

## Reliability Evaluation of Assembly Manufacturing System based on Stochastic Flow Network and Data Modeling

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### Abstract

*In this paper, the author analyzes the reliability of the assembly manufacturing system by using stochastic flow network, in order to evaluate the capability of the assembly system and satisfy the order. The graph based model is effective, but the research object is the general as processing equipment based serial manufacturing system, ignoring an important and special system in manufacturing system, namely the assembly manufacturing system. It is characterized by that the working time is random and the cycle time is the main evaluation index and the non-serial structure. The author establishes the reliability evaluation model of assembly manufacturing system, and calculate the actual cycle of the assembly manufacturing system to meet the demand of the order, that is, the system reliability. The results provide effective help for the production planning and decision making of manufacturing enterprises.*

**Keywords:** *Assembly manufacturing system, Reliability evaluation, Stochastic flow network, Minimum capability vector*

### 1. Introduction

The research of manufacturing system can be divided into two kinds of basic problems: the design and evaluation of manufacturing system, the operation management of manufacturing system [1]. The research scope of this paper is the reliability evaluation of manufacturing system. From the point of view of decision-making and production management, the reliability of manufacturing system is an important performance measure [4, 14, 15]. Manufacturing system reliability is used to evaluate the ability of manufacturing system to complete its expected function, which is usually described in the form of probability. Borty [2] and other manufacturing system reliability (MSR) is defined as the probability of completion of the parts of the production order within a given date. The reliability on the flexible manufacturing system (FMSR) evaluation has been studied and put forward the reliability of flexible manufacturing system (FMSR) can be understood as the system is able to complete all the requirements given steady state probability [3]. Tan Min on the computer integrated manufacturing system (CIMS) reliability was studied. The stable availability of system reliability as the evaluation index of the system, namely a production line of products output probability [5-6]. Lin defines the manufacturing system reliability as the probability of the manufacturing system to complete the customer's demand [7-8], the system reliability is used to evaluate the production capacity and the capacity of the manufacturing system. To reduce by unreliable machine caused by serial production line production efficiency, the methods of Li and Meerkov using Bernoulli statistics to determine the reliability of the machine, studied the change of production system performance, the delivery of manufacturing system was studied, with the introduction of buffer in manufacturing system [9-10].

According to the research literature, scholars on the concept of the definition of manufacturing system reliability can be divided into two categories: one is based on the traditional definition of reliability, the manufacturing system at time  $t$  the normal work of the steady-state probability; the other is from the point of view of the function of manufacturing system, the manufacturing system to meet customer needs or the corresponding system function. In this paper, the definition of the concept of manufacturing system reliability is based on the latter, that is, from the point of view of the function of the manufacturing system to define its reliability. The function of manufacturing system is mainly reflected in the quality, productivity and delivery time, which is different from the traditional product reliability research.

For assembly manufacturing system of performance evaluation, previous studies have focused on productivity evaluation and line balancing problems. Gershwin, studied the performance of assembly line with buffer, the approximate decomposition method is used to decompose the assembly line into two machines, in the middle of which is a buffer. This simple model is used to evaluate the system buffer level and productivity, and the relationship between the two [11]. Nils Boysen, *et al.*, classify the problem of assembly line balancing and give a detailed account of how to choose the appropriate line-balance model [12, 13]. While the literature from the perspective of reliability or probability is rare. K. Agpak, *et al.*, established a chance-constrained programming model for the balance between traditional linear assembly lines and U-shaped assembly lines, and emphasized the stochastic of workstation capacity. The concept of reliability of assembly line system is proposed, and the reliability is defined as the product of the probability of each workstation on the assembly line to complete the assigned tasks within the takt time [21].

Assembly line is a typical manufacturing system, its connotation relative to the general manufacturing system has certain reduction, it can be said that it is a part of the manufacturing system, but also different from other manufacturing systems. Generally speaking, the difference is mainly reflected as follows: the operation time of assembly line of each workstation has strong randomness, especially to assembly line workers to operate mainly; assembly line is often used as a performance index to the cycle, and the workstation ability can also be used to represent the length of operation, in the actual production process, statistics the data about time can be obtained easily, easy to make evaluation on the ability of the system; non serial feature structure with, namely the assembly line and assembly line.

The function of reliability in a manufacturing system, the research method in the past, is to use more effective method mainly has the following three types: Based on mathematical analysis method, based on Markov method, graph theory, computer network simulation method. For the performance evaluation of manufacturing system, network analysis has become a mature tool manufacturing system as an input and output system, it is easy to convert it into a manufacturing network [6, 17, 18], the maximum flow is the pursuit of the goal. In a manufacturing network, the side can be seen as a machine or a station, and the node can be seen as a inspection station at the back of the station. Due to the failure or part of the equipment or workers failure, product rework, leading to the workstation system provided in the capacity of a random state or multi state, the overall performance of the system is in a random state. Therefore, this multi-state manufacturing network, we can call the stochastic flow network. Due to the existence of edge failure and rework path, may lead to network flow is non-conservation, therefore, the manufacturing system is not a typical network. In order to evaluate the ability of manufacturing network to meet customer needs, Lin calculates the production line reliability with rework based on the minimum path / cut (MP, Minimal Path / Cut) method [7]. Ming J. Zuo calls the minimum path vector in system state  $d$  as  $d$ -MP and gives the RSDP (Recursive Sum of Disjoint Products) algorithm to find all  $d$ -MPs that meet the requirements [19].

Based on the theory of stochastic flow network, the paper first analyzes the assembly manufacturing system topology as the manufacturing network, describes the assembly path, and builds the reliability evaluation model of the assembly manufacturing system based on the time factor, and then analyzes the output whether to meet customer needs, to evaluate the reliability of manufacturing systems. The calculation process of reliability will determine the input amount of each workstation, which not only provides the decision support for the production manager, but also indicates the weak link in the system which the manager should pay attention to. In this paper, the reliability of assembly manufacturing system is studied to evaluate the capacity of manufacturing system to meet the order. Because of the stochastic capacity, pass rate and production cycle of the workstation, the evaluation process comprehensively considers the reliability of assembly manufacturing system in terms of quality, capacity and delivery.

## 2. Problem Description and Assumptions

Assembly manufacturing system is studied in this paper refers to the production line very stressed time constraints, different from other large fixed assembly lines, such as aircraft, enterprise decision makers tend to order the assembly line to produce qualified products in a certain amount within the stipulated time, if not timely enough to meet the order requirements, the system is not reliable. At the same time, the time element of the assembly manufacturing system is obvious, such as the processing time of the workstation and the cycle time of the assembly line, and the working states of the assembly workers is more polymorphic and stochastic than the machines.

For a certain product, assembly manufacturing system usually has a certain production rhythm, called cycle time, which is divided into three kinds in this paper: Design cycle time, Takt time, (actual)production cycle time. The design cycle time refers to the theoretical minimum cycle time that the production line can achieve; the Takt time is the cycle time according to changes in market demand or order to calculate the assembly line should be provided; the production cycle time is the actual production interval of the final output product of the assembly line. The working hours of each working station on the assembly line will be equal to the cycle time to the greatest extent, to ensure the balance of the entire production line. However, in the actual production process, due to the reliability of parts and assembly workers operating proficiency, every workstation assembly equipment timely supply situation, the real production of 1 units of qualified product time is not necessarily equal to the cycle time. In order to make the evaluation more intuitive, this paper from the angle of time to the definition of reliability of assembly manufacturing systems, designed according to the above mentioned cycle the minimum ideal cycle will be installed as a product assembly line assembly manufacturing network input I, output O system for actual production rhythm. This paper attempts to find the relationship between I and O, to determine the system in each of the station's input capacity, and then compared with the capacity of each workstation, and then come to the probability of the reliability of the value of R. A given distance producing cycle time  $\mu$ , the reliability for the assembly manufacturing system can be defined as  $R_{\mu} = P_r(O \leq \mu)$ .

However, in order to apply the stochastic flow network theory to calculate the reliability of the assembly system, using cycle time as the research objective is not good for modeling and calculation, because stochastic flow networks generally study the problem of maximum flow, and for the manufacturing system, the pursuit of the goal is to cycle time the smaller the better. Because productivity and takt-time are reciprocal, if the delivery cycle T orders as per unit time, then the delivery quantity of customer demand period T products D is productivity. Therefore, in this

paper, the theory of minimum cycle conversion of productivity as the input of the system, the actual productivity of the system as the output, and then the results are translated into the cycle expressed manufacturing system reliability evaluation value.

The research content of this paper includes the reliability evaluation of single product assembly manufacturing system and the evaluation of the reliability of the multi-product assembly manufacturing system. Based on the input  $I$ , output  $O$ , inspection station (with node), machine or workstation  $e$ , the success rate of  $P$  workstation (failure rate  $q$ ,  $p+q=1$ ), these six elements were the ability for each workstation is described and assumptions, establish the assembly manufacturing system stochastic flow network model. In the assembly manufacturing system with rework path, rework is often due to some parts of the assembly error, but only the components for re assembly, therefore, the first hypothesis only exists between two adjacent stations of the assembly line and rework action, this study only with a path of rework.

Other necessary assumptions:

1. each node (inspection station) is absolutely reliable;
2. the capacity of each edge (workstation) is a random variable according to a given probability distribution;
3. the capacity of different edges (workstations) are statistically independent;
4. each defective WIP is reworked at most one time by the same workstation.

In summary, the first key point of this research is: the assembly manufacturing system into the process of topology of stochastic flow network, how to deal with non-serial structure.

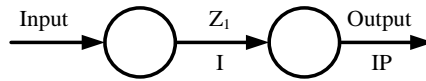
### **3. Construction of Stochastic Flow Network Model for Assembly Manufacturing System**

#### **3.1. Network diagram construction of General Assembly Manufacturing System**

The use of AOA (Activity-On-Arrow) network graph assembly manufacturing system into an assembly of the network graph, where each edge (arrow) is regarded as a machine and each node denotes an inspection station following the machine [12]. There is a need to explain that there are two kinds of work on the assembly line, assembly work and adjustment work. Michael, in his paper, called the workstations with assembly operations are performed as assembly station [20]. Michael said in the paper with assembly line workstation assembly station, and stations besides assembly station are called adjustment stations. The so-called assembly refers to the assembly process to assemble the components to complete the assembly process; and adjustment namely the workstation does not exist sub assembly line, do not need to add parts to the line of semi-finished products, only to adjust the line of semi-finished products, such as fastening bolts and adjustment. So the workstation will be corresponding to the two, the assembly station, this paper uses  $e_i$  ( $i=1,2,\dots n_1$ ); adjust the workstation, using  $s_m$  ( $m=1,2,\dots n_2$ ), in which  $n_1+n_2=n$ ,  $n$  for the assembly line of the total number of workstations. Before the network model of assembly manufacturing system is established, the construction process of the manufacturing network of the simple serial manufacturing system is explained.

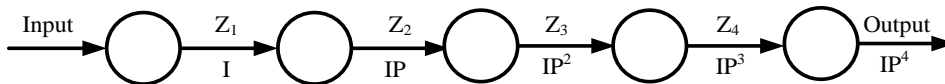
For the simplest manufacturing system, we only consider one workstation, as shown in Figure 1. According to the input amount  $I$  of workstation  $Z_1$ , since the success rate of the workstation itself is not 100%, the actual output will be  $I_p$ , that

output  $O=Ip$ ,  $P = 0 \sim 1$ . In this way, we find the relationship between I and O through the success rate P, once the I is given, then O is I divided by the P of the business. While  $O \geq D$ ,  $O=Ip \geq d$ , then  $I \geq d/p$ .



**Figure 1. Manufacturing network for a workstation**

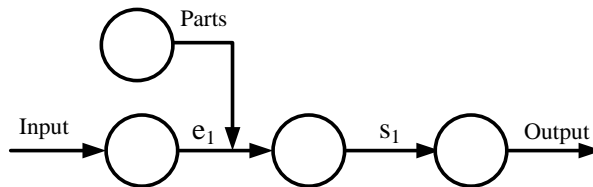
Further, we will expand the system into a number of workstations. Assume that a manufacturing network has 4 stations ( $n=4$ ) ( $Z_1, Z_2, Z_3, Z_4$ ), each workstation had the same success rate of P, as shown in Figure 2.



**Figure 2. Manufacturing network for four workstations**

Obviously, we can draw the following:

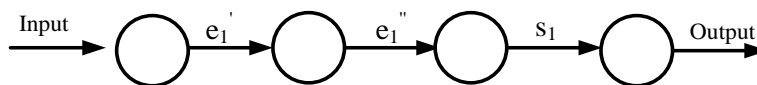
**Corollary 1:** in general, a serial manufacturing system ( $z_1, z_2, \dots, z_n$ ) for the production of unit product in delivery time within T, requires the input of  $I \geq d/p^n$ , which requires the first workstation must complete a product in the  $t=Tp^n/d$  time. The workstation  $Z_i$  ( $1 \leq i \leq n$ ) as input  $Ip^{(i-1)}$ , denoted as  $w_i=Ip^{(i-1)}$ , which is in order to meet the delivery requirements, must be in the  $t=Tp^{i-1}/d$  workstation completed. Further, for the assembly manufacturing system, modeled on the above ideas, we are still starting from the simplest case discussion, namely the existence of an assembly workstation  $e_1$ , and an adjustment of the workstation  $s_1$ . As shown in Figure 3.



**Figure 3. Assembly line manufacturing network**

**Corollary 2:** add  $e_i$  type assembly workstation can be transformed into two serial  $e_i'$  workstation with  $e_i''$ .

The  $e_i'$  workstation complete view and take part in the work, and  $e_i''$  workstation to complete the installation work.  $e_i'$  workstation must be tight before the  $e_i''$  workstation, because the actual sequence must be first after installation, so they actually the same workstation, so each other, between the two cannot be added other workstation. Therefore, we can convert into a serial form in Figure 4.



**Figure 4. The serial form of split workstation  $e_i$**

### 3.2. Assembly manufacturing system with rework path

The assembly workstation  $e_i$ ,  $e_i'$  attribute inherited the supply of spare parts, and assembly work,  $e_i'$  also has a success rate of two and the random property, its success rate refers to the qualified rate of parts,  $g$  said here, the failure rate of  $h$ ,  $g+h=1$ . The random capacity refers to the logistics department or the assembly line of sufficient supply of parts, spare parts supply probability in different time is different, such as the supply of 1 and an average of 30 seconds 40 seconds average supply probability of 1 is different, finally given in the form of probability distribution. The  $e_i''$  attribute inherit its assembly work, it is an adjustment workstation. So the definition of the success rate and the adjustment of the workstation  $s_m$  the same, it is defined as  $P$ , the failure rate is defined as  $q$ ,  $p+q=1$ . Its stochastic capability is the probability that the work station has completed the operation in different time.

When the properties of  $e_i$  and assembly workstation can be determined, to the completion of the assembly work to add parts, in selected parts immediately after the need to assemble the product,. If an assembly manufacturing network ( $n=4$ ) has 4 stations ( $Z_1, Z_2, Z_3, Z_4$ ),  $Z_1$  and  $Z_3$  respectively represent the workstation to add workstation  $e_1$  and  $e_2$ , and  $Z_2, Z_4$  respectively represent the adjustment of  $s_1$  and  $s_2$  workstation. Its assembly line network model is shown in Figure 5.

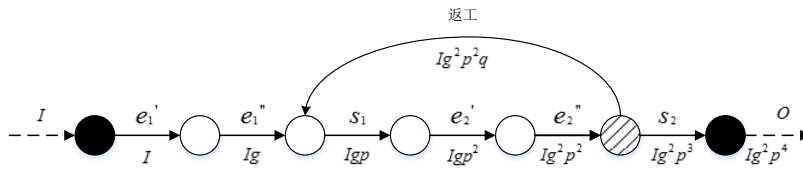


Figure 5. Assembly manufacturing network with rework path

But it is difficult to tell whether the input is the input of the normal processing path or the input of the rework path. The model of Figure 6 is decomposed into the normal processing route map and the rework route map, as shown in Figure 7.

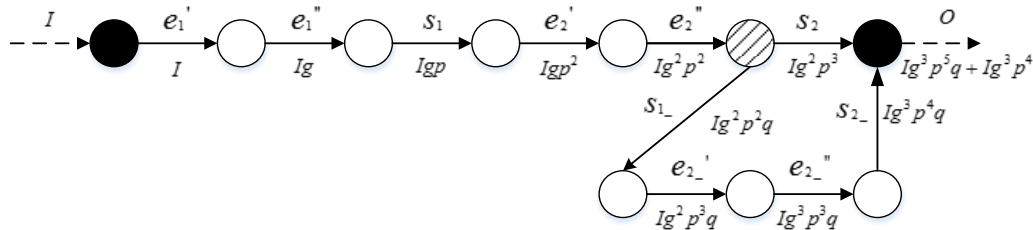
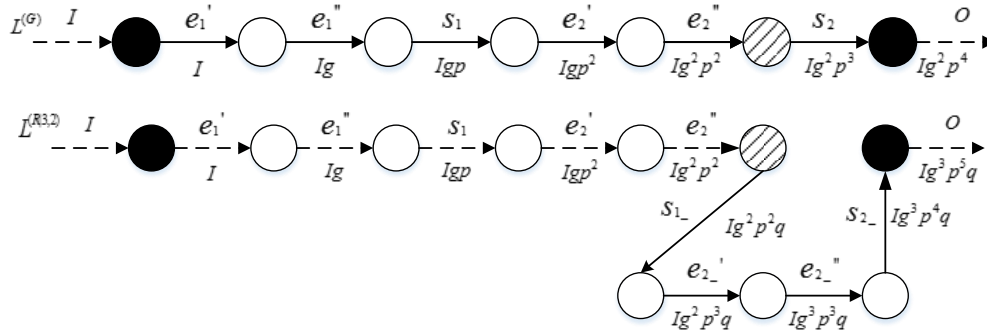


Figure 6. Another form of Figure 5

### 3.3. Single product assembly manufacturing system

Because of the conflicts between flow conservation principle rework path existence and stochastic flow network, at the same time in order to better analyze the assembly assembly path of the network graph, the assembly line in Figure 6 needs to be decomposed into normal assembly path  $L^{(G)}$  and  $L^{(R|r,r-k)}$ , need to rework products from the first  $r$  to adjust the output from the input  $k$  workstation, adjust the workstation before  $r$ . We use figure 6 to do further research, set the normal assembly line  $L^{(G)} = \{e_1', e_1'', s_1, e_2', e_2'', s_2\}$ , The rework route is  $L^{(R|3,3-1)} = \{e_1', e_1'', s_1, e_2', e_2'', s_{1-}, e_{2-}', e_{2-}'', s_{2-}\}$ , Here  $(R|3,3-1)$  indicates the need to rework the product from 3 ( $r=3$ ) to adjust the workstation output, from its first 1 ( $k=1$ ) to adjust the workstation ( $s_1$  workstation) input, we

assume that the workstation for assembly rework products. Therefore,  $L^{(R|3,2)} = \{s_1, e_2', e_2'', s_2\}$ , We use the dotted line to represent this workstation in the network breakdown diagram of the assembly line, the single product network model is shown in Figure 7.

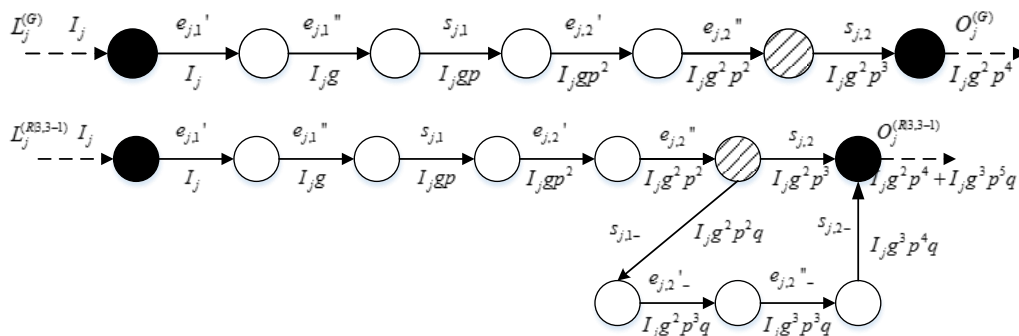


**Figure 7. Single product assembly manufacturing system**

So far, the assembly manufacturing system of stochastic flow network model has the general sense of the construction is completed, or this is a single product assembly network model, the opposite is the multi-product network model, the two main difference is that the assembly manufacturing system of multiple products, each workstation with respect to the random ability of different products are different, so if an order contains several products, need to be in the same assembly line, assembly line will evaluate whether can delivery schedule becomes relatively difficult.

**3.4. Stochastic flow network model for multi-product assembly**

Suppose there are  $\alpha$  ( $\alpha \geq 2$ ) for product assembly, the demand vector  $D = (d_1, d_2, \dots, d_a)$ , the assembly line for each product is decomposed into normal assembly path and rework path  $L_j^{(G)}, L_j^{(R|r,r-k)}$  ( $j = 1, 2, \dots, a$ ). From this, we can know that products have  $\alpha$  normal assembly path and  $\alpha$  rework path, a total of  $2\alpha$  path. The same with the example of Figure 6, the  $j$  normal assembly line  $L_j^{(G)} = \{e_{j,1}', e_{j,1}'', s_{j,1}, e_{j,2}', e_{j,2}'', s_{j,2}\}$ , rework  $L_j^{(R|3,3-1)} = \{s_{j,1}, e_{j,2}', e_{j,2}'', s_{j,2}\}$ . Multi product assembly line network diagram model is shown in Figure 8.



**Figure 8. Stochastic flow network model for multi-product assembly manufacturing system**

## 4. Reliability Model Construction

### 4.1. Determine the input amount

In order to better meet the needs of customers for delivery, and to determine the time to install the various workstations of the cycle. Originally planned to complete a product assembly to the theoretical minimum time as input, but because of the stochastic flow network is generally the maximum flow problem, and it is difficult to explain, is the time as input is difficult to elaborate it in the network transmission, so as to time is not in line with the random input stream network model. Therefore, we can only indirectly calculate the cycle time, the ratio of input here with delivery time T and each workstation that required a product of the time the workstation assembled, calculate the time equal time condition.

In order to meet the needs d of customers in the delivery time T, prior need to prepare enough raw materials. Here you can set the number of I raw materials can be assembled out of the number of O products. It must meet the conditions of  $O \geq d$ , in order to meet customer's requirement. Through the above stochastic flow network model, it is easy to deduce the output of the finished product  $O = (I g^{n_1} p^n + I g^{n_1+1} p^{n+k} q)$ . The formula  $I g^{n_1} p^n$  is the output from the normal assembly path  $L^{(G)}$  and  $I g^{n_1+1} p^{n+k} q$  is the output of the rework path  $L^{(R|r,r-k)}$ . In order to meet the demand of D, the assembly line of the 1 position of the input I can be expressed by the following formula:

$$I = d / (g^{n_1} p^n + g^{n_1+1} p^{n+k} q)$$

### 4.2. Input amount of each workstation

In order to determine the input of each workstation  $w_i(m)$ , the use of  $f_i^{(G)}$ ,  $f_i^{(R|r,r-k)}$  representing the normal assembly path  $L^{(G)}$  and rework path  $L^{(R|r,r-k)}$  of the input.

$$w_{i(m)} = f_i^{(G)} + f_i^{(R|r,r-k)}$$

For three different kinds of workstation  $e_i'$ ,  $e_i''$ ,  $s_m$ , the input formula is different, here by the stochastic flow network model are concluded respectively.

$$w_i(e_i') = f_i^{(G)}(e_i') + f_i^{(R|r,r-k)}(e_i') = I g^{i-1} p^{i-1+i_m} + I g^i p^{i-1+i_m+k} q$$

$$w_i(e_i'') = f_i^{(G)}(e_i'') + f_i^{(R|r,r-k)}(e_i'') = I g^i p^{i-1+i_m} + I g^{i+1} p^{i-1+i_m+k} q$$

$$w_m(s_m) = f_m^{(G)}(s_m) + f_m^{(R|r,r-k)}(s_m) = I g^{m_i} p^{m-1+m_i} + I g^{m_i+1} p^{m-1+m_i+k} q$$

### 4.3. Determine the input amount of the multi-product assembly line

The assembly line with respect to the random ability of different products are different, so the application method of stochastic flow network is difficult, this paper will do is simplified to a certain extent, in a variety of products in the product is a benchmark product, its weight is 1, the assembly line relative to other products of random ability are based on weight for example, the A product weight is 1, the B products weight is 2.

In order to meet the customer's demand vector D, we must meet the demand of T in the delivery time  $O_j^{(G)} + O_j^{(R|r,r-k)} \geq d_j$ . Therefore, you can determine the number of products in the input  $I_j$ .



$$I_j = d_j / (g^{n_1} p^n + g^{n_1+1} p^{n+k} q)$$

After the determination of the input of each workstation:

$$w_{i(m)} = \sum_{j=1}^a \alpha_{j,i(m)} (f_{j,i(m)})^{(G)} + b \beta_{j,i(m)} f_{j,i(m)}^{(R|r,r-k)}$$

Among them,  $\alpha_{j,i(m)}$  for the production of products for j unit and workstation i loss ability weight m; b is a factor, if the first path is b=2 workstation, rework, rework or b=1 (due to back the product, for the first time in the assembly process to remove the original components the assembly further down the assembly, consumed capacity increased to 2 times of the original);  $\beta_{j,i(m)}$  is also a factor, if the workstation exists rework input, while  $\beta_{j,i(m)=1}$  or  $\beta_{j,i(m)=0}$ .

$$w_i(e_i') = \sum_{j=1}^a \alpha_{j,i(m)} (I g^{i-1} p^{i-1+i_m} + b \beta_{j,i(m)} I g^i p^{i-1+i_m+k} q)$$

$$w_i(e_i') = \sum_{j=1}^a \alpha_{j,i(m)} (I g^i p^{i-1+i_m} + b \beta_{j,i(m)} I g^{i+1} p^{i-1+i_m+k} q)$$

$$w_m(s_m) = \sum_{j=1}^a \alpha_{j,i(m)} (I g^{m_i} p^{m-1+m_i} + b \beta_{j,i(m)} I g^{m_i+1} p^{m-1+m_i+k} q)$$

However, the input of each machine should meet the maximum capacity vector of each workstation, each workstation assembly unit product time  $t_{i(m)}$  should be greater than the minimum required per unit of product assembly workstation time  $M_{i(m)}$ , i.e.

$$t_{i(m)} = T / w_{i(m)} \geq M_{i(m)}$$

Where the use of  $x_{i(m)}$  to add or adjust the ability of each workstation  $e_i$  or  $s_m$  workstation, based on the assumption that the ability of each workstation is a random variable that can be  $x_{i(m)} \in [M_{i(m)}, +\infty)$ , in the state of the system  $X(x_1, x_2, \dots, x_{n_1}, x_{(1)}, x_{(2)}, \dots, x_{(n_2)})$ , in order to meet the  $t_{i(m)}$ , needs to have the following constraints:

$$x_{i(m)} \leq t_{i(m)} = T / w_{i(m)}$$

#### 4.4. System reliability evaluation

At a given delivery time T time, as well as the customer demand vector D, the system reliability  $R_d$  is the assembly system in the time T output is not less than the probability of D. Therefore, the reliability of the system is  $\Pr \{X|V(X) \leq W(D)\}$ , where  $V(X)$  is defined as in the X state, the average of each assembly system by the end of the assembly with a product of the time;  $W(D)$  is defined as in the X state, in the end of each assembly assembly system most of the time allowed for a finished product.

$$W(D) = T / (\sum_{j=1}^a d_j)$$

This formula means that each workstation should provide sufficient capacity, and ultimately can get enough output in the specified time O. But each meet  $V(X) \leq W(D)$  state of X one by one to find out is a waste of time, not wise, therefore, can try to find the set of  $\{X|V(X) \leq W(D) Y\}$  in minimum capacity vector  $Y(y_1, y_2, \dots, y_{n_1}, y_{(1)}, y_{(2)}, \dots, y_{(n_2)})$ . Finally, the formula of system reliability can be expressed as:

$$R_d = \Pr\{X|V(X) \leq W(D)\} = \Pr\{X|X \leq (y_1, y_2, \dots, y_{n_1}, y_{(1)}, y_{(2)}, \dots, y_{(n_2)})\}$$

## 5. Solution Steps and Algorithm Examples

### 5.1. Solving steps

Step 1: according to the formula to determine the original input  $I_j$ , and then according to the formula to determine the amount of each work station  $w_{i(m)}$ .

Step 2: find the minimum time to complete all the products.

$$O_{\min} = \max \{ w_{i(m)} M_{i(m)} \mid i, m \in L \}$$

Here L refers to the assembly model established in the previous paper.

Step 3: determine the minimum time required to complete the product is less than the delivery time, that is, to meet the next type

$$O_{\min} \leq T$$

Step 4: determine the minimum power vector Y

In the above identified input  $w_{i(m)}$  each workstation, the minimum time to find work each workstation to meet  $x_{i(m)(c)} \leq T/w_{i(m)} < x_{i(m)(c+1)}$ ,  $Y_i = x_{i(m)(c)}$ . This determines the minimum power vector  $Y(Y_1, Y_2, \dots, Y_{n_1}, Y(1), Y(2), \dots, Y(n_2))$ .

Step 5: solving the reliability of the assembly system.

### 5.2. Algorithm example

In order to explain the application of stochastic flow network theory in the reliability evaluation of assembly manufacturing system, this chapter will make use of the examples in the factory to solve the reliability evaluation method. Figure 8 the situation as an example. The assembly system requirements within a day of time (8 hours of work), the assembly of the  $d_1=100$  parts,  $d_2=150$  parts. Given the success rate of each workstation  $p=0.95$ , the required components to add the pass rate of  $g=0.99, \alpha_1=1, \alpha_2=1.2$ , given the random capacity of each station as shown in Table 1.

**Table 1. Random capacity of each workstation**

Serial	Ability	probability	Serial	Ability	probability
e1'	+∞	0.001	e2'	+∞	0.001
	92	0.002		100	0.002
	88	0.004		95	0.01
	84	0.012		90	0.012
	82	0.979		85	0.95
e1''	+∞	0.001	e2''	+∞	0.001
	90	0.006		100	0.004
	88	0.01		95	0.01
	86	0.012		90	0.015
	84	0.971		85	0.97
S1	+∞	0.001	S2	+∞	0.001
	90	0.004		104	0.004
	86	0.006		102	0.006
	82	0.01		100	0.009
	78	0.979		98	0.01
			96	0.012	
			94	0.958	

In this case, the workstation  $z_3$  (i.e.,  $e_2, r=3$ ) appears after the problem, from its previous ( $k=1$ ) workstation  $z_2$  ( $s_1$ ) began to rework. Analysis of the random network from the front, can be divided into general assembly path  $L_j^{(G)} = \{e_{j,1}', e_{j,1}'', s_{j,1}, e_{j,2}', e_{j,2}'', s_{j,2}\}$  and rework path  $L_j^{(R|3,3-1)} = \{s_{j,1}, e_{j,2}', e_{j,2}'', s_{j,2}\}$ .

Step one:

$$I_1 = d_1 / (g^{n_1} p^n + g^{n_1+1} p^{n+k} q) = 100 / (0.99^2 0.95^4 + 0.99^3 0.95^5 0.05) = 119.64$$

$$I_2 = d_2 / (g^{n_1} p^n + g^{n_1+1} p^{n+k} q) = 150 / (0.99^2 0.95^4 + 0.99^3 0.95^5 0.05) = 179.46$$

Determine the input of each workstation  $w_{i(m)}$ , see Table 2.

**Table 2. The input of each workstation**

	$w_1(e_1')$	$w_1(e_1'')$	$w_1(s_1)$	$w_2(e_2')$	$w_2(e_1'')$	$w_2(s_2)$
$w_{i(m)}$	334.992	331.64	344.69	313.38	310.24	294.74
$T/w_{i(m)}$	85.8	87	83.4	91.8	93	97.8

Step two: find the minimum time to complete all the products.

$$\begin{aligned}
 O_{min} &= \max\{w_{i(m)}M_{i(m)} | i, m \in L\} \\
 &= \max\{334.992 \times 82, 331.64 \times 84, 344.69 \times 78, 313.38 \times 85, 310.24 \\
 &\quad \times 85, 294.74 \times 94\} \\
 &= \max\{27469.3, 27857.8, 26885.8, 26673, 26370.4, 27705.6\} \\
 &= 27857.8
 \end{aligned}$$

Step three: determine the minimum time required to complete the product less than the delivery time, so as to ensure the completion of the delivery time. Eight hours a day is 28800 seconds, that is, to meet the next type.

$$O_{min} = 27857.8 \leq T = 28800.$$

Step four: determine the minimum power vector  $Y$

In the above identified input each workstation  $w_{i(m)}$ , the minimum time to find work each workstation to meet  $x_{i(m)(c)} \leq T/w_{i(m)} < x_{i(m)(c+1)}$ ,  $Y_i = x_{i(m)(c)}$ . This determines the minimum power vector  $Y$  (84,86,82,90,90,96).

Step five: solving the reliability of the assembly system

$$\begin{aligned}
 R_d &= \Pr\{X|V(X) \leq W(D)\} = 0.991 \times 0.983 \times 0.989 \times 0.987 \times 0.985 \times 0.97 \\
 &= 0.9085
 \end{aligned}$$

It can be known that the assembly system can complete the reliability of the order product in the delivery time of 0.9085.

## 6. Conclusion

Based on the stochastic flow network theory, this paper attempts to rebuild a scientific network model based on the theory of assembly manufacturing system. Based on the network model of general assembly manufacturing system, this paper establishes a reliability evaluation model for single-product assembly line and multi-product assembly line with rework path, and gives the general algorithm for model solving, and obtains a complete assembly manufacturing system reliability evaluation method. This paper mainly completed the following research work.

- (I) redefine the reliability of the assembly manufacturing system;
- (II) redefine the model elements of the stochastic flow network to fit the specific situation of the assembly manufacturing system;
- (III) re-construct the stochastic flow network diagram for the assembly manufacturing system, and split the work station according to their characteristics;
- (IV) considering the existence of the rework path of the assembly manufacturing system, the stochastic flow network graph is decomposed, and the formula of the input amount of each work station for the assembly manufacturing system is obtained;

(V) establish a complete assembly manufacturing system reliability evaluation method, and give a numerical example.

The article makes a preliminary exploration on the research of assembly manufacturing system reliability, the future will continue to general application level research in order to achieve the application and solution of the model, at the same time considering more practical production environment and constraints, including the following points: one is to consider how to use the method of stochastic flow network reliability evaluation of assembly line with buffer ;the two is whether the use of time as the input ;third, extension of other conditions, is to expand the rework path, as well as the different workstations have different pass rate. In addition, this method can calculate the specific input amount of each workstation in the system, therefore, the practical significance of this method can be further excavated from the perspective of system reliability allocation, and not limited to the reliability evaluation.

## References

- [1] Z. Li, "The challenge and frontier of manufacturing system research", Chinese Journal of mechanical engineering, vol.11, (2010), pp. 124-136.
- [2] S. Chakra and B. Ankiah, "Assessment of Manufacturing System Reliability: A Case Study", Journal of the Operational Research Societ, vol.40, no.1, (1989), pp. 21-24.
- [3] K. Miriyala and N. Viswanadham, "Reliability Analysis of Flexible Manufacturing Systems", The International Journal of Flexible Manufacturing Systems, vol.2, (1989), pp. 145-162.
- [4] T. Min, "Loose GUI. CIMS production line reliability model", Manufacturing auto., (1990), pp. 4-10.
- [5] T. Min. "CIMS serial production line reliability modeling and analysis", Control and decision making, vol.8, no.6, (1993), pp. 45-48.
- [6] Y. K. Lin and P. Chang, "Reliability evaluation for a manufacturing network with multiple production lines", Computers & Industrial Engineering, vol.2, (2012), pp. 15-18.
- [7] Y. K. Lin and P. Chang, "Evaluate the system reliability for a manufacturing network with reworking actions", Reliability Engineering and System Safety, (2012), pp. 22-28.
- [8] Y. K. Lin, "Reliability evaluation for a waste-reduction parallel-line manufacturing system", Journal of Cleaner Production, vol.1, (2012), pp. 44-46.
- [9] J. Li and S. M. Meerkov, "Production variability in manufacturing systems: Bernoulli reliability case", Annals of Operations Research, vol.3, (2000), pp. 299-324.
- [10] J. Li, E. Emre and S. M. Meerkov, "Random Demand Satisfaction in Unreliable Production Inventory Customer Systems", Annals of Operations Research, vol.2, (2004), pp. 159-175.
- [11] S. B. Gershwin and M. H. Burman, "A decomposition method for analyzing inhomogeneous assembly/disassembly systems", Annals of Operations Research, vol.3, no.93, (2000), pp. 91-115.
- [12] N. Boysen, M. Fliedner and A. Scholl, "A classification of assembly line balancing problems", European Journal of Operational Research, vol.183, (2007), pp. 674-693.
- [13] N. Boysen, M. Fliedner and A. Scholl, "Assembly line balancing: Which model to use when", International Journal of Production Economics, vol.11, (2008), pp. 509-528.
- [14] A. R. Rezvani, H. Rahnejat and H. Bera, "Reliability analysis of a flexible machining cell", The International Journal of Advanced Manufacturing Technology, vol.1, (1986), pp. 55-68.
- [15] A. Adamyany and D. He, "Analysis of sequential failures for assessment of reliability and safety of manufacturing systems", Reliability Engineering and System Safety, (2002), pp. 227-236.
- [16] A. Philip and R. Sharma, "A stochastic reward net approach for reliability analysis of a flexible manufacturing module", Int'l Journal of Systems Assurance Eng'g and Mgt., (2013), pp. 293-302.
- [17] L. Garcia-Diaz, "A network flow approach to solve clustering problems in group technology", International Journal of Production Research, vol.3, (1993), pp. 603-612.
- [18] D. Francas and S. Minner, "Manufacturing network configuration in supply chains with product recovery", Omega, vol.37, (2009), pp. 757-769.
- [19] M. J. Zuo and Z. Tian, "An efficient method for reliability evaluation of multistate networks given all minimal path vectors", IIE Transactions, vol.39, (2007), pp. 811-817.
- [20] M. Manitz, "Queueing-model based analysis of assembly lines with finite buffers and general service times", Computers & Operations Research, vol.35, (2008), pp. 2520-2536.
- [21] K. Agpak and H. Gokcen, "A chance-constrained approach to stochastic line balancing problem", European Journal of Operational Research, vol. 180, (2007), pp. 1098-1115.