtained from a gamma distributed vector ***y*** by taking the square root of each individual element of the latter, namely

$$\mathrm{Z} = \mathrm{y}^{\odot 1/2} \tag{4.4}$$

The gamma distributed vector ***y*** can be easily described by its characteristic function (CF)

$$\phi_y(s) = \det(1 - SA)^{-m} \tag{4.5}$$

where ***S*** is the diagonal matrix of the variables in the transform domain; that is, ***S***= diag(jt_1, , jt_n) The positive-definite matrix ***A*** is determined by the covariance matrix of ***y*** , which is denoted by $\boldsymbol{R}_y$. The (*k*, *l*) th entry of is related to the covariance matrix of ***y*** by

$$A(k,l) = \sqrt{\frac{R_y(k,l)}{m}} \tag{4.6}$$

which, again, can be simply denoted as

$$R_y = mA^{\odot 2} \tag{4.7}$$

Before we are able to implement the idea given in (4.3), we need to determine the relationship among $\boldsymbol{R}_x$, $\boldsymbol{R}_y$, and $\boldsymbol{R}_z$

The logical relation can be shown as

$$R_z \rightarrow R_y \rightarrow R_x \tag{4.8}$$

Clearly, given $\boldsymbol{R}_z$, we need a systematic procedure to determine $\boldsymbol{R}_y$, and $\boldsymbol{R}_z$, whereby desired Nakagami sequence can be synthesized.

4.2 Determination of Ry from Rz:

The covariance matrix of a random vector is uniquely determined by the variances and cross-correlations of its random components. The determination of the variances and cross-correlation coefficients follow different approaches and we therefore address them separately.[3]

4.2.1 Variance

Suppose we want to determine the variance of the component of ***y*** , *y(i)* , from the corresponding component of *z(i)* . Since index is not important in the derivation, it will be dropped for brevity. To begin with, we relate the variance of a gamma variable to its counterpart for Nakagami variate. Consider the moments of the Nakagami distribution NK(m, Ω)Using the probability density function (PDF) of NK (m, Ω) , it follows that

$$\mathrm{E}[\mathrm{Z}^{\alpha}] = \frac{2}{\Gamma(\mathrm{m})}\left(\frac{\mathrm{m}}{\Omega}\right)^{\mathrm{m}} \int_0^{\infty} \mathrm{z}^{2\mathrm{m}+\alpha-1} \exp\left(-\frac{\mathrm{m}}{\Omega}\mathrm{z}^2\right) \mathrm{dz} \tag{4.9}$$

which, after making change of variable yields

$$E[Z^{\alpha}] = \frac{\Gamma\left(m + \frac{\alpha}{2}\right)}{\Gamma(m)} \left(\frac{\Omega}{m}\right)^{\alpha/2} \tag{4.10}$$

The Gamma function $\Gamma(m)$ is defined by

$$\Gamma(m) = \int_0^{\infty} x^{m-1} e^{-x}\, dx \tag{4.11}$$

which, for a positive integer m , can be simplified to $\Gamma(m) = (m-1)!$.

The average power Ω is an intermediate parameter, which we do not need to specify. With this expression, it is easy to obtain

$$var[m] = E[z^2] - (E[z])^2$$

$$= \left[1 - \frac{1}{m}\frac{\Gamma^2\left(m + \frac{1}{2}\right)}{\Gamma^2(m)}\right]\Omega$$

$$\mathrm{Var}[z^2] = \frac{\Omega^2}{m} \tag{4.12}$$

Recall that $y=z^2$. Hence, we can represent the variance of the gamma variable, var[y] , in terms of var[z]

$$\text{var[y]} = \frac{var^2[z]}{m}\left[1 - \frac{1}{m}\frac{\Gamma^2\left(m+\frac{1}{2}\right)}{\Gamma^2(m)}\right]^{-2} \tag{4.13}$$

This equation will be used to determine the variance of squared Nakagami signal. When using the results obtained above, appropriate indices should be added. For example, var[$y(i)$] should be understood to be var[$y(i)$]= $R_y(i, i)$.

4.2.2 Cross Correlation:

Given $\boldsymbol{R}_Z$, the (i, j) th correlation coefficient is equal to

$$\rho(i,j) = R_Z(i,j)[R_Z(i,i)R_Z(j,j)]^{-\frac{1}{2}} \tag{4.14}$$

Let us show how to obtain $\rho(i,j)$ from its counterpart for the gamma vector $\boldsymbol{y}$. The latter is denoted by $v(i, j)$ for distinction and is given by

$$v(i,j) = R_y(i,j)[R_y(i,i)R_y(j,j)]^{-\frac{1}{2}} \tag{4.15}$$

The relation between the correlation coefficients of the Nakagami and the Gamma vectors is revealed by

$$\rho(i,j) = corr(z(i)^n, z(j)^n)$$

$$= \varphi(m,n)\{{}_2\text{F}_1(-\tfrac{n}{2},\tfrac{n}{2}; m; v(i,j) - 1\} \tag{4.16}$$

Where

$$\varphi(a,b) = \frac{\Gamma^2(a+\frac{b}{2})}{\Gamma(a)\Gamma(a+b)-\Gamma^2(a+\frac{b}{2})} \tag{4.17}$$

and the hyper-geometric function is defined by

$${}_2\text{F}_1(a,b;c,z) = \sum_{n=0}^{\infty}\frac{(a)_n(b)_n z^n}{(c)_n n!} \tag{4.18}$$

With $(a)_n = a(a+1)\ldots(a+n-1)$ and $(a)_o = 1$. In this expression, ρ_n represents the order-n correlation of the Nakagami vector. The physical significance of (4.16) is clear, indicating that an arbitrary-order correlation of two Nakagami variables can always be expressed in terms of the correlation of the gamma variables $y(i)$ and $y(j)$. Our interest is, however, to determine the correlation $v(i, j)$ between the two gamma variables from the given value of Nakagami correlation $\rho_1(i,j)$.
Namely, given $\rho_1(i,j)$, we need to solve the equation

$$\rho_1(i,j) = \varphi(m,1)\{{}_2\text{F}_1(-\tfrac{1}{2},-\tfrac{1}{2}; m; v(i,j) - 1\} \tag{4.19}$$

for the unknown v(i,j).

4.2.3 Computational Issues :

It remains for us to find a simple method for solving (4.19). To simplify notation, we will drop the indices and simply write $\rho_1(i, j)$ and *v* (*i*, *j*) as ρ and *v*, respectively. Let us define

$$f(v) \equiv \varphi(m,1)\left\{\left(-\tfrac{1}{2},-\tfrac{1}{2};m;v\right)-1\right\}-\rho \qquad (4.20)$$

Since we can easily obtain the derivative of the hypergeometric function, we will use the Newton Raphson method.

$$f'(v) = \frac{\varphi(m,1)}{4m}\left\{\left(\tfrac{1}{2},\tfrac{1}{2};m+1;v\right)\right\} \qquad (4.21)$$

whereby the iterative algorithm is obtained as shown.

$$v_{i+1} = v_i - \frac{f(v_i)}{f'(v_i)} \qquad (4.22)$$

We need an initial value to start the iteration. It has been shown that a good approximation to *v* is ρ . Thus, we can set

$$v_o = \rho \qquad (4.23)$$

The expressions (4.16) to (4.23) constitute the iterative algorithm for determining the desired value of *v*. The algorithm usually con-verges in a few steps. The iterative technique described above is also applied to the special case of Rayleigh fading channels. The hyper geometric function is approximated in one way or another to simplify the procedure for solving the nonlinear equation. This approximation, however, is unnecessary since the Newton Raphson method can provide a more accurate result at the rate of geometrical convergence.[4]

Techniques for generating a Gaussian vector with a specified covariance matrix are well developed. A natural way to generate a gamma vector is therefore to represent the gamma vector in terms of a set of Gaussian vectors. To gain inspiration, suppose

$\boldsymbol{x}$ ~N(0, $\boldsymbol{R}_x$) and consider the CF of the vector

$$u = X^{\Theta 2} \qquad (4.24)$$

Denote the elements of $\boldsymbol{u}$ given by $u_1, u_2 \dots\dots\dots\dots u_n$. By definition, its CF is given by

$$\emptyset_u(s) = E[\exp(ju_1t_1 + \cdots + ju_nt_n)]$$

$$= \int 2\pi^{(-1/2)} \det(R_x)^{(-1/2)} \exp\left(-\tfrac{1}{2}\mathrm{x}^{\mathrm{T}}\mathrm{R}_x^{-1}x + x^TSx\right)dx$$

$$= \det(\mathrm{I} - 2\mathrm{SR}_x)^{(-1/2)} \qquad (4.25)$$

where $\boldsymbol{S}$**,** the diagonal matrix of transform variables, has been defined right after (4.5). Observe that (4.5) and (4.25) have a similar form, suggesting that we can factor (4.5) such that

$$\emptyset_y(S) = \prod_{k=1}^{2m} \det(I - 2SR)^{(-1/2)} \quad (4.26)$$

where $\boldsymbol{R}_x$ needs to be determined, this is an important expression.

First, (4.26) allows us to relate the covariance matrix of ***y*** to that of ***x***. By applying (4.7) to the right-hand side, it follows that the covariance matrix of ***y*** equals

$$R_y = m(2R_x)^{\Theta 2} = 4mR_x^{\Theta 2} \quad (4.27)$$

It implies that $\boldsymbol{R}\, x$ must meet the following condition:

$$R_x = \frac{1}{2\sqrt{m}} R_y^{\Theta(1/2)} \quad (4.28)$$

From (4.26), it also follows that ***y*** has a simple direct-sum decomposition. This is easy to understand if we consider the case in which 2*m* takes an integer value. Recall that the probability density function of independent random variables is Fourier transformed to the product in the CF domain. When transformed back to the original domain, the left-hand side of (4.26) is simply equal to ***y*** , whereas the right-hand side leads to the sum of $\boldsymbol{u}_k$ **.** Accordingly, we obtain

$$y = \sum_{k-1}^{2m} U_k = \sum_{k-1}^{2m} X_k^{\Theta 2} \quad (4.29)$$

We use the subscript *k* to indicate that ***x*** *k* are independent sequences. The above expression reassures us that a Nakagami vector has the same direct sum decomposition as that of its scalar counterpart, and that the correlation is uniquely deter-mined by the correlation of the generating Gaussian vector. Correspondingly, for the Nakagami vector, we have

$$Z = y^{\Theta 1/2} = \left(\sum_{k=1}^{2m} U_k\right)^{\Theta 1/2} = \left(\sum_{k=1}^{2m} x_k{}^{\Theta 2}\right)^{\Theta 1/2} \quad (4.30)$$

In this notation, the *i* th entry of ***z*** is given by

$$Z(i) = \left[\sum_{k=1}^{2m} x_k^2(i)\right]^{1/2} \quad (4.31)$$

This expression forms a basis for synthesizing a correlated Nakagami fading channel from a set of independent Gaussian vectors.

Finally, from (4.28) and (4.13), it follows that $\forall k, l = \{1, \dots \dots \dots \dots, n\}$

$$\begin{aligned} R_x(k,l) &= \{\varsigma\, var[Z(k)]\} \quad , k = l \\ &= \varsigma\{var[Z(k)]var[Z(l)]v(k,l)\}^{1/2} \quad , k \neq l \end{aligned} \quad (4.32)$$

Where

$$Var[\mathrm{Z(k)}] = R_k(k,k)$$

$$\varsigma = \frac{1}{2m}\left[1 - \frac{1}{m}\frac{\Gamma^2(m+1/2)}{\Gamma^2(m)}\right]^{-1} \quad (4.33)$$

This expression shows the way to determine $\boldsymbol{R}_x$ directly from the covariance matrix of ***Z.***

4.3 Generation of the Nakagami Random Vector, x_k

Given the covariance matrix R_x , it is easy to generate a random vector x_k . We identify two cases. If the size of x is not very large (say, for the application in diversity reception), we can use the technique of Cholesky or eigen decomposition. We only consider the former. Cholesky decompose R_x such that

$$\boldsymbol{R}_x = \boldsymbol{L}\boldsymbol{L}^T \tag{4.34}$$

with denoting Hermitian transposition. We next generate independent identically distributed (iid) Gaussian sequence with zero mean and unit variance

$$e_k \sim N(0, \boldsymbol{I}) \tag{4.35}$$

Then the vector

$$X_k = \boldsymbol{L}e_k \tag{4.36}$$

will be the one we want.

If the size of $\boldsymbol{x}_k$ is very large, decomposition of $\boldsymbol{R}_x$ will be very time consuming. In this case, it is more efficient to fit the time sequence with an autoregressive (AR) model. It can be shown that any sequence with a rational spectrum can be fit by using an AR model with arbitrary accuracy as long as the model order is sufficiently long. The model order can be chosen by using an information-theoretic criterion. Suppose we are able to determine the AR model, say q . The AR coefficients $\boldsymbol{a}$ can be determined from the $(q+1)\times(q+1)$ submatrix $\boldsymbol{R}_{q+1}$ of $\boldsymbol{R}_x$.

$$\boldsymbol{R}_{q+1} = \begin{bmatrix} R_x(1,1) & p^T \\ p & R_q \end{bmatrix} \tag{4.37}$$

where $\boldsymbol{p}$ is the q -by-1 vector. Then the AR coefficient vector is given by

$$\boldsymbol{a} = R_q^{-1}\,\boldsymbol{p} \tag{4.38}$$

and the desired sequence x is obtained as

$$x_k(n) = \sum_{i=1}^{q} a(i)\, x_k(n-q) + e_k(n) \tag{4.39}$$

where $e_k(k)$ is the iid Gaussian error sequence with zero and variance given by

$$\sigma_e^2 = \frac{\det(R_{q+1})}{\det(R_q)} \tag{4.40}$$

Besides AR modeling, one may also consider using autoregressive moving average (ARMA) models especially when the model order q is large. Generally speaking, the Cholesky decomposition method guarantees meeting the requirement on the covariance structure. On the other hand, the AR technique, although efficient, only provides an approximation to the specified correlation function.

4.4 Decomposition Form of Real Value :

In the previous discussion of direct-sum decomposition, we assumed that m was integer-valued. Let us extend the results to the general case of an arbitrary fading parameter. Denote the integral part of $2m$ by

$$p = [2m] \tag{4.41}$$

For the general case 2*m*, the decimal of is nonzero and we cannot express ***y*** in the same form as (4.29). However, we can add a correction terms to (4.29) such that

$$y = \alpha \sum_{k=1}^{p} X_k^{\Theta 2} + \beta X_{p+1}^{\Theta 2} \tag{4.42}$$

The second term is the correction term. The vectors x_k are the Gaussian vectors having the same characteristics as we previously discussed. The constants α and β are the only unknowns to be determined. The idea behind this expression is a principle in statistics; namely, the summation of independent chi-square variables can be accurately approximated by a single gamma variable . This technique has been widely used in statistical theory and engineering .

The expression given in (4.42) can be considered to be a direct-sum decomposition for the general case in which *m* takes on anarbitrary value.

It remains for us to determine the unknown coefficients α and β . Consider a single component of ***y*** and $\boldsymbol{x}_k$, say *r* th, and hence we can write

$$y(r) = \alpha \sum_{k}^{p} x_k^2(r) + \beta x_{p+1}^2(r) \tag{4.43}$$

Both gamma and squared Gaussian variables can be ex-pressed in terms of chi-square variables. In particular, by applying (4.5) we obtain

$$y(r) \sim \frac{A(r,r)}{2} \chi^2(2m)$$

$$x_k^2(r) \sim \sigma_x^2(r) \chi^2(2m) \tag{4.44}$$

We then combine (4.6) and (4.28) to obtain

$$A(r,r) = 2\sigma_x^2(r) \tag{4.45}$$

Applying these results to (4.43) allows us to write

$$\chi^2(2m) \stackrel{d}{=} \alpha \sum_{k=1}^{p} \chi_k^2(1) + \beta \chi_{p+1}^2(1) \tag{4.46}$$

the symbol $\stackrel{d}{=}$ means the both sides have the same distribution. We take the first two moments of both sides and note that

$$E[\chi^2(m)] = m$$

And

$$var[\chi^2(m)] = 2m$$

yielding

$$p\alpha + \beta = 2m$$

$$p\alpha^2 + p\beta^2 = 2m \tag{4.47}$$

Solving the equations we obtain,

$$\alpha = \frac{2pm \pm \sqrt{2pm(p+1-2m)}}{p(p+1)} \tag{4.48}$$

There are two possible values for α . We suggest choosing the one closer to unity. The reason becomes clear if we employ the closure property of independent χ^2 variables to rewrite (4.46) as

$$\chi^2(2m) \overset{d}{\approx} \alpha\chi^2(p) + \beta\chi^2(1) \tag{4.49}$$

For illustration, suppose m =1.8. Then, the two solutions for are 1.0732 and 0.7268. Choosing the former, we have

$$\chi^2(3.6) \overset{d}{\approx} 1.0732\chi^2(3) + \ 0.3804\chi^2(1) \tag{4.50}$$

The first term provides a coarse approximation to $\chi^2(2m)$, while the second term provides a fine correction. In general, the weighting coefficients can be determined by

$$\alpha = \frac{2pm \pm \sqrt{2pm(p+1-2m)}}{p(p+1)}$$

$$\beta = 2m - p\alpha \tag{4.51}$$

where p = [2m]. It is interesting to examine the case of m = $\frac{1}{2}, 1, \frac{3}{2}, 2, \ldots$ under this situation, we have p = [2m] = 2m which, when inserted into (4.51), gives α =1 and β =0 . Namely, no correction component is required. This is exactly the result we obtained in the previous sections.[5]

A drawback of our proposed decomposition method is that it is not an exact pdf model, so minor inaccuracy can be expected. However, inaccuracy in this method is fairly small for most practical purposes.

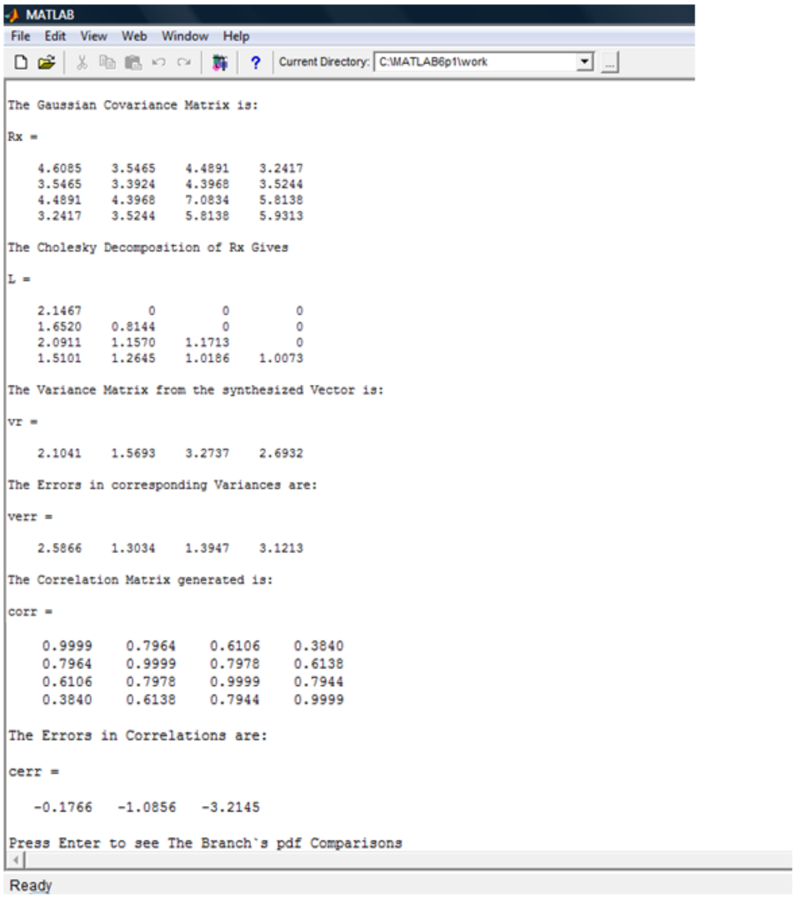
5. Implementation and Simulation Results for Decomposition Method:

Let us implement the use of the new decomposition technique through an example for multi branch channels as encountered in diversity reception in a fading environment.
We would like to generate a multi branch channel, which suffers from a Nakagami fading with the envelope correlation specified by

$$C_z = \begin{bmatrix} 1.000 & 0.795 & 0.604 & 0.372 \\ 0.795 & 1.000 & 0.795 & 0.604 \\ 0.604 & 0.795 & 1.000 & 0.795 \\ 0.372 & 0.604 & 0.795 & 1.000 \end{bmatrix}$$

and variance vector specified by P_Z = [2.16,1.59,3.32,2.78]. We need to generate the vector channel for m =2.18, m =2.5, m= 3, m=3.5 and m=4 respectively.

FOR m = 2.18



```
The Gaussian Covariance Matrix is:

Rx =

    4.6085    3.5465    4.4891    3.2417
    3.5465    3.3924    4.3968    3.5244
    4.4891    4.3968    7.0834    5.8138
    3.2417    3.5244    5.8138    5.9313

The Cholesky Decomposition of Rx Gives

L =

    2.1467         0         0         0
    1.6520    0.8144         0         0
    2.0911    1.1570    1.1713         0
    1.5101    1.2645    1.0186    1.0073

The Variance Matrix from the synthesized Vector is:

vr =

    2.1041    1.5693    3.2737    2.6932

The Errors in corresponding Variances are:

verr =

    2.5866    1.3034    1.3947    3.1213

The Correlation Matrix generated is:

corr =

    0.9999    0.7964    0.6106    0.3840
    0.7964    0.9999    0.7978    0.6138
    0.6106    0.7978    0.9999    0.7944
    0.3840    0.6138    0.7944    0.9999

The Errors in Correlations are:

cerr =

   -0.1766   -1.0856   -3.2145

Press Enter to see The Branch`s pdf Comparisons
```

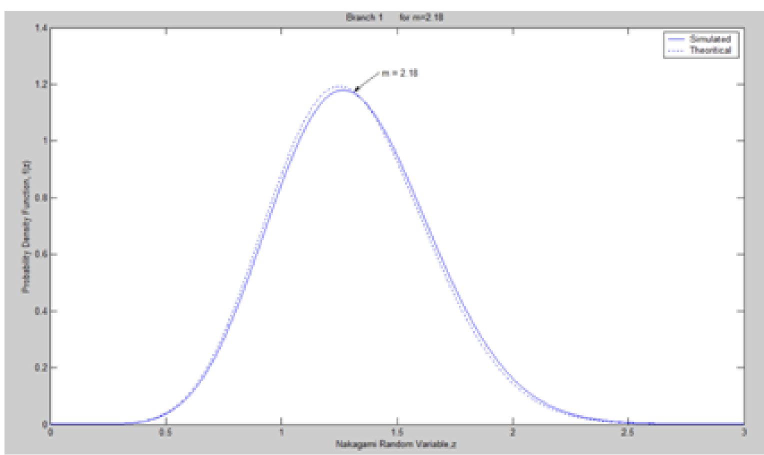


Fig (5.a) Generated pdf Using Decomposition Method vs. Theoretical pdf for Branch1

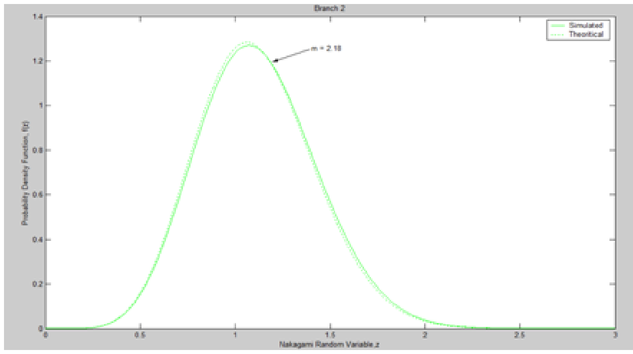


Fig (5.b) Generated pdf Using Decomposition Method vs. Theoretical pdf for Branch2

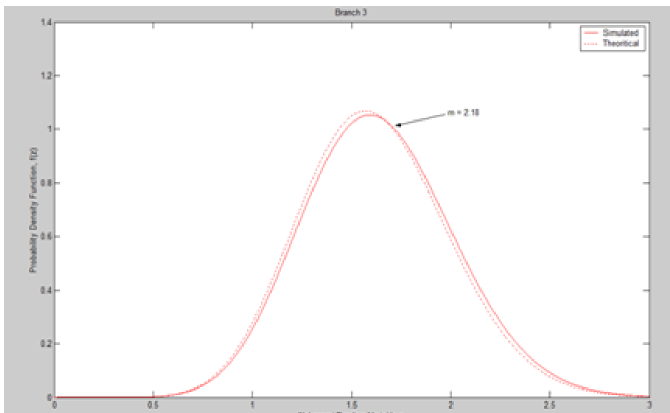


Fig (5.c) Generated pdf Using Decomposition Method vs. Theoretical pdf for Branch3

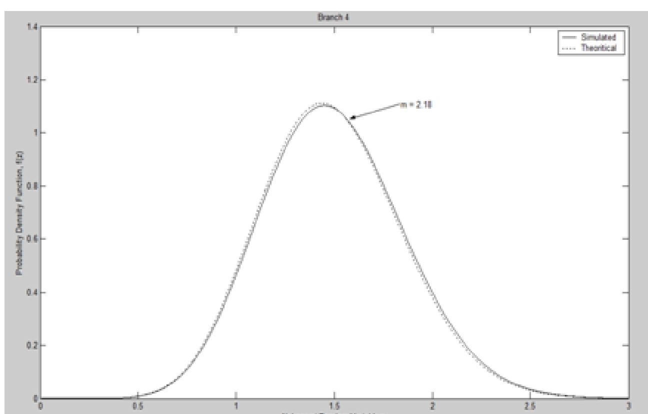


Fig (5.d) Generated pdf Using Decomposition Method vs. Theoretical pdf for Branch4

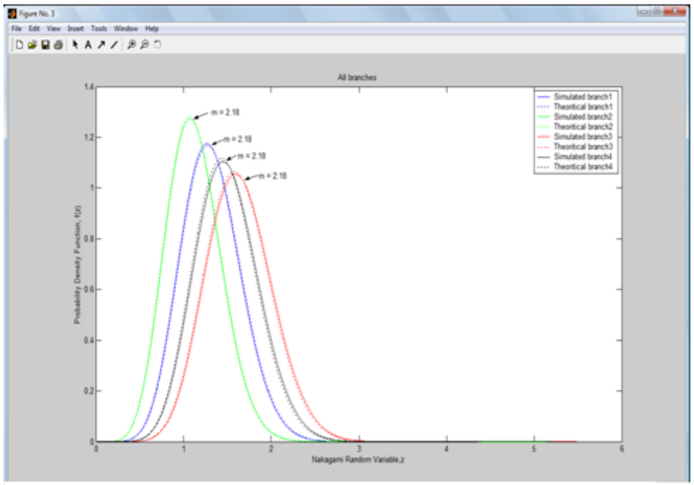


Fig (5.e) Generated pdf Using Decomposition Method vs. Theoretical pdf for All Branches

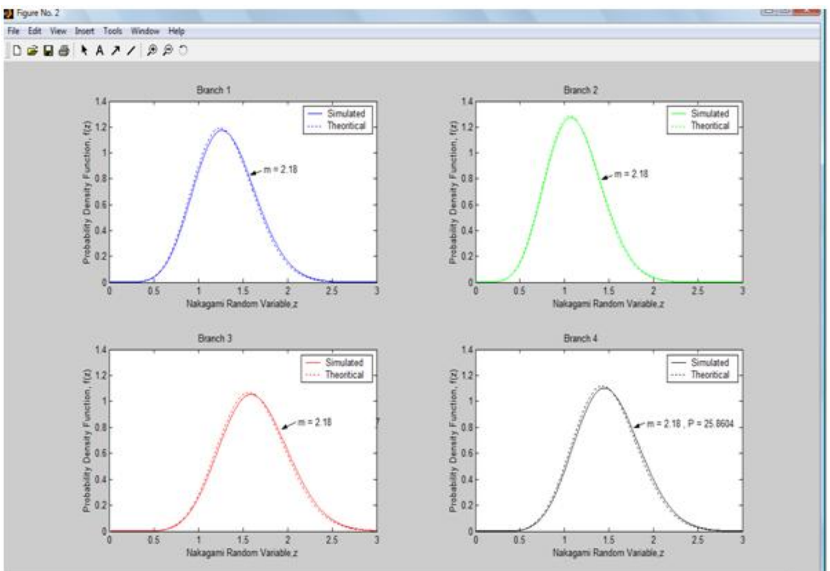
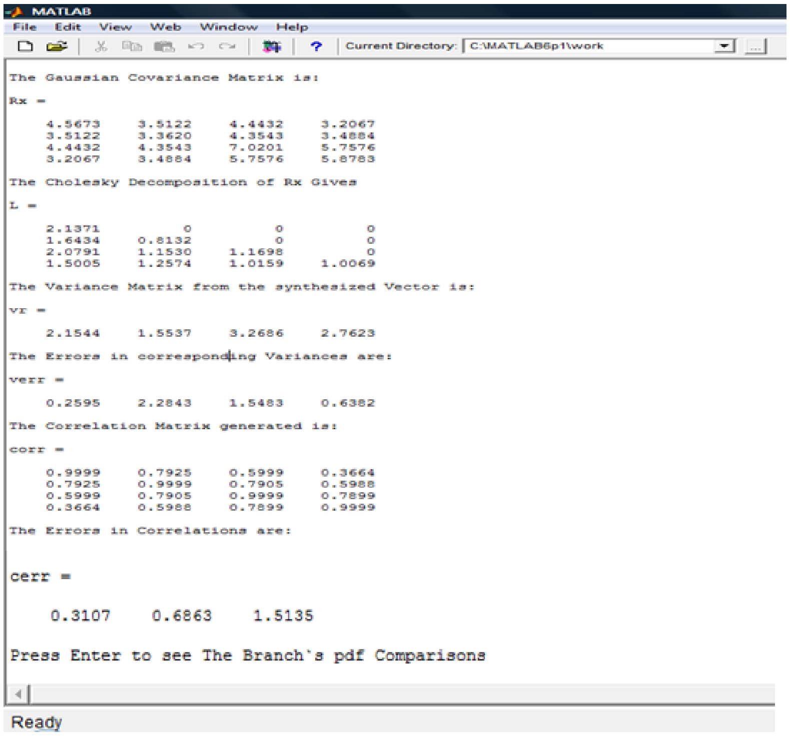


Fig (5.f) Generated pdf Using Decomposition Method vs. Theoretical pdf for All Branches in Single Plot for m=2.18

FOR m =2.5



```
MATLAB
File  Edit  View  Web  Window  Help
Current Directory: C:\MATLAB6p1\work

The Gaussian Covariance Matrix is:

Rx =

    4.5673    3.5122    4.4432    3.2067
    3.5122    3.3620    4.3543    3.4884
    4.4432    4.3543    7.0201    5.7576
    3.2067    3.4884    5.7576    5.8783

The Cholesky Decomposition of Rx Gives

L =

    2.1371         0         0         0
    1.6434    0.8132         0         0
    2.0791    1.1530    1.1698         0
    1.5005    1.2574    1.0159    1.0069

The Variance Matrix from the synthesized Vector is:

vr =

    2.1544    1.5537    3.2686    2.7623

The Errors in corresponding Variances are:

verr =

    0.2595    2.2843    1.5483    0.6382

The Correlation Matrix generated is:

corr =

    0.9999    0.7925    0.5999    0.3664
    0.7925    0.9999    0.7905    0.5988
    0.5999    0.7905    0.9999    0.7899
    0.3664    0.5988    0.7899    0.9999

The Errors in Correlations are:

cerr =

    0.3107    0.6863    1.5135

Press Enter to see The Branch`s pdf Comparisons

Ready
```

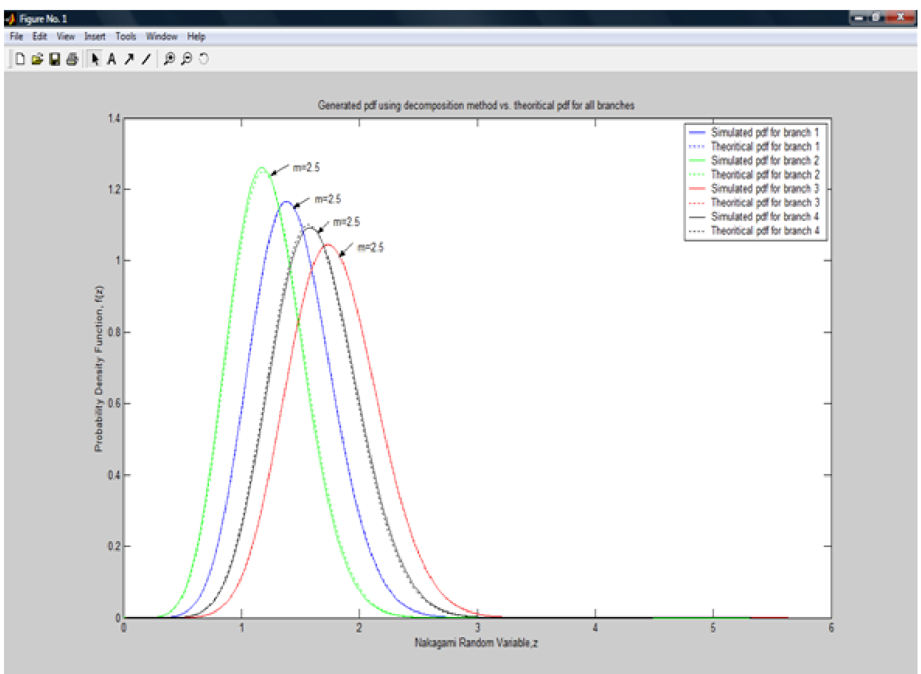
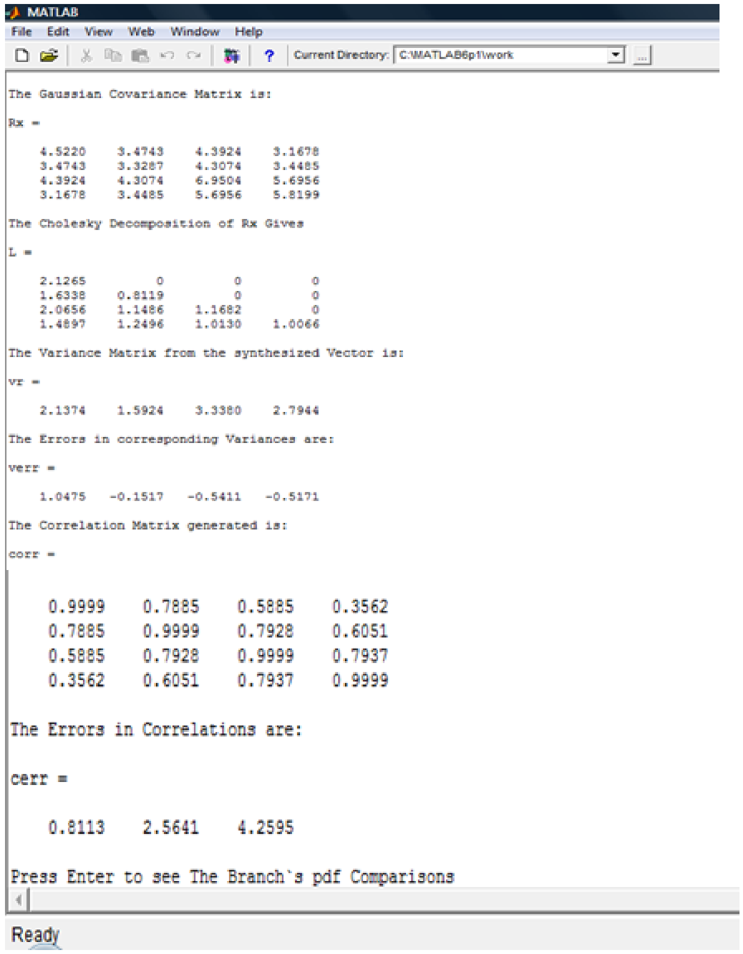


Fig (5.g) Generated pdf Using Decomposition Method vs. Theoretical pdf for All Branches in Single Plot for m=2.5

FOR m= 3




```
The Gaussian Covariance Matrix is:

Rx =

    4.5220    3.4743    4.3924    3.1678
    3.4743    3.3287    4.3074    3.4485
    4.3924    4.3074    6.9504    5.6956
    3.1678    3.4485    5.6956    5.8199

The Cholesky Decomposition of Rx Gives

L =

    2.1265         0         0         0
    1.6338    0.8119         0         0
    2.0656    1.1486    1.1682         0
    1.4897    1.2496    1.0130    1.0066

The Variance Matrix from the synthesized Vector is:

vr =

    2.1374    1.5924    3.3380    2.7944

The Errors in corresponding Variances are:

verr =

    1.0475   -0.1517   -0.5411   -0.5171

The Correlation Matrix generated is:

corr =

    0.9999    0.7885    0.5885    0.3562
    0.7885    0.9999    0.7928    0.6051
    0.5885    0.7928    0.9999    0.7937
    0.3562    0.6051    0.7937    0.9999

The Errors in Correlations are:

cerr =

    0.8113    2.5641    4.2595

Press Enter to see The Branch`s pdf Comparisons
```

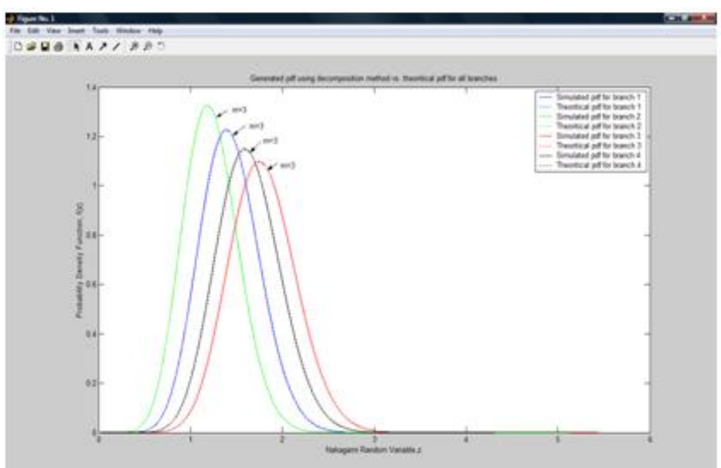



Fig (5.h) Generated pdf Using Decomposition Method vs. Theoretical pdf for All Branches in Single Plot for m=3

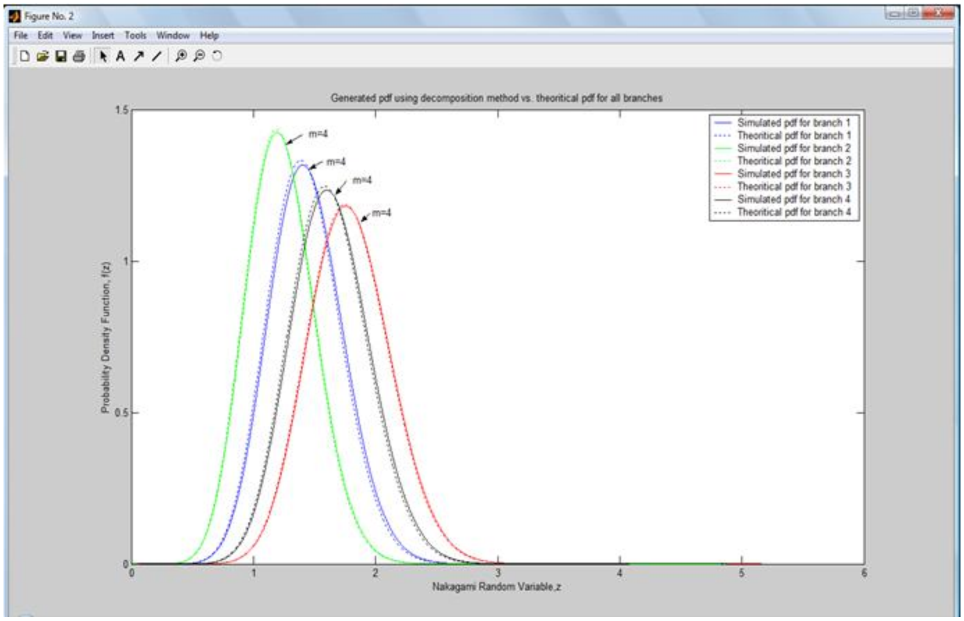


Fig (5.i) Generated pdf Using Decomposition Method vs. Theoretical pdf for All Branches in Single Plot for m=4

6. Conclusion

The paper discusses the computer generation of correlated Nakagami RVs with arbitrary fading parameters and correlations.

A new approach to generate correlated Nakagami fading signals with arbitrary fading parameters and correlations is the decomposition method. Basically we obtain the Nakagami signals by taking the square root of correlated Gamma RVs. It is shown that the correlation coefficient between Gaussian is derived from Gamma which is in turn obtained from the correlation coefficients of Nakagami RVs. For generating correlated Gamma RVs, we propose the Cholesky decomposition method to transform correlated Gamma RVs into weighted sum of independent Gamma RVs.

As the m Parameter is increasing from 2.18 to 4 the Nakagami Probability Density Function (PDF), f(Z) peak value is increasing. PDF indicates the probability of occurrence of the Nakagami random variable and since the quality of the received signal increases with increasing m parameter we find the pdf increasing.

The advantage of our proposed Decomposition Method is its capability of generating correlated Gamma RVs for any non-integer fading parameter and cross-correlation. It is versatile and also simpler to implement as it requires only the generation of independent Gamma RVs.

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