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Directivity Gain Improvement in Phase Array Antenna via a Novel L2-Norm Adaptive Beamforming Algorithm

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Abstract

The performance of phased array antenna radiation pattern can be enhanced either by employing the optimum logical weights to each segment of the array, or by optimizing the physical geometry of the array. Intensive works have been presented in the literature to optimize physical geometry of antenna array. However, because of complexities and high computational overheads; optimization of antenna weights (or adaptive beamforming) got relatively less attention. A severe issue of adaptive beamforming is the directivity gain. The paper proposes a new adaptive beamforming algorithm that is more effective in improving the directivity gain of phase array antenna in comparison with the existing adaptive beamformers. The algorithm shows further effectiveness when number of radiators is increased in the array. The comparative results are very promising.

***Keywords**: Phased array antenna, antenna geometry optimization, antenna array weighting, mobile and wireless communication, antenna radiation pattern, beamforming, adaptive antenna array*, adaptive beamforming

1. Introduction

Wireless technology is an essential requirement of today's communication specially to achieve high degree of mobility. The antenna and its radiation pattern are very important in any wireless system. The performance of phased array antenna can be enhanced either by assigning the optimum weight to each segment of the antenna, or by optimizing the geometry of an array. In the literature, researchers have presented intensive work to optimize the geometry of antenna array. However, since optimization of antenna array weights is a multi-constraint and complex problem, it got relatively less attention in the literature. The classical numerical computing techniques are not appropriate to handle these problems. However, with the ascent growth of soft computing approaches, maturity of evolutionary/heuristic algorithms, and advancement in adaptive filtering; optimization of antenna array weights is the principle focus these days. Moreover, researchers are also working on the hybridization of these approaches.

The idea of designing the phase array antenna using geometry optimization was first proposed in early 1960's. Initially, it was described that the antenna system performance could be enhanced by keeping constant segment weights and varying elements position [1], or by removing the grating lobes through components position in an array [2], or by selecting small segment approximation to synthesize the required array pattern [3]. Though the scenario apparently looks benefiting, however marginal improvement was

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reported in the system performance. In the same year, this area was further refined with the evolution of 'thinned array antenna'. It is the method of varying the geometry of an array to reach the desired performance [4, 5]. To niche down the computational cost, a new approach based on Gauss-quadrature was proposed [6]. The computational cost issue was well handled in this approach, but this was found to be constrained in handling directivity issue.

Subsequently, the directivity issue was further highlighted by various researchers. They have identified that a periodic array has better spatial-spectrum estimation ability then other geometry [7]. They have likewise recognized that steering vectors are for distinct signal directions [8]. Researchers have also suggested to employ evolutionary algorithms for antenna optimization in order to have higher directivity. A variety of research work presented in the literature employing numerous evolutionary algorithms on antenna and array geometry optimization. The evolutionary algorithms used are genetic algorithm (GA), particle swarm optimization (PSO), ant colony optimization (ACO) and others [9-15].

The principle focus of the above approaches is to devise adaptive beam-former by adjusting antenna design parameters logically. An adaptive beam-former can adjust its weight automatically according to the given criteria. However, beam-former's performance highly degrades in case of array imperfectionalities such as time delay error, steering direction error, multipath propagation effects, phase errors of the sensors and wave front distortions [16-20]. An optimal beamforming at either/both end of communication system results in enhanced received power, relatively lower bit error rate (BER) and higher signal to noise plus interference ratio (SNIR).

Primarily, researchers have proposed two major approaches for beamforming *i.e.* fixed beamforming and adaptive beamforming [21-23]. The fixed approach computes statistically optimum weight vectors for beamforming [24, 25]. The adaptive beamforming however, employ a variety of adaptive algorithms such normalized least mean square (NLMS), maximum likelihood estimation (MLE), auto-regressive (AR), super resolution beamforming technique, estimation of direction of arrival (DOA) based on multiple signal classification (MUSIC) algorithm, recursive least squares (RLS), constrained constant modulus (CCM) and linearly constrained constant modulus algorithm (LCCMA) *etc*. [26-34]. These approached are efficient to improve DOA, BER and system complexity. However, there is still a room for improvement to have low computational cost with improved directionality. This paper proposes a novel adaptive beamforming algorithm that improves the directivity of phase array antenna. The comparative results show that the proposed algorithm has outperformed to handle the antenna directivity issues as compared to existing adaptive beamforming algorithm.

2. Proposed Algorithm

The weight adaptation is the basic process in adaptive beamforming algorithms to be used in phase array antenna systems. However, very limited work have been done and investigated on the algorithms that have simultaneous weight update of both the transmitter and receiver ends.

Here we are proposing a novel algorithm namely *L2 norm constrained least mean square (L2-CLMS)*. This algorithm not only has better performance but at the same time it has higher gain factor for the beam pattern in the desired direction. It is the variant of constrained least mean square [35] and therefore its cost function is very similar to the constrained least mean square with the difference of penalty of L2–norm to a certain gain factor g. The proposed algorithm searches for the solution of the following constrained optimization problem:

$$min_w E[|e(n)|]^2 \ s.t \begin{cases} \boldsymbol{C}^H \boldsymbol{w}(n) = \boldsymbol{z} \\ ||\boldsymbol{w}(n)||^2 = g \end{cases} \tag{1}$$

where $\boldsymbol{C}$ is a constraint matrix and $\boldsymbol{z}$ is the respective constraint vector having number of constraints elements. g represents the gain factor. The Lagrange multipliers are introduced in the objective function as follow:

$$\xi_{\boldsymbol{w}} = E[|e(n)|^2] + \boldsymbol{\lambda}_1^H(\boldsymbol{C}^H\boldsymbol{w}(n) - \boldsymbol{z}) + \lambda_2\left(\left||\boldsymbol{w}(n)|\right|^2 - g\right) \quad (2)$$

For calculating the derivative of equation (2), we go term by term. First we are going to calculate the derivative of the first term on the right side of equation (2). Using instantaneous estimation, we get,

$$\nabla_{\boldsymbol{w}}E[|e(n)|^2] = -2e^*(n)\boldsymbol{x}(n) \quad (3)$$

Now the derivative of the second term is,

$$\nabla_{\boldsymbol{w}}\boldsymbol{\lambda}_1^H(\boldsymbol{C}^H\boldsymbol{w} - \boldsymbol{z}) \quad = \boldsymbol{C}\boldsymbol{\lambda}_{\mathbf{1}} \quad (4)$$

The derivative of last term is,

$$\nabla_{\boldsymbol{w}}\lambda_2\left(\left||\boldsymbol{w}(n)|\right|^2 - g\right) = \lambda_2\boldsymbol{w}(n) \quad (5)$$

Hence the derivative of equation (2) can be computed by substituting all the derivatives of equations (3), (4) and (5), we get

$$\nabla_{\boldsymbol{w}}\xi_{\boldsymbol{w}} = -2e^*(n)\boldsymbol{x}(n) + \boldsymbol{C}\boldsymbol{\lambda}_{\mathbf{1}} + \lambda_2\boldsymbol{w}(n) \quad (6)$$

The weight-update equation of L2 norm will be

$$\boldsymbol{w}(n+1) = \left(1 - \frac{1}{2}\mu\lambda_2\right)\boldsymbol{w}(n) + \mu e^*(n)\boldsymbol{x}(n) - \frac{1}{2}\mu\boldsymbol{C}\boldsymbol{\lambda}_1 \quad (7)$$

Re-arranging equation (7) after multiplying it by $\boldsymbol{C}^H$, we obtain

$$\boldsymbol{\lambda}_1 = \frac{2}{\mu}\left(1 - \frac{1}{2}\mu\lambda_2\right)(\boldsymbol{C}^H\boldsymbol{C})^{-1}\boldsymbol{C}^H\boldsymbol{w}(n) + 2e^*(n)(\boldsymbol{C}^H\boldsymbol{C})^{-1}\boldsymbol{C}^H\boldsymbol{x}(n) - \frac{2}{\mu}(\boldsymbol{C}^H\boldsymbol{C})^{-1}\boldsymbol{C}^H\boldsymbol{w}\,(\,+1) \quad (8)$$

Let $(\boldsymbol{C}^H\boldsymbol{C})^{-1} = \boldsymbol{G}$, $\boldsymbol{C}^H\boldsymbol{w}(n+1) = \boldsymbol{z}$ and $(\boldsymbol{C}^H\boldsymbol{C})^{-1}\boldsymbol{C}^H = \boldsymbol{H}$, then equation (8) becomes

$$\boldsymbol{\lambda}_1 = \left(\frac{2}{\mu} - \lambda_2\right)\boldsymbol{H}\boldsymbol{w}(n) + 2e^*(n)\boldsymbol{H}\boldsymbol{x}(n) - \frac{2}{\mu}\boldsymbol{G}\boldsymbol{z} \quad (9)$$

Now to find the value of λ_2, multiplying both sides of equation (7) by $\boldsymbol{w}^H(n)$, we obtain

$$\boldsymbol{w}^H(n)\boldsymbol{w}(n+1) = \left(1 - \frac{1}{2}\mu\lambda_2\right)\boldsymbol{w}^H(n)\boldsymbol{w}(n) + \mu e^*(n)\boldsymbol{w}^H(n)\boldsymbol{x}(n) - \frac{1}{2}\mu\boldsymbol{w}^H(n)\boldsymbol{C}\boldsymbol{\lambda}_1 \quad (10)$$

In order to eliminate the unknown *priori coefficient vector* $\boldsymbol{w}(n+1)$ on the left side of equation (10), we propose an approximation as $\boldsymbol{w}^H(n)\boldsymbol{w}(n+1) \approx g$. This approximation is based on an assumed convergence of the algorithm, when $\boldsymbol{w}(n+1) \approx \boldsymbol{w}(n)$. Also, we define $g(n) = \boldsymbol{w}^H(n)\boldsymbol{w}(n)$ as the L2 norm at instant n; therefore

$$g = \left(1 - \frac{1}{2}\mu\lambda_2\right)g(n) + \mu e^*(n)\boldsymbol{w}^H(n)\boldsymbol{x}(n) - \frac{1}{2}\mu\boldsymbol{w}^H(n)\boldsymbol{C}\boldsymbol{\lambda}_1$$

$$= g(n) - \frac{1}{2}\mu\lambda_2 g(n) + \mu e^*(n)\boldsymbol{w}^H(n)\boldsymbol{x}(n) - \frac{1}{2}\mu\boldsymbol{w}^H(n)\boldsymbol{C}\boldsymbol{\lambda}_1$$

$$g - g(n) = -\frac{1}{2}\mu\lambda_2 g(n) + \mu e^*(n)\boldsymbol{w}^H(n)\boldsymbol{x}(n) - \frac{1}{2}\mu\boldsymbol{w}^H(n)\boldsymbol{C}\boldsymbol{\lambda}_1$$

$$e_{L_2} = -\frac{1}{2}\mu\lambda_2 g(n) + \mu e^*(n)\boldsymbol{w}^H(n)\boldsymbol{x}(n) - \frac{1}{2}\mu\boldsymbol{w}^H(n)\boldsymbol{C}\boldsymbol{\lambda}_1$$

$$\lambda_2 = \frac{2}{g(n)}e^*(n)\boldsymbol{w}^H(n)\boldsymbol{x}(n) - \frac{2}{\mu g(n)}e_{L_2} - \frac{1}{g(n)}\boldsymbol{w}^H(n)\boldsymbol{C}\boldsymbol{\lambda}_1 \tag{11}$$

Assuming following variables for the terms in equations (9) and (11),

$$A = 2e^*(n)\boldsymbol{H}\boldsymbol{x}(n) - \frac{2}{\mu}\boldsymbol{G}\boldsymbol{z} + \frac{2\boldsymbol{H}\boldsymbol{w}(n)}{\mu} \tag{12}$$

$$B = \boldsymbol{H}\boldsymbol{w}(n) \tag{13}$$

$$C = \frac{2}{g(n)}e^*(n)w^H(n)x(n) - \frac{2}{\mu g(n)}e_{L_2} \tag{14}$$

$$D = \frac{1}{g(n)}\boldsymbol{w}^H(n)\boldsymbol{C} \tag{15}$$

Now equation (9) and (11) can be written as,

$$\boldsymbol{\lambda}_1 = A - \lambda_2 B \tag{16}$$

$$\lambda_2 = C - D\boldsymbol{\lambda}_1 \tag{17}$$

Simultaneously solving equations (16) and (17), we get,

$$\boldsymbol{\lambda}_1 = \frac{A - ADB - CB + DAB}{1 - DB} \tag{18}$$

$$\lambda_2 = \frac{C - DA}{1 - DB} \tag{19}$$

Now let $(1 - DB) = 1 - \frac{\boldsymbol{w}^H(n)\boldsymbol{C}\boldsymbol{H}\boldsymbol{w}(n)}{g(n)} = Q(n)$. By substituting the values of A, B, C, D from equations (12) to (15) into equation (18) and (19), we obtain

$$\begin{aligned}\boldsymbol{\lambda}_1 &= \frac{2}{Q(n)}e^*(n)\boldsymbol{H}\boldsymbol{x}(n) - \frac{2}{\mu\, Q(n)}\boldsymbol{G}\boldsymbol{z} + \frac{2}{\mu\, Q(n)}\boldsymbol{H}\boldsymbol{w}(n) \\ &- \frac{2}{g(n)Q(n)}e^*(n)\,\boldsymbol{H}\,\boldsymbol{w}^H(n)\boldsymbol{C}\boldsymbol{H}\boldsymbol{w}(n) + \frac{2}{\mu g(n)Q(n)}\boldsymbol{G}\boldsymbol{z}\boldsymbol{C}\boldsymbol{H}\boldsymbol{w}(n) \\ &- \frac{2}{\mu g(n)Q(n)}\boldsymbol{H}\boldsymbol{w}(n)\boldsymbol{w}^H(n)\boldsymbol{C}\boldsymbol{H}\boldsymbol{w}(n) - \frac{2}{g(n)Q(n)}e^*(n)\boldsymbol{w}^H(n)\boldsymbol{x}(n)\boldsymbol{H}\boldsymbol{w}(n) \\ &+ \frac{2}{\mu g(n)Q(n)}e_{L_2}\boldsymbol{H}\boldsymbol{w}(n) + \frac{2}{g(n)Q(n)}e^*(n)\boldsymbol{w}^H(n)\boldsymbol{C}\boldsymbol{H}\,\boldsymbol{x}(n)\boldsymbol{H}\boldsymbol{w}(n) \\ &- \frac{2}{\mu g(n)Q(n)}\boldsymbol{w}^H(n)\boldsymbol{C}\boldsymbol{G}\boldsymbol{z}\boldsymbol{H}\boldsymbol{w}(n) \\ &+ \frac{2}{g(n)Q(n)}e^*(n)\boldsymbol{w}^H(n)\boldsymbol{C}\boldsymbol{H}\,\boldsymbol{w}(n)\boldsymbol{H}\boldsymbol{w}(n)\end{aligned} \tag{20}$$

$$\lambda_2 = \frac{2}{g(n)Q(n)} e^*(n)\boldsymbol{w}^H(n)\boldsymbol{x}(n) - \frac{2}{\mu g(n)Q(n)} e_{L_2} - \frac{2}{g(n)Q(n)} e^*(n)\boldsymbol{w}^H(n)\boldsymbol{CHx}(n) + \frac{2}{\mu g(n)Q(n)} \boldsymbol{w}^H(n)\boldsymbol{CGz} - \frac{2}{\mu g(n)Q(n)} \boldsymbol{w}^H(n)\boldsymbol{CHw}(n) \quad (21)$$

Now substituting the values of $\boldsymbol{\lambda_1}$and λ_2 from equations (20) and (21) into equation (7), we ge

$$\boldsymbol{w}(n+1) = \boldsymbol{w}(n) - \frac{\mu}{2}\left(\frac{2}{g(n)Q(n)} e^*(n)\boldsymbol{w}^H(n)\boldsymbol{x}(n) - \frac{2}{\mu g(n)Q(n)} e_{L_2} - \frac{2}{g(n)Q(n)} e^*(n)\boldsymbol{w}^H(n)\boldsymbol{CHx}(n) + \frac{2}{\mu g(n)Q(n)} \boldsymbol{w}^H(n)\boldsymbol{CGz} - \frac{2}{\mu g(n)Q(n)} \boldsymbol{w}^H(n)\boldsymbol{CHw}(n)\right)\boldsymbol{w}(n) + \mu e^*(n)\boldsymbol{x}(n) - \frac{\mu}{2}\boldsymbol{C}\left(\frac{2}{Q(n)} e^*(n)\boldsymbol{Hx}(n) - \frac{2}{\mu\, Q(n)}\boldsymbol{Gz} + \frac{2}{\mu\, Q(n)}\boldsymbol{Hw}(n) - \frac{2}{g(n)Q(n)} e^*(n)\,\boldsymbol{H}\,\boldsymbol{w}^H(n)\boldsymbol{CHw}(n) + \frac{2}{\mu g(n)Q(n)}\boldsymbol{GzCHw}(n) - \frac{2}{\mu g(n)Q(n)}\boldsymbol{Hw}(n)\boldsymbol{w}^H(n)\boldsymbol{CHw}(n) - \frac{2}{g(n)Q(n)} e^*(n)\boldsymbol{w}^H(n)\boldsymbol{x}(n)\boldsymbol{Hw}(n) + \frac{2}{\mu g(n)Q(n)} e_{L_2}\boldsymbol{Hw}(n) + \frac{2}{g(n)Q(n)} e^*(n)\boldsymbol{w}^H(n)\boldsymbol{CH}\,\boldsymbol{x}(n)\boldsymbol{Hw}(n) - \frac{2}{\mu g(n)Q(n)}\boldsymbol{w}^H(n)\boldsymbol{CGzHw}(n) + \frac{2}{g(n)Q(n)} e^*(n)\boldsymbol{w}^H(n)\boldsymbol{CH}\,\boldsymbol{w}(n)\boldsymbol{Hw}(n)\right) \quad (22)$$

The final weight update equation for L2-CLMS is therefore

$$\boldsymbol{w}(n+1) = \boldsymbol{w}(n) + \mu e^*(n)\boldsymbol{x}(n) - \frac{\mu}{g(n)Q(n)} e^*(n)\boldsymbol{w}^H(n)\boldsymbol{x}(n)\boldsymbol{w}(n) + \frac{1}{g(n)Q(n)} e_{L_2}\boldsymbol{w}(n) + \frac{\mu}{g(n)Q(n)} e^*(n)\boldsymbol{w}^H(n)\boldsymbol{CHx}(n)\boldsymbol{w}(n) - \frac{1}{g(n)Q(n)}\boldsymbol{w}^H(n)\boldsymbol{CGzw}(n) + \frac{1}{g(n)Q(n)}\boldsymbol{w}^H(n)\boldsymbol{CHw}(n)\boldsymbol{w}(n) - \frac{\mu}{Q(n)} e^*(n)\boldsymbol{CHx}(n) + \frac{1}{Q(n)}\boldsymbol{CGz} + \frac{1}{Q(n)}\boldsymbol{CHw}(n) + \frac{\mu}{g(n)Q(n)} e^*(n)\boldsymbol{C}\,\boldsymbol{H}\,(n)\boldsymbol{w}^H(n)\boldsymbol{CHw}(n) - \frac{1}{g(n)Q(n)}\boldsymbol{CGzw}^H(n)\boldsymbol{CHw}(n) + \frac{1}{g(n)Q(n)}\boldsymbol{CHw}(n)\boldsymbol{w}^H(n)\boldsymbol{CHw}(n) + \frac{\mu}{g(n)Q(n)} e^*(n)\boldsymbol{Cw}^H(n)\boldsymbol{x}(n)\boldsymbol{Hw}(n) - \frac{1}{g(n)Q(n)} e_{L_2}\boldsymbol{CHw}(n) - \frac{\mu}{g(n)Q(n)} e^*(n)\boldsymbol{Cw}^H(n)\boldsymbol{CH}\,\boldsymbol{x}(n)\boldsymbol{Hw}(n) + \frac{1}{g(n)Q(n)}\boldsymbol{Cw}^H(n)\boldsymbol{CGzHw}(n) - \frac{\mu}{g(n)Q(n)} e^*(n)\boldsymbol{Cw}^H(n)\boldsymbol{CH}\,\boldsymbol{w}(n)\boldsymbol{Hw}(n) \quad (23)$$

3. Simulation Results

The simulation results can be obtained with various sets of system parameters. The proposed L2-CLMS algorithm is found to be outclass over the existing same family algorithm L1-CLMS [35] for all sets of system parameters. The results of at-least two sets of parameters are discussed here.

Figure 1(a-d) shows the beam pattern of the antenna array having 25 elements with λ/2 spacing between them. The patterns are plotted for unity gain factor for the desired directions of 40°, 90°, 120° and 150°. These patterns are obtained by employing the existing adaptive beamforming algorithm L1-CLMS and the proposed adaptive beamforming algorithm L2-CLMS.

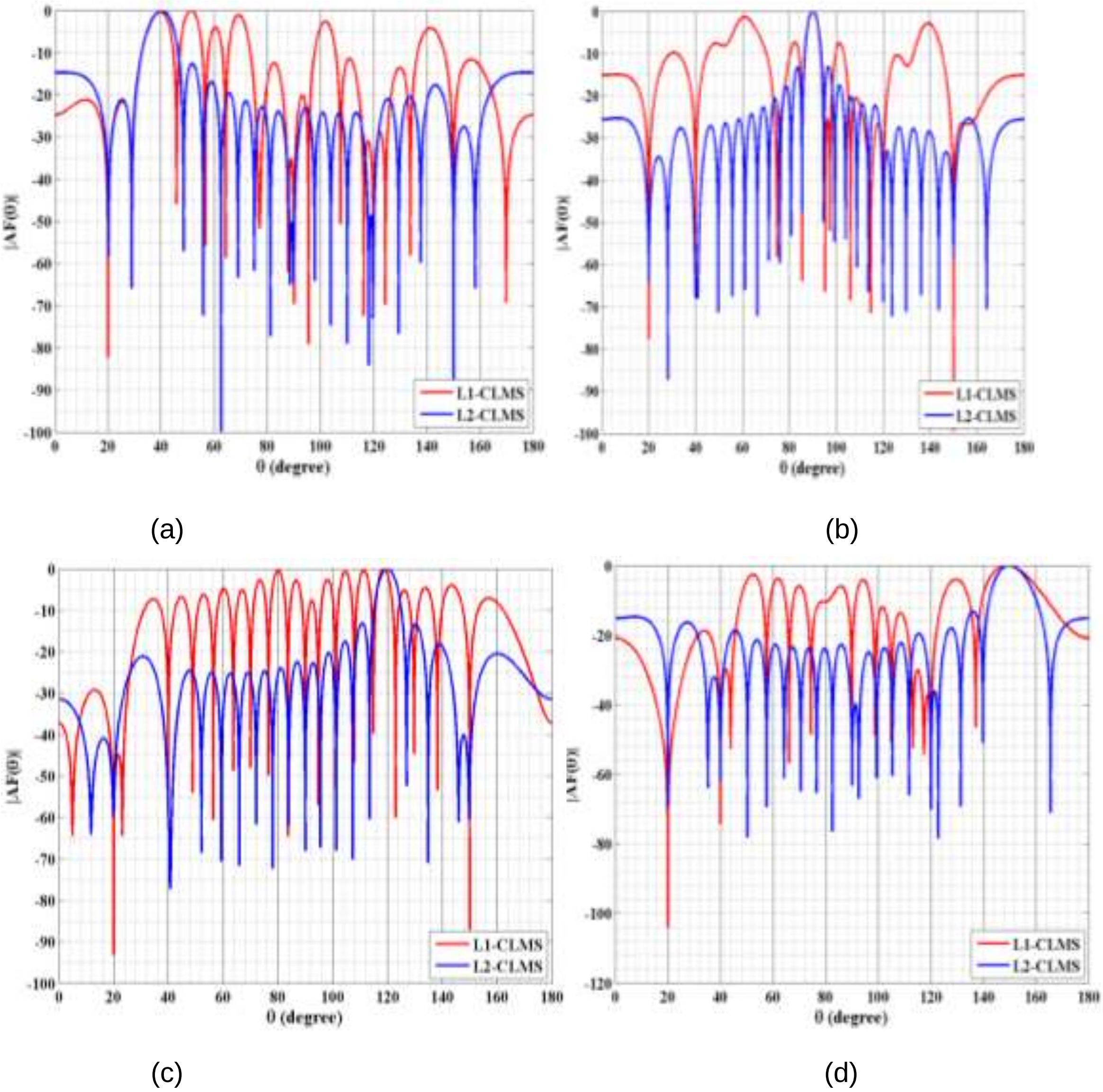


Figure 1. Beam Pattern of Unity Gain, λ/2 spaced, 25 Elements Phase Antenna Array for existing L1-CLMS and proposed L2-CLMS Algorithms at the Desired Directions of (a) 40° (b) 90° (c) 120° and (d) 150°

It is evident from the plots of Figure 1(a-d) that the proposed algorithm has higher value of directivity in the desired direction (main lobe) as compared to the existing algorithm. Moreover, the beam patterns of L2-CLMS for all the undesired directions (side lobes) have lower values of directivity as compared to the existing algorithm. This clearly proves the better performance of L2-CLMS over the existing L1-CLMS.

Figure 2(a-d) shows the polar plot of the antenna array with same specifications as above. Again, the plots are for unity gain factor with the desired directions of 40°, 90°, 120° and 150° by employing the existing adaptive beamforming algorithm L1-CLMS and the proposed adaptive beamforming algorithm L2-CLMS.

The polar plots clearly indicate the supremacy of proposed algorithm over the existing algorithm. The major lobes achieved from both the algorithms are almost same with respect to gain and directionality. However, the proposed algorithm gives smallest minor lobes or null lobes that help saving maximum radiating energy in the undesired directions. In contrast, the existing algorithm spreads most of the radiating energy in the undesired directions through minor lobes.

Figure 3(a) shows the beam pattern for 100 equally spaced antenna elements in an array for the desired direction of 90°. In this case, the performance of L2-CLMS is further improved. It gives the unity gain in the desired direction only. Gains in all the undesired directions are negligible.

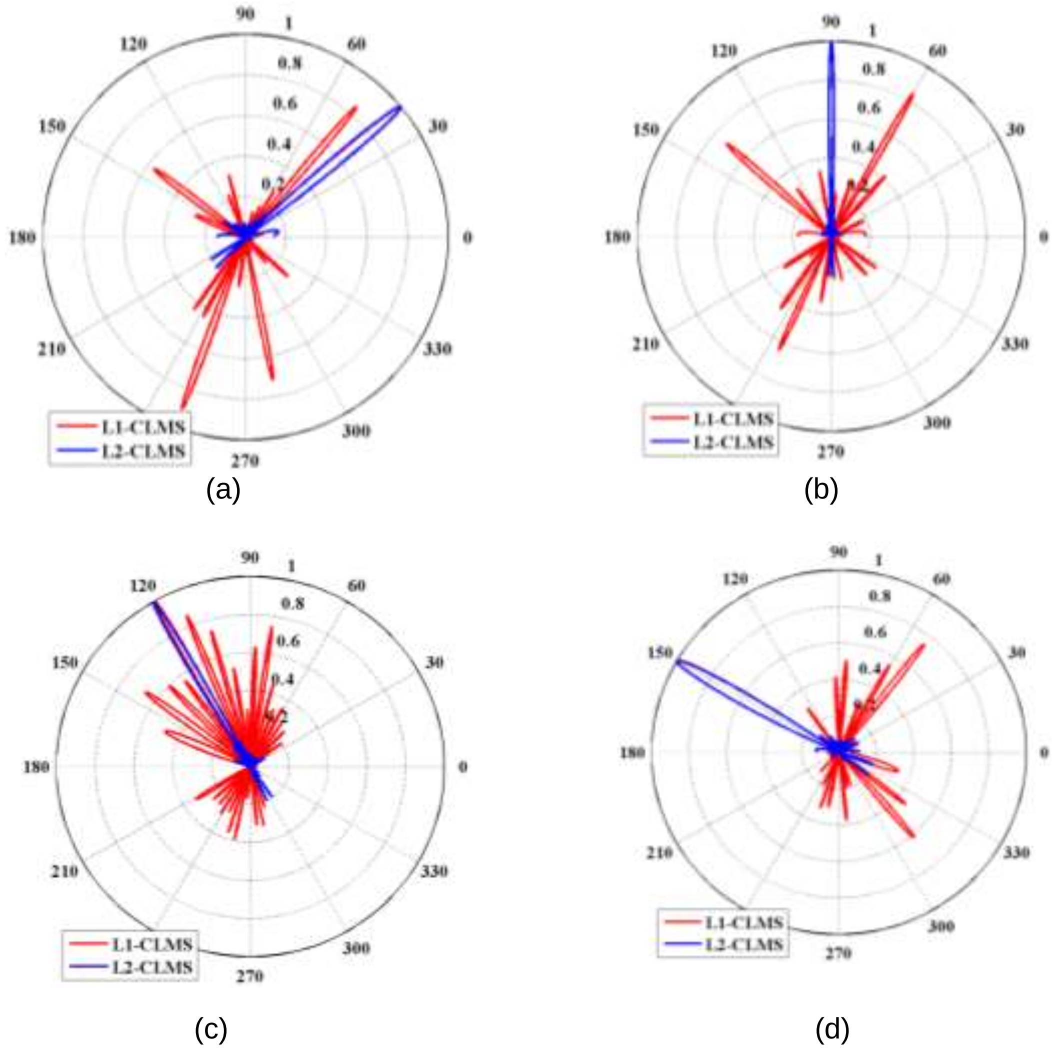
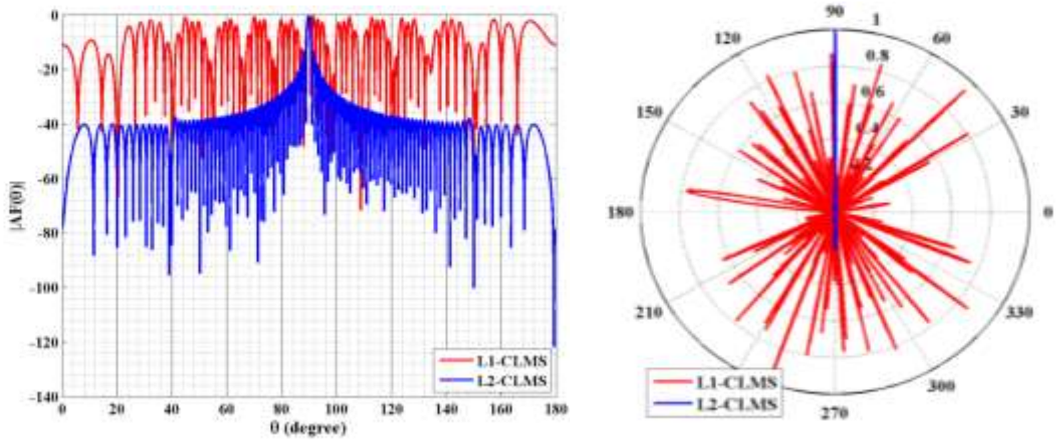


Figure 2. Polar Plot of Unity Gain, λ/2 spaced, 25 Elements Phase Antenna Array for existing L1-CLMS and proposed L2-CLMS Algorithms at the Desired Directions of (a) 40° (b) 90° (c) 120° and (d) 150°



(a) **(b)**

Figure 3. Radiation Characteristics of Unity Gain, λ/2 spaced, 100 Elements Phase Antenna Array for existing L1-CLMS and Proposed L2-CLMS Algorithms at the Desired Direction of 90° (a) Beam Pattern (b) Polar Plot

Unlike the proposed algorithm, the existing algorithm gives almost same gain factor in all the directions whether desired or undesired. It not only shows the superiority of proposed algorithm but at the same time it is evident that the performance can further be improved by increasing the number of elements in an array.

Figure 3(b) shows the polar plot of the antenna array with 100 elements and unity gain for the desired direction of 90°. Again, the major lobes achieved from both the algorithms are almost same with respect to gain and directionality. However, the proposed algorithm gives almost no minor lobe. It helps to conserve maximum radiating energy in the undesired directions. On the other hand, the existing algorithm spreads most of the radiating energy in the undesired directions through minor lobes.

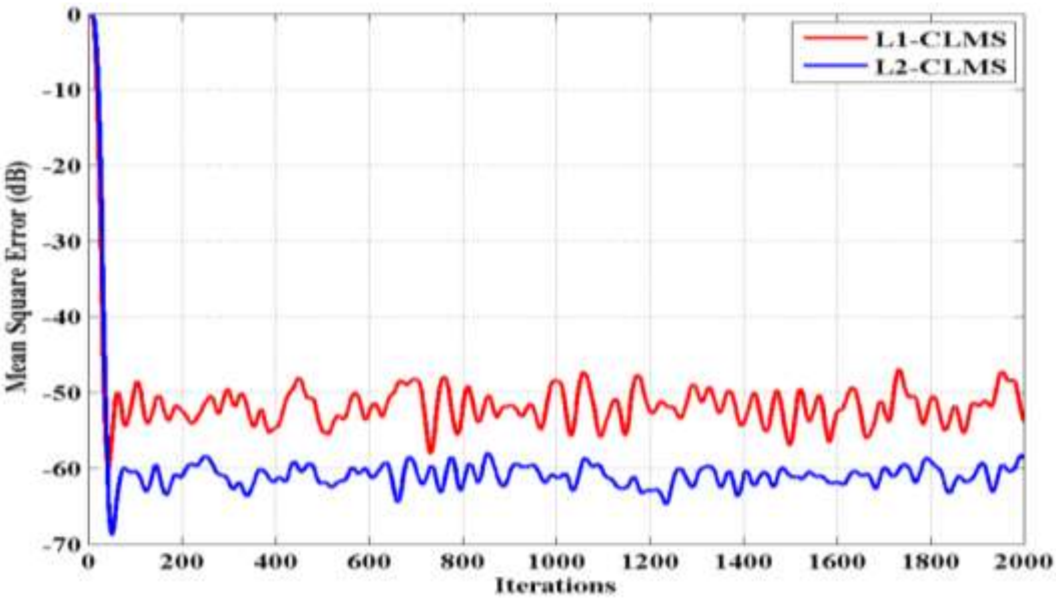


Figure 4. Mean Square Error of Adaptation from Existing L1-CLMS and proposed L2-CLMS

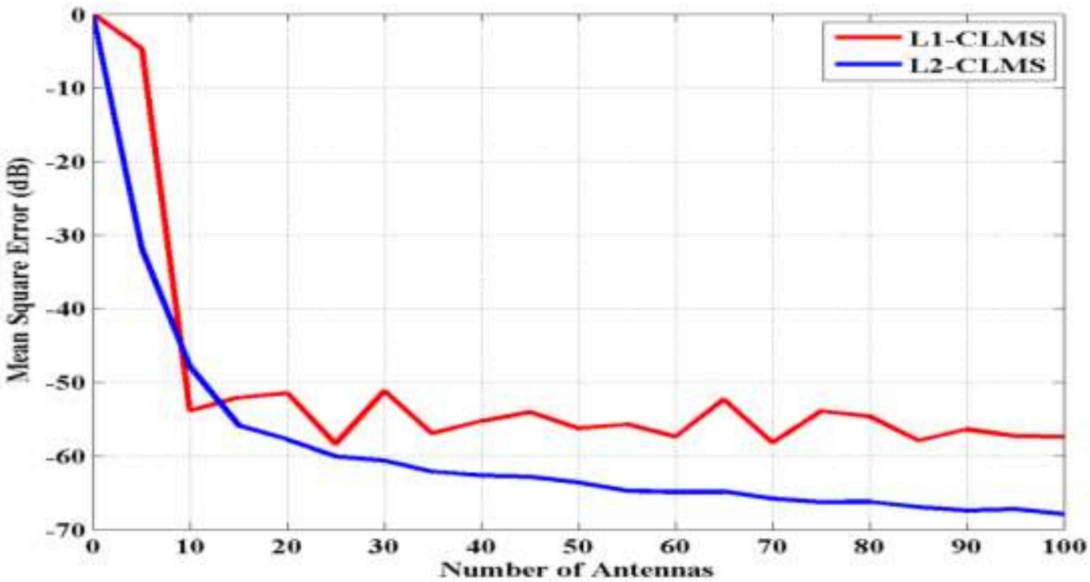


Figure 5. Mean Square Error of Adaptation for Variable Number of Array Elements from existing L1-CLMS and proposed L2-CLMS

The mean square errors (MSE) of adaptive beamforming by employing existing L1-CLMS and proposed L2-CLMS are shown in Figure 4. It is evident that MSE of L2-CLMS is the lowest as compared to the MSE of existing algorithm. This too confirms the better performance of L2-CLMS over the existing L1-CLMS algorithm.

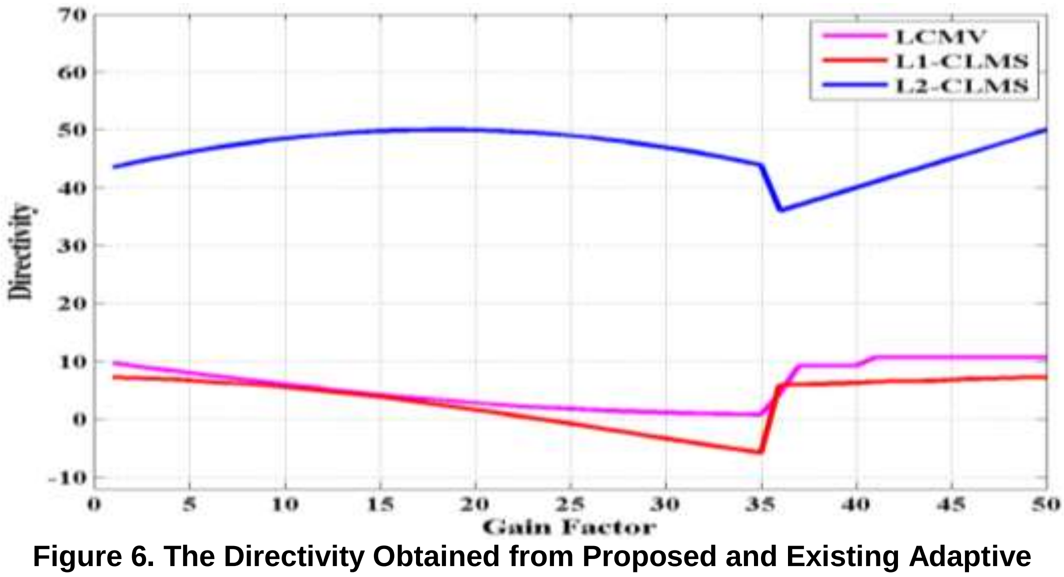


Figure 6. The Directivity Obtained from Proposed and Existing Adaptive Beamforming Algorithms for Variable Gain Factor

Figure 5 confirms that the performance of proposed L2-CLMS algorithm can further be improved with the increase in number of antenna array elements.

It is evident from Figure 6 that for the same value of gain factor, the proposed algorithm gives highest value of directivity. This is due to the conservation of radiation energy in the undesired directions as discussed in the explanation of Figure 2(a-d) and Figure 3(b). The proposed algorithm, in Figure 6; is additionally compared with another existing algorithm *linearly constrained minimum variance (LCMV)*.

Conclusion

A novel adaptive beamforming algorithm *L2 norm constrained least mean square (L2-CLMS)* has been proposed. It is the variant of constrained least mean square and therefore its cost function is very similar to it with the difference of penalty of L2-norm to a certain gain. The proposed algorithm is far more appropriate to achieve maximum directivity of phase array antenna with variable gain factor. Moreover, the proposed algorithm helps conserving the radiating energy in all the undesired directions. Additionally, the performance of proposed algorithm can further be increased by increasing the number of array elements. The comparative analysis of existing L1-CLMS and proposed L2-CLMS shows the supremacy of L2 norm constrained least mean square adaptive beamforming algorithm.

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