

A Method for Dealing with Multi-granularity and Multi-scale Linguistic Information based on Modified Proportional 2-tuples

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Abstract

Multi-granularity linguistic information has been studied by researchers in many disciplines, in which the scale of linguistic term sets is usually restricted in domain $[0, 1]$. In this paper, we consider the multiple attribute group decision making (MAGDM) problems, in which the evaluation of each alternative with respect to each attribute is provided by several experts of the corresponding field. In order to convey the preferences of different experts exactly and to describe the characteristics of different attributes, linguistic term sets with different granularities and different scales have been proposed to express their evaluation values. Moreover, the performance values given by the experts take the form of modified proportional 2-tuples. In this process, the modified proportional 2-tuple, its comparative rules as well as several aggregation operators have been proposed. A new method has been proposed to achieve a basic modified linguistic term set (BMLTS) and a transformation function has been defined to make the performance values uniform. An example is given to illustrate the applicability and flexibility of the method.

Keywords: *Multiple attribute group decision making; Computing with words; Linguistic variables; Linguistic modeling; Multi-granularity linguistic term sets; Multiple scale linguistic term sets*

1. Introduction

Due to time constraint or computational costs, human decisions are usually evaluated by linguistic variables rather than numerical ones. A possible way to deal linguistic variables is through the use of linguistic modeling [1-5] and techniques of computing with words (CW). In 1986, Bonissone and Decker [3] presented a linguistic computational model based on extension principle. This model was operated on fuzzy numbers, which supported the semantics of linguistic terms. In 1993, Delgado and Verdegay [4] developed another model based on a symbolic method. Through this model, computations could be done on the indexes of linguistic terms. However, the primary limitations of both the above-mentioned approaches were that their results usually did not exactly match with any of the initial linguistic terms. Therefore an approximation process was developed to express the results within the initial expression domain. However, it produced a loss in information and hence the lacked in precision. In 2001, Herrera and Martínez [5] established 2-tuple linguistic representation model to overcome this limitation. The 2-tuple was composed of a linguistic term $s_i \in \tilde{S}$ and a real number $\alpha' \in [-0.5, 0.5]$. This model received a great deal of attention from researchers in many disciplines, such as: multiple decision making [6-14], supply performance [15], computer network security [16], classification problem [17], emergency response capacity [18] and failure mode and effect analysis [19]. However, as Herrera and Martínez pointed out, the 2-tuple linguistic representation model is only suitable for linguistic variables with equidistant labels. In order to break through this limitation, Wang and Hao [20] introduced a proportional 2-tuple fuzzy linguistic representation model which represented the linguistic information

by means of proportional 2-tuple (s_i, α) $s_i \in S$. Their model could be applied to linguistic variables which did not have to be uniformly and symmetrically distributed around a medium label and without the traditional requirement “equidistant labels” between them. However, the aggregation operators of this approach were based on canonical characteristic values (CCVs) of linguistic labels and they could only be used in symmetrical trapezoidal fuzzy numbers $[b - \delta, b, c, c + \delta]$. The author also suggested generalization of their proposal by considering proportional 2-tuples under more general contexts. In this regard, Yucheng Du et al studied the relationship between 2-tuple and proportional 2-tuple based on the concept of numerical scale and found that when different numerical scales were set different versions of 2-tuple fuzzy linguistic representation model could be obtained [21].

In this work, we study the relationship between 2-tuple and proportional 2-tuple from a different perspectives, and put forward modified proportional 2-tuple fuzzy linguistic representation model. In our model, the first proportional 2-tuple is represented by piecewise form. Then based on the concept of modified proportional symbolic translation (see Definition 13), proportional 2-tuple (s_i, α) is expressed as (s_i, α) . (s_i, α) is called as the modified proportional 2-tuple. The modified proportional 2-tuple is a modification of proportional 2-tuple. The benefits of such modification are- when linguistic information is denoted by the modified proportional 2-tuple, its expression is consistent with that of 2-tuples. Therefore it has the character of 2-tuple, but its aggregations operator was not limited to only symmetrical trapezoidal fuzzy number $[b - \delta, b, c, c + \delta]$ contexts. Furthermore, the modified proportional 2-tuple is another form of representation of the proportional 2-tuple. In other words, it has the character of proportional 2-tuple and can be used in linguistic term sets, whose terms are not with equidistant labels. Therefore, the modified proportional 2-tuple does not have the limitation of the 2-tuple. Generally speaking, the modified proportional 2-tuple has advantages of both the 2-tuple and proportional 2-tuple, but does not suffer from their individual limitations. In fact, the modified version forms a link between them.

The problems of dealing with linguistic information belonging to various term sets with different granularity received extensive attention of researchers [22-29]. Typically, the scale used in methodology is usually restricted to $[0,1]$. In MAGDM problems, each attribute was valued by experts in the corresponding field. Depending upon the degree of uncertainty or preference of an expert to qualify an attribute, the linguistic term set chosen have different number of terms and scales different length (long or short). To the best of our knowledge, no methodology have been reported to-date that could simultaneously involve term sets with different granularity and different scales. The aim of this work is to develop a method that can manage multiple attribute group decision making problems, in which the performance values are in the form of modified proportional 2-tuple, and the linguistic term sets to which they belong have different granularity and different scale at the same time. Through this process, we obtain BMLTS and define a transaction function to make the performance values uniform.

The rest of the paper is arranged as follows. Section 2 reviews 2-tuple and proportional 2-tuple fuzzy linguistic representation model. Section 3 introduces notions of modified proportional 2-tuple. Section 4 presents a method managing multi-granularity and multi-scale linguistic information based on modified proportional 2-tuples. A real example is given in section 5. Important conclusions are made in section 6.

2. Preliminaries

2.1. Notions of 2-tuple Linguistic Representation Model

In this section, we review notions of 2-tuple. Consider $S = \{s_0, s_1, \dots, s_g\}$ as a finite and totally ordered discrete term set with odd cardinality, where s_i represents a possible value for a linguistic variable.

Definition 1 ([5]). Let $S = \{s_0, s_1, \dots, s_g\}$ be a linguistic term set, and $\beta \in [0, g]$ be the result of an aggregation of the indices of a set of labels assessed in linguistic term set S . Then the function Δ , used to obtain the 2-tuple linguistic information equivalent to β , is defined as

$$\Delta: [0, g] \rightarrow S \times [-0.5, 0.5],$$

$$\Delta(\beta) = (s_i, \alpha) \quad , \quad \text{with} \quad \begin{cases} s_i, i = \text{round}(\beta) \\ \alpha = \beta - i, \alpha \in [-0.5, 0.5] \end{cases} \quad ,$$

(1)

where $\text{round}(\cdot)$ is the usual round operation, and s_i has the closest subscript to β , and α is called symbolic translation.

Definition 2 ([5]). Let $S = \{s_0, s_1, \dots, s_g\}$ be a linguistic term set, and (s_i, α) be a linguistic 2-tuple. There is always a function Δ^{-1}

$$\Delta^{-1}: [0, g] \rightarrow S \times [-0.5, 0.5] \quad ,$$

$$\Delta^{-1}(s_i, \alpha) = i + \alpha = \beta \quad .$$

(2)

Definition 3 ([5]). Let (s_k, α_k) and (s_l, α_l) be 2-tuples, whose comparative rules are defined by ordinary lexicographic order:

- (1) If $k < l$, then $(s_k, \alpha_k) < (s_l, \alpha_l)$.
 - (2) If $k > l$, then $(s_k, \alpha_k) > (s_l, \alpha_l)$.
 - (3) If $k = l$, then
 - (a) If $\alpha_k = \alpha_l$, then $(s_k, \alpha_k) = (s_l, \alpha_l)$.
 - (b) If $\alpha_k < \alpha_l$, then $(s_k, \alpha_k) < (s_l, \alpha_l)$.
 - (c) If $\alpha_k > \alpha_l$, then $(s_k, \alpha_k) > (s_l, \alpha_l)$.
- (3)

Negation operator of 2-tuple (s_i, α) is defined as :

$$\text{Neg}((s_i, \alpha)) = \Delta(g - \Delta^{-1}(s_i, \alpha)) \quad .$$

(4)

Definition 4 ([5]). Let $x = \{(\gamma_1, \alpha_1), (\gamma_2, \alpha_2), \dots, (\gamma_n, \alpha_n)\}$ be a set of 2-tuples, then the 2-tuple arithmetic mean \bar{x}^σ is computed as

$$\bar{x}^\varepsilon = \Delta \left(\sum_{i=1}^n \frac{1}{n} \Delta^{-1}(\gamma_i, \alpha_i) \right)$$

(5)

Definition 5 ([5]). Let $x = \{(\gamma_1, \alpha_1), (\gamma_2, \alpha_2), \dots, (\gamma_n, \alpha_n)\}$ be a set of 2-tuples and $W = \{w_1, w_2, \dots, w_n\}$ be their associated weights. The 2-tuple weighted average \bar{x}^ε is computed as

$$\bar{x}^\varepsilon = \Delta \left(\frac{\sum_{i=1}^n \Delta^{-1}(\gamma_i, \alpha_i) \cdot w_i}{\sum_{i=1}^n w_i} \right) = \Delta \left(\frac{\sum_{i=1}^n \beta_i \cdot w_i}{\sum_{i=1}^n w_i} \right)$$

(6)

Definition 6 ([5]). Let $x = \{(\gamma_1, \alpha_1), (\gamma_2, \alpha_2), \dots, (\gamma_n, \alpha_n)\}$ be a set of 2-tuples and $W = \{w_1, w_2, \dots, w_n\}$ be their associated weights that satisfies: 1) $w_i \in [0,1]$, 2) $\sum_{i=1}^n w_i = 1$.

The 2-tuple OWA operator F^ε is

$$F^\varepsilon((\gamma_1, \alpha_1), (\gamma_2, \alpha_2), \dots, (\gamma_n, \alpha_n)) = \Delta \left(\sum_{j=1}^n w_j \cdot \beta_j^* \right)$$

(7)

where β_j^* is the j th largest of the β_i values.

2-tuple linguistic information is treated as a continuous range instead of discrete one. This approach has no loss of information, and can be used in linguistic terms with equidistant labels.

2.2. Proportional 2-tuple Linguistic Representation Model

Wang and Hao proposed proportional 2-tuple fuzzy linguistic representation model based on the concept proportional 2-tuple $(\alpha s_i, (1-\alpha)s_{i+1})$. The model can deal with linguistic terms which did not have equidistant labels.

Definition 7 ([20]). Let $S = \{s_0, s_1, \dots, s_g\}$ be a ordinal linguistic term set, $I = [0,1]$, and

$$IS \equiv I \times S = \{(\alpha, s_i) : \alpha \in [0,1], i = 0, 1, \dots, g\}$$

(8)

Given a pair (s_i, s_{i+1}) of two successive ordinal terms in S , any two elements (α, s_i) , (β, s_{i+1}) of IS is called a symbolic proportion pair, and α, β are called a pair of symbolic proportions of pair (s_i, s_{i+1}) , if $\alpha + \beta = 1$. A symbolic proportion pair $(\alpha, s_i) (1-\alpha, s_{i+1})$ will be denoted by $(\alpha s_i, (1-\alpha)s_{i+1})$ and the set of all the symbolic proportion pairs is denoted by \bar{S} , i.e., $\bar{S} = \{(\alpha s_i, (1-\alpha)s_{i+1}) : \alpha \in [0,1], i = 0, 1, \dots, g-1\}$, where \bar{S} is called ordinal proportional 2-tuple set generated by S .

Definition 8 ([20]). Let $S = \{s_0, s_1, \dots, s_g\}$ be a linguistic term set, and \bar{S} is the ordinal proportional 2-tuple set. The function $\pi : \bar{S} \rightarrow [0, g]$ is defined by

$$\pi((\alpha s_i, (1-\alpha)s_{i+1})) = i + (1-\alpha) \quad , \quad (9)$$

where $i = 1, 2, \dots, g-1$, $\alpha \in [0, 1]$ and π is called the position index function of ordinal proportional 2-tuples.

Definition 9 ([20]). Let $S = \{s_0, s_1, \dots, s_g\}$ be a linguistic term set, and \bar{S} be the ordinal proportional 2-tuple set and $x \in [0, g]$ represented the result of the aggregation. The function $\pi^{-1} : [0, g] \rightarrow \bar{S}$ is defined as

$$\pi^{-1}(x) = ((1-\beta)s_i, \beta s_{i+1}), x \in [0, g] \quad , \quad (10)$$

where $i = E(x)$, E is the integer part function, $\beta = x - i$.

Definition 10 ([20]). Let $S = \{s_0, s_1, \dots, s_g\}$ be a linguistic term set, \bar{S} is the ordinal proportional 2-tuple set. For any $(\alpha s_i, (1-\alpha)s_{i+1}), (\beta s_j, (1-\beta)s_{j+1}) \in \bar{S}$, the comparative rules are defined as-

- 1) If $i < j$, then
 - a) $(\alpha s_i, (1-\alpha)s_{i+1})$ and $(\beta s_j, (1-\beta)s_{j+1})$ represents the same information when $i = j - 1$ and $\alpha = 0, \beta = 1$.
 - b) otherwise $(\alpha s_i, (1-\alpha)s_{i+1}) < (\beta s_j, (1-\beta)s_{j+1})$.
- 2) If $i = j$, then
 - a) If $\alpha = \beta$, then $(\alpha s_i, (1-\alpha)s_{i+1})$ and $(\beta s_j, (1-\beta)s_{j+1})$ represent the same information.
 - b) If $\alpha < \beta$, then $(\alpha s_i, (1-\alpha)s_{i+1}) > (\beta s_j, (1-\beta)s_{j+1})$.
 - c) If $\alpha > \beta$, then $(\alpha s_i, (1-\alpha)s_{i+1}) < (\beta s_j, (1-\beta)s_{j+1})$.

Wang and Hao also presented several aggregation operators based on CCV . Let s_i be symmetrical trapezoidal fuzzy number, where $s_i = (a_i, b_i, c_i, d_i)$. Then the canonical characteristic value is defined as $CCV(s_i) = \frac{(b_i + c_i)}{2}$. The authors proposed CCV of proportional 2-tuple as follows.

Definition 11 ([20]). S , \bar{S} and CCV on S are defined as previously. The \overline{CCV} for a proportional 2-tuple $(\alpha s_i, (1-\alpha)s_{i+1})$ is defined as

$$\overline{CCV}((\alpha s_i, (1-\alpha)s_{i+1})) = \alpha CCV(s_i) + (1-\alpha)CCV(s_{i+1}) \quad . \quad (12)$$

Let $CCV(s_i) = l_i, i = 0, 1, 2, \dots, g$, and $l_0 < l_1 < \dots < l_g$. CCV is a bijection function, and the inverse of CCV is defined as-

$$(CCV)^{-1}(x) = (CCV)^{-1}(c_i + \beta(c_{i+1} - c_i)) = (CCV)^{-1}((1-\beta)c_i + \beta c_{i+1}) = (1-\beta)s_i + \beta s_{i+1} \quad . \quad (13)$$

Definition 12 ([20]). S , \bar{S} , CCV and \overline{CCV} have the same previous definitions. Let $y = \{y_1, y_2, \dots, y_n\}$ be a set of proportional 2-tuples in \bar{S} and $w = \{w_1, w_2, \dots, w_n\}$ be their associated weights with $0 \leq w_i \leq 1$, for $i = 1, 2, \dots, n$, and $\sum_{i=1}^n w_i = 1$, the proportional 2-tuple weighted average (based on CCV) \bar{y}^w can be defined as-

$$\bar{y}^w = (CCV)^{-1} \left(\sum_{i=1}^n w_i CCV(y_i) \right)$$

(14) Definition 12 ([20]). S , \bar{S} , CCV and \overline{CCV} are defined as previously. Let $y = \{y_1, y_2, \dots, y_n\}$ be a set of proportional 2-tuples in \bar{S} , and $w = \{w_1, w_2, \dots, w_n\}$ be their associated weights with $0 \leq w_i \leq 1$, for $i = 1, 2, \dots, n$, and $\sum_{i=1}^n w_i = 1$, the proportional 2-tuple ordered weighted average (based on CCV) \bar{y}^{ow} is defined as-

$$\bar{y}^{ow} = (CCV)^{-1} \left(\sum_{i=1}^n w_i CCV(y_{(i)}) \right)$$

(15)

where $y_{(1)}, y_{(2)}, \dots, y_{(n)}$ is a permutation of y_1, y_2, \dots, y_n , so that $y_{(1)} \geq y_{(2)} \geq \dots \geq y_{(n)}$.

From the 2-tuple linguistic representation model and the proportional 2-tuple linguistic representation model, we found the comparative rules and their aggregation operators of them were quite different. However, as Wang and Hao have pointed out in [20] that since the proportional 2-tuple is just an extension of 2-tuple, the difference between them should not be too much. Interestingly, we found that just by modifying the proportional 2-tuple linguistic representation model, the above-mentioned difference can be minimized.

3. Modified Proportional 2-tuple Fuzzy Linguistic Representation Model

In this section, the modified proportional 2-tuple is described to establish a bridge between 2-tuple and proportional 2-tuple.

3.1. Notions for Modified Proportional 2-tuple

Let $\tilde{S} = \{s_{\bar{1}}, s_{\bar{2}}, \dots, s_{\bar{i}}, \dots, s_{\bar{n}}\}$ be an ordinal fuzzy linguistic term set, in that the subscript i represent the ordinal number of the linguistic term in \tilde{S} and \bar{i} represent the specific value of the subscript. In \tilde{S} the distance of label indexes of two successive terms are not equidistant.

Definition 13. Let $\tilde{S} = \{s_{\bar{1}}, s_{\bar{2}}, \dots, s_{\bar{n}}\}$ be a finite and totally ordered discrete fuzzy linguistic term set with odd cardinality n , and $\beta' \in [\bar{i}, \overline{i+1}) \subset [\bar{1}, \bar{n}]$ be the result of an aggregation of the indexes of a set of labels assessed in linguistic term set \tilde{S} . If $\beta' - \bar{i} = \alpha(\overline{i+1} - \bar{i})$, hence $\alpha \in [0, 1)$ indicates the ratio of $\beta' - \bar{i}$ to $\overline{i+1} - \bar{i}$. The same information of β' can be represented by the following function $\tilde{\Delta}$.

$$\tilde{\Delta}: [\bar{1}, \bar{n}] \rightarrow \tilde{S} \times [-0.5, 0.5),$$

$$\tilde{\Delta}(\beta') = \begin{cases} (s_{\bar{i}}, \alpha), \alpha \in [0, 0.5) \\ (s_{\overline{i+1}}, \alpha - 1), \alpha \in [0.5, 1) \end{cases} = \begin{cases} (s_{\bar{i}}, \alpha'), \alpha' \in [0, 0.5) \\ (s_{\overline{i+1}}, \alpha'), \alpha' \in [-0.5, 0) \end{cases}, \quad (16)$$

where $s_{\bar{i}}$ or $s_{\overline{i+1}}$ has the closest subscript to β' . $\alpha' = \begin{cases} \alpha, \alpha \in [0, 0.5) \\ \alpha - 1, \alpha \in [0.5, 1) \end{cases}$ is called modified proportional symbolic translation and $(s_{\bar{i}}, \alpha')$ ($i = 1, 2, \dots, n$) is called the modified proportional 2-tuple. In the following, the set $\tilde{S}' = \{(s_{\bar{i}}, \alpha') | s_{\bar{i}} \in \tilde{S}, \alpha' \in [-0.5, 0.5)\}$ will be called the modified proportional 2-tuple set generated by \tilde{S} . The function $\tilde{\Delta}$ is a mapping from $[\bar{1}, \bar{n}]$ to \tilde{S}' . This model is called as the modified proportional 2-tuple fuzzy linguistic representation model.

Remark 1. It is obvious that the linguistic term $s_{\bar{i}} \in \tilde{S}$ can be expressed by modified proportional 2-tuple as $(s_{\bar{i}}, 0) \in \tilde{S}'$.

Example 1. Let $\tilde{S} = \{s_{\bar{1}}, s_{\bar{2}}, \dots, s_{\bar{5}}\} = \{s_{-1.2}, s_{-0.5}, s_0, s_{0.5}, s_{1.2}\}$, and $\beta' = 0.3$. We denote $\beta' = 0.3$ by modified proportional 2-tuple as following -

$\beta' = 0.3 \in [0, 0.5)$, therefore $\bar{i} = 0$, $\overline{i+1} = 0.5$. The formula $\beta' - \bar{i} = \alpha(\overline{i+1} - \bar{i})$ can be written as $(0.3 - 0) = \alpha(0.5 - 0)$, then and $\alpha = 0.6 \in [0.5, 1)$. $\alpha' = \alpha - 1 = 0.6 - 1 = -0.4$. Then

$$\tilde{\Delta}(0.3) = (s_{\overline{i+1}}, \alpha') = (s_{0.5}, -0.4)$$

We can represent the same information of $\beta' = 0.3$ by modified proportional 2-tuple $(s_{0.5}, -0.4)$.

The inverse function of $\tilde{\Delta}$ is denoted by $\tilde{\Delta}^{-1}$, which is defined as follows.

Definition 14. Let $\tilde{S} = \{s_{\bar{1}}, s_{\bar{2}}, \dots, s_{\bar{n}}\}$ be a linguistic term set, and $(s_{\bar{i}}, \alpha')$ be modified proportional 2-tuple. There is always a function $\tilde{\Delta}^{-1}$:

$$\tilde{\Delta}^{-1}: \tilde{S} \times [-0.5, 0.5) \rightarrow [\bar{1}, \bar{n}],$$

$$\tilde{\Delta}^{-1}(s_{\bar{i}}, \alpha') = \begin{cases} \bar{i} + \alpha'(\overline{i+1} - \bar{i}), \alpha' \in [0, 0.5) \\ \bar{i} + \alpha'(\overline{i-1} - \bar{i}), \alpha' \in [-0.5, 0) \end{cases}. \quad (17)$$

Example 2. Let $\tilde{S} = \{s_{\bar{1}}, s_{\bar{2}}, \dots, s_{\bar{5}}\} = \{s_{-1.2}, s_{-0.5}, s_0, s_{0.5}, s_{1.2}\}$, and $(s_{0.5}, -0.4)$ be modified proportional 2-tuple. We denote $(s_{0.5}, -0.4)$ by a numerical value $\beta' \in [-1.2, 1.2]$ as follows:

From the expression of $(s_{\bar{i}}, \alpha') = (s_{0.5}, -0.4)$, we get $\bar{i} = 0.5, \overline{i-1} = 0, \alpha' = -0.4 \in [-0.5, 0)$. By function $\tilde{\Delta}^{-1}$

$$\tilde{\Delta}^{-1}(s_{0.5}, -0.4) = \bar{i} + \alpha'(\overline{i-1} - \bar{i}) = 0.5 + (-0.4)(0.5 - 0) = 0.3.$$

$(s_{0.5}, -0.4)$ can be transformed to numerical value $\beta' = 0.3$ by function $\tilde{\Delta}^{-1}$.

3.2. The Comparative Rules and Aggregation Operators of Modified Proportional 2-tuples

Definition 15. Let $\tilde{S} = \{s_{\bar{1}}, s_{\bar{2}}, \dots, s_{\bar{n}}\}$, $(s_{\bar{i}}, \alpha'_i)$ and $(s_{\bar{j}}, \alpha'_j)$ are the modified proportional 2-tuples. The comparative rules for them were as follows:

(1) If $\bar{i} < \bar{j}$, then $(s_{\bar{i}}, \alpha'_i) < (s_{\bar{j}}, \alpha'_j)$.

(2) If $\bar{i} > \bar{j}$, then $(s_{\bar{i}}, \alpha'_i) > (s_{\bar{j}}, \alpha'_j)$.

(3) If $\bar{i} = \bar{j}$, then

(a) If $\alpha'_i = \alpha'_j$ then $(s_{\bar{i}}, \alpha'_i) = (s_{\bar{j}}, \alpha'_j)$.

(b) If $\alpha'_i < \alpha'_j$ then $(s_{\bar{i}}, \alpha'_i) < (s_{\bar{j}}, \alpha'_j)$.

(18)

(c) If $\alpha'_i > \alpha'_j$ then $(s_{\bar{i}}, \alpha'_i) > (s_{\bar{j}}, \alpha'_j)$.

Definition 16. The negation operator over modified proportional 2-tuple is defined as

$$Ne((s_{\bar{i}}, \alpha'_i)) = \tilde{\Delta}(g - \tilde{\Delta}^{-1}(s_{\bar{i}}, \alpha'_i))$$

(19)

Definition 17. Let $x = \{(\gamma_1, \alpha'_1), (\gamma_2, \alpha'_2), \dots, (\gamma_n, \alpha'_n)\}$ be a set of the modified proportional 2-tuples, the arithmetic mean of the modified proportional 2-tuples is defined as

$$\bar{x}'^e = \tilde{\Delta}\left(\sum_{i=1}^n \frac{1}{n} \tilde{\Delta}^{-1}(\gamma_i, \alpha'_i)\right)$$

(20)

Definition 18. Let $x = \{(\gamma_1, \alpha'_1), (\gamma_2, \alpha'_2), \dots, (\gamma_n, \alpha'_n)\}$ be a set of the modified proportional 2-tuples. $W = \{w_1, w_2, \dots, w_n\}$ be their associated weights, and $\beta'_i = \tilde{\Delta}^{-1}(\gamma_i, \alpha'_i)$. Then the weighted average, \bar{x}'^e of the modified proportional 2-tuples is computed as

$$\bar{x}'^e = \tilde{\Delta}\left(\frac{\sum_{i=1}^n \tilde{\Delta}^{-1}(\gamma_i, \alpha'_i) \cdot w_i}{\sum_{i=1}^n w_i}\right) = \tilde{\Delta}\left(\frac{\sum_{i=1}^n \beta'_i \cdot w_i}{\sum_{i=1}^n w_i}\right)$$

(21)

Definition 19. Let $x' = \{(\gamma_1, \alpha'_1), (\gamma_2, \alpha'_2), \dots, (\gamma_n, \alpha'_n)\}$ be a set of modified proportional 2-tuples, $W = \{w_1, w_2, \dots, w_n\}$ be their associated weights that satisfies: 1) $w_i \in [0, 1]$, 2)

$\sum_{i=1}^n w_i = 1$. The modified proportional 2-tuples OWA operator F'^e is

$$F'^e((\gamma_1, \alpha'_1), (\gamma_2, \alpha'_2), \dots, (\gamma_n, \alpha'_n)) = \tilde{\Delta}\left(\sum_{j=1}^n w_j \cdot \beta_j'^*\right)$$

(22)

where $\beta'_i = \tilde{\Delta}^{-1}(\gamma_i, \alpha'_i)$ and $\beta_j'^*$ is the j th largest of the β'_i values.

The above section shows that the operations on the modified proportional 2-tuple,

such as comparative rules, negation operator and aggregation operators are exactly the same with respect to the corresponding item of 2-tuple. So, the modified proportional 2-tuple does not have the limitation that the aggregation operators have to be only used in symmetrical trapezoidal fuzzy number $[b - \delta, b, c, c + \delta]$ contexts. Further more, all the existing aggregation operators of 2-tuple all can be applied to this modified proportional 2-tuple. Therefore, the application of the modified proportional 2-tuple fuzzy linguistic representation model can be extended to a much wider range.

4. Fusion Approach for Managing Multi-granularity and Multi-scale Linguistic Information based on Modified Proportional 2-tuple

$\tilde{S}_j = \{s_{1j}, s_{2j}, \dots, s_{n_jj}\}$ ($j=1,2,\dots,m$) are linguistic term sets which have different granularity and scale. The performance value is $u_{ij} \in \tilde{S}_j$ ($i=1,2,\dots,n; j=1,2,\dots,m$). In order to obtain the collective performance values of alternatives, we must transform u_{ij} into linguistic information in a specific linguistic term set to make the information uniform. The specific linguistic term set denoted by $\tilde{S}_T = \{s_{1T}, s_{2T}, \dots, s_{n_T T}\}$ is called as basic modified fuzzy linguistic term set (BMLTS).

4.1. Setting up basic Modified Linguistic Term Set (BMLTS)

In this subsection, we will describe how to obtain BMLTS. Let $\tilde{S}_j = \{s_{1j}, s_{2j}, \dots, s_{n_jj}\}$ ($j=1,2,\dots,m$) are linguistic term sets whose granularity and scale are finite. Let $\tilde{S}_T = \{s_{1T}, s_{2T}, \dots, s_{n_T T}\}$ represent the BMLTS we are going to establish. There are three essential factors that need to be determined.

(1) Determination of the granularity of BMLTS. For the sake of maintaining uncertain degree of every experts, the maximum granularity of \tilde{S}_j ($j=1,2,\dots,m$) is chosen as the granularity of BMLTS. i.e. if \tilde{S}_k has the maximum granularity, then $n_T = n_k = \max\{n_1, n_2, \dots, n_m\}$.

(2) To find the range of BMLTS. The range of BMLTS should cover all of \tilde{S}_j ($j=1,2,\dots,m$). The minimum linguistic term in \tilde{S}_j ($j=1,2,\dots,m$) is selected as the smallest linguistic term $s_{1T} \in \tilde{S}_T$, i.e. if $1T = \min\{\overline{1}, \overline{12}, \dots, \overline{1m}\}$, then the smallest linguistic term of \tilde{S}_T is s_{1T} . The maximum linguistic term in \tilde{S}_j ($j=1,2,\dots,m$) is chosen as the largest linguistic term $s_{n_T T} \in \tilde{S}_T$, i.e. If $n_T T = \max\{\overline{n_1 1}, \overline{n_2 2}, \dots, \overline{n_m m}\}$, then the largest linguistic term of \tilde{S}_T is $s_{n_T T}$.

(3) Determination of other linguistic terms in BMLTS. Let $\tilde{S}_k = \{s_{1k}, s_{2k}, \dots, s_{n_k k}\}$ has the maximum granularity, so $n_T = n_k = \max\{n_1, n_2, \dots, n_m\}$. Using function $\tau_{\tilde{S}_k, \tilde{S}_T}$ an one to one mapping was made from $s_{ik} \in \tilde{S}_k$, ($i=1,2,3,\dots,n_k$) to $s_{iT} \in \tilde{S}_T$, ($i=1,2,3,\dots,n_T$), to obtain the other linguistic terms of $s_{iT} \in \tilde{S}_T$, ($i=1,2,3,\dots,n_T$). The mapping $\tau_{\tilde{S}_k, \tilde{S}_T}$ is presented in the next subsection. From the above steps, the granularity, scale and linguistic terms of \tilde{S}_T are obtained to set up the BMLTS.

4.2. Getting Linguistic Terms of BMLTS

In this subsection, we will develop a transformation function from \tilde{S}'_k to \tilde{S}'_T to get the linguistic terms of BMLTS.

The function $\tilde{\Delta}:[\bar{1},\bar{n}]\rightarrow\tilde{S}'$ is a mapping which can transform numeric values $\beta' \in [\bar{1},\bar{n}]$ to modified proportional 2-tuple depending on its definition domain $[\bar{1},\bar{n}]$ and transformation rule. However there are $m+1$ different term sets $\tilde{S}_j = \{s_{1j}, s_{2j}, \dots, s_{n_jj}\} (j=1,2,\dots,m,T)$ in our group decision making problem. For this process, we need $m+1$ mapping functions according $m+1$ term sets $\tilde{S}_j = \{s_{1j}, s_{2j}, \dots, s_{n_jj}\} (j=1,2,\dots,m,T)$. In order to show the difference, we have represented the mapping function by $\tilde{\Delta}_j$.

Note 1: Function $\tilde{\Delta}_j (j=1,2,\dots,m,T)$ is as follows:

$$\tilde{\Delta}_j: [\bar{1}j, \bar{n}_jj] \rightarrow \tilde{S}'_j, \quad (23)$$

$$\tilde{\Delta}_j(\beta'_j) = \begin{cases} (s_{ij}, \alpha) & \alpha \in [0, 0.5) \\ (s_{(i+1)j}, \alpha - 1) & \alpha \in [0.5, 1) \end{cases} = \begin{cases} (s_{ij}, \alpha') & \alpha' \in [0, 0.5) \\ (s_{(i+1)j}, \alpha') & \alpha' \in [-0.5, 0) \end{cases}$$

The inverse function of $\tilde{\Delta}_j$ is denoted by $\tilde{\Delta}_j^{-1} (j=1,2,\dots,m,T)$

$$\tilde{\Delta}_j^{-1}: \tilde{S}'_j \rightarrow [\bar{1}j, \bar{n}_jj], \quad (24)$$

$$\tilde{\Delta}_j^{-1}(s_{ij}, \alpha') = \begin{cases} \bar{ij} + \alpha'((i+1)j - \bar{ij}), & \text{if } \alpha' \in [0, 0.5) \\ \bar{ij} + \alpha'(\bar{ij} - (i-1)j), & \text{if } \alpha' \in [-0.5, 0) \end{cases}$$

Definition 20. Let $\tilde{S}_k = \{s_{1k}, s_{2k}, \dots, s_{n_kk}\}$ is the term set which has the maximum granularity, $\tilde{S}_T = \{s_{1T}, s_{2T}, \dots, s_{n_rT}\}$ is the BMLTS. \tilde{S}'_k and \tilde{S}'_T are ordinal modified proportional 2-tuple term sets generated by \tilde{S}_k and \tilde{S}_T respectively. A transformation function from \tilde{S}'_k to \tilde{S}'_T is defined as-

$$\tau'_{\tilde{S}'_k \tilde{S}'_T}: \tilde{S}'_k \rightarrow \tilde{S}'_T, \quad (25)$$

$$\tau'_{\tilde{S}'_k \tilde{S}'_T}(s_{ik}, 0) = (s_{iT}, 0)$$

$$\bar{iT} = \beta'_{iT} = \left(\frac{\tilde{\Delta}_k^{-1}(s_{ik}, 0) - \tilde{\Delta}_k^{-1}(s_{1k}, 0)}{\tilde{\Delta}_k^{-1}(s_{n_kk}, 0) - \tilde{\Delta}_k^{-1}(s_{1k}, 0)} \cdot (n_rT - \bar{1T}) + \bar{1T} \right) \quad (i = 2, 3, \dots, n_{k-1})$$

where

From the value of transformation $(s_{iT}, 0)$, the i th linguistic term s_{iT} in \tilde{S}_T is computed. Next, the other terms of BMLTS, besides s_{iT} and s_{n_rT} are also calculated. In this way, after all the terms in $\tilde{S}_T = \{s_{1T}, s_{2T}, \dots, s_{n_rT}\}$ are obtained, the BMLTS is finally set up.

Note 2: If a linguistic term set simultaneously has the maximum granularity and the greatest scale, it can be viewed as BMLTS \tilde{S}_T .

4.3. Transforming Performance Values into Linguistic Information of BMLTS

In this section, we consider a multiple attribute group decision making problem as follows:

Let $X = \{x_1, x_2, \dots, x_n\}$ ($n \geq 2$) be a finite set of alternatives, $U = \{u_1, u_2, \dots, u_m\}$ ($m \geq 2$) be a finite set of attributes, $E = \{E_1, E_2, \dots, E_m\}$ ($m \geq 2$) be a group of experts and E_j be the j th expert corresponding to the field of the j th attribute U_j . The performance value of the i th alternative with regard to the j th attribute given by the j th expert is denoted by u_{ij} ($i=1,2,\dots,n; j=1,2,\dots,m$). Assume that each expert may use different linguistic term sets.

The performance values of alternative are defined by different linguistic term sets \tilde{S}'_j ($j=1,2,\dots,m$). In the interest of getting the collective performance values of alternatives, first we denoted the performance values u_{ij} ($i=1,2,\dots,n; j=1,2,\dots,m$) by modified proportional 2-tuple $u_{ij} = (s_{ij}, \alpha')$, $\alpha' \in [-0.5, 0.5]$. Using the function $\tau'_{\tilde{S}'_j, \tilde{S}'_T}$, we can transform the performance value $(s_{ij}, \alpha') \in \tilde{S}'_j$ ($j=1,2,\dots,m$) to the modified proportional 2-tuple $p_{ij} = (s_{k}, \alpha'_k) \in \tilde{S}'_T$.

$$\tau'_{\tilde{S}'_j, \tilde{S}'_T} : \tilde{S}'_j \rightarrow \tilde{S}'_T,$$

$$\tau'_{\tilde{S}'_j, \tilde{S}'_T} (s_{ij}, \alpha') = (s_{kT}, \alpha'_{kT}) = \tilde{\Delta}_T \left(\frac{\tilde{\Delta}_j^{-1}(s_{ij}, \alpha') - \tilde{\Delta}_j^{-1}(s_{1j}, 0)}{\tilde{\Delta}_j^{-1}(s_{nj}, 0) - \tilde{\Delta}_j^{-1}(s_{1j}, 0)} \cdot (\tilde{\Delta}_T^{-1}(s_{nT}, 0) - \tilde{\Delta}_T^{-1}(s_{1T}, 0)) + \tilde{\Delta}_T^{-1}(s_{1T}, 0) \right).$$

(26)

Example 3: Let $\tilde{S}_1 = \{s_{11}, s_{21}, \dots, s_{51}\} = \{s_{-1.2}, s_{-0.5}, s_0, s_{0.5}, s_{1.2}\}$ and $\tilde{S}_T = \{s_{1T}, s_{2T}, \dots, s_{7T}\} = \{s_{-5}, s_{-3}, s_{-1}, s_0, s_1, s_3, s_5\}$ be two linguistic term sets. $(s_{0.5}, -0.4) \in \tilde{S}'_1$ is a modified proportional 2-tuple. Transform $(s_{0.5}, -0.4)$ to a linguistic information in \tilde{S}'_T as follows:

$$\begin{aligned} \tau'_{\tilde{S}'_1, \tilde{S}'_T} (s_{0.5}, -0.4) &= \tilde{\Delta}_T \left(\frac{\tilde{\Delta}_1^{-1}(s_{0.5}, -0.4) - \tilde{\Delta}_1^{-1}(s_{-1.2}, 0)}{\tilde{\Delta}_1^{-1}(s_{1.2}, 0) - \tilde{\Delta}_1^{-1}(s_{-1.2}, 0)} \cdot (\tilde{\Delta}_T^{-1}(s_5, 0) - \tilde{\Delta}_T^{-1}(s_{-5}, 0)) + \tilde{\Delta}_T^{-1}(s_{-5}, 0) \right) \\ &= \tilde{\Delta}_T \left(\frac{0.3 - (-1.2)}{1.2 - (-1.2)} (5 - (-5)) + (-5) \right) = \tilde{\Delta}_T (6.25 + (-5)) = \tilde{\Delta}_T (1.25) = (s_{1T}, 0.125) \end{aligned}$$

Remark 2: Let $(s_{k}, \alpha'_k) \in \tilde{S}'_T$, ($k=1,2,\dots,n_T$), we can get its corresponding value in $(s_{ij}, \alpha'_{ij}) \in \tilde{S}'_j$ ($j=1,2,\dots,m$) by the function $\tau'_{\tilde{S}'_j, \tilde{S}'_T}$

$$\tau'_{\tilde{S}'_j, \tilde{S}'_T} (s_{k}, \alpha'_k) = (s_{ij}, \alpha'_{ij}) = \tilde{\Delta}_j \left(\frac{\tilde{\Delta}_T(s_{k}, \alpha'_k) - \tilde{\Delta}_T(s_{1T}, 0)}{\tilde{\Delta}_T(s_{n_T}, 0) - \tilde{\Delta}_T(s_{1T}, 0)} (\tilde{\Delta}_j^{-1}(s_{nj}, 0) - \tilde{\Delta}_j^{-1}(s_{1j}, 0)) + \tilde{\Delta}_j^{-1}(s_{1j}, 0) \right)$$

(27)

Example 4: Let $\tilde{S}_1 = \{s_{11}, s_{21}, \dots, s_{51}\} = \{s_{-1.2}, s_{-0.5}, s_0, s_{0.5}, s_{1.2}\}$ and

$\tilde{S}_T = \{s_1, s_2, \dots, s_7\} = \{s_{-5}, s_{-3}, s_{-1}, s_0, s_1, s_3, s_5\}$. $(s_1, 0.125) \in \tilde{S}'_T$ is a modified proportional 2-tuple, its correspondence value in \tilde{S}'_T was computed as follows-

$$\begin{aligned} \tau'_{\tilde{S}'_T}(s_1, 0.125) &= \tilde{\Delta}_1 \left(\frac{\tilde{\Delta}_T(s_1, 0.125) - \tilde{\Delta}_T(s_{-5}, 0)}{\tilde{\Delta}_T(s_5, 0) - \tilde{\Delta}_T(s_{-5}, 0)} (\tilde{\Delta}_1(s_{1.2}, 0) - \tilde{\Delta}_1(s_{-1.2}, 0)) + \tilde{\Delta}_1(s_{-1.2}, 0) \right) \\ &= \tilde{\Delta}_1 \left(\frac{1.25 - (-5)}{5 - (-5)} (1.2 - (-1.2)) + (-1.2) \right) = \tilde{\Delta}_1(1.5 + (-1.2)) = \tilde{\Delta}_1(0.3) = (s_{0.5}, -0.4) \end{aligned}$$

5. Illustrative Examples

An advanced technological achievement is waiting to be transformed into products. The problem is to find the best or a suitable company with respect to various company's capabilities. After pre-evaluation, four potential companies were selected as alternatives for further evaluation-

- x_1 is a car company
- x_2 is a computer company
- x_3 is a shipbuilding company
- x_4 is detector production company

After a detailed discussion with the experts about what criteria to consider for evaluating the companies' comprehensive abilities, four performance attributes were chosen-

- u_1 is the transformation ability
- u_2 is the financial strength
- u_3 is the management ability
- u_4 is the ability of marketing

And the weights according to four attributes given by experts were

$$w = (0.3 \ 5 \ 1, \mathbf{0.1} \ 5 \ 7, \mathbf{0.1} \ 8 \ 1, \mathbf{0.3} \ 0 \ 9).$$

There four group of experts, each group is in charge of the evaluation of an attribute u_j ($j=1,2,3,4$). Each group of experts used different fuzzy linguistic term set \tilde{S}_j ($j=1,2,3,4$):

$$\begin{aligned} \tilde{S}_1 &= \{s_{11} = \text{very.poor}, s_{21} = \text{poor}, s_{31} = \text{medium}, s_{41} = \text{good}, s_{51} = \text{very.good}\} = \{s_{-3}, s_{-2}, s_0, s_2, s_3\} \\ \tilde{S}_2 &= \{s_{12} = \text{very.poor}, s_{22} = \text{poor}, s_{32} = \text{medium}, s_{42} = \text{good}, s_{52} = \text{very.good}\} = \{s_{-2}, s_{-1}, s_0, s_1, s_2\} \\ \tilde{S}_3 &= \{s_{13} = \text{extremely.poor}, s_{23} = \text{very.poor}, s_{33} = \text{poor}, s_{43} = \text{medium}, \\ &\quad s_{53} = \text{good}, s_{63} = \text{very.good}, s_{73} = \text{extremely.good}\} = \{s_{-5}, s_{-3}, s_{-1}, s_0, s_1, s_3, s_5\} \\ \tilde{S}_4 &= \{s_{14} = \text{extremely.poor}, s_{24} = \text{very.poor}, s_{34} = \text{poor}, s_{44} = \text{medium}, \end{aligned}$$

$$s_{\bar{54}} = good, s_{\bar{64}} = very.good, s_{\bar{74}} = extremely.good\} = \{s_{-3}, s_{-2}, s_{-1}, s_0, s_1, s_2, s_3\}$$

The decision making process consisted the following steps-

Step 1: Establish decision matrix. Experts choose linguistic term sets \tilde{S}_j according to

the characteristic of the attributes and their preferences. Then they give linguistic evaluation on alternatives with regard to the special attribute of their research fields. Construct decision matrix A .

$$A = \begin{pmatrix} u_{11} & u_{12} & u_{13} & u_{14} \\ u_{21} & u_{22} & u_{23} & u_{24} \\ u_{31} & u_{32} & u_{33} & u_{34} \\ u_{41} & u_{42} & u_{43} & u_{44} \end{pmatrix} = \begin{pmatrix} (s_0,0) & (s_0,0) & (s_2,0) & (s_1,0) \\ (s_2,0) & (s_1,0) & (s_{-1},0) & (s_2,0) \\ (s_{-2},0) & (s_0,0) & (s_5,0) & (s_1,0) \\ (s_3,0) & (s_2,0) & (s_3,0) & (s_0,0) \end{pmatrix}$$

Step 2: Setting up the BMLTS \tilde{S}_T . Determine three essential factors of BMLTS: granularity, range and linguistic terms by the Process stated in subsection 4.1.

- (1) The granularity of \tilde{S}_T . We choose the maximum granularity of the fourth linguistic term sets $\tilde{S}_j (j=1,2,3,4)$ as the granularity of \tilde{S}_T ,

$$T = m \text{ a } \{5,5,7,7\} = 7.$$

- (2) The range of \tilde{S}_T . The minimum linguistic term of \tilde{S}_T is

$$\bar{1} = \min\{s_{\bar{11}}, s_{\bar{12}}, s_{\bar{13}}, s_{\bar{14}}\} = \min\{-3, -2, -5, -3\} = -5$$

the maximum linguistic term of \tilde{S}_T is

$$\bar{7} = \max\{s_{\bar{51}}, s_{\bar{52}}, s_{\bar{73}}, s_{\bar{74}}\} = m \text{ a } \{3, 2, 5, 3\} = 5.$$

- (3) The other linguistic terms of \tilde{S}_T . \tilde{S}_3 has simultaneously the maximum granularity and the greatest scale, so \tilde{S}_3 was considered as BMLTS

$$\tilde{S}_T = \{s_{-5}, s_{-3}, s_{-1}, s_0, s_1, s_3, s_5\}.$$

Step 3: Make the performance values uniform.

$$\tau'_{\tilde{S}_1\tilde{S}_T}(u_{11}) = \tau'_{\tilde{S}_1\tilde{S}_T}(s_0,0) = \tilde{\Delta}_T \left(\frac{\tilde{\Delta}_1^{-1}(s_0,0) - \tilde{\Delta}_1^{-1}(s_{-3},0)}{\tilde{\Delta}_1^{-1}(s_3,0) - \tilde{\Delta}_1^{-1}(s_{-3},0)} \cdot (\tilde{\Delta}_T^{-1}(s_5,0) - \tilde{\Delta}_T^{-1}(s_{-5},0)) + \tilde{\Delta}_T^{-1}(s_{-5},0) \right)$$

$$= \tilde{\Delta}_T(0) = (s_0,0) = p_{11}$$

$$\tau'_{\tilde{S}_1\tilde{S}_T}(u_{21}) = (s_3,0.1667) = p_{21} \quad \tau'_{\tilde{S}_1\tilde{S}_T}(u_{31}) = (s_{-3},-0.1667) = p_{31} \quad \tau'_{\tilde{S}_1\tilde{S}_T}(u_{41}) = (s_5,0) = p_{41}$$

$$\tau'_{\tilde{S}_2\tilde{S}_T}(u_{12}) = (s_0,0) = p_{12} \quad \tau'_{\tilde{S}_2\tilde{S}_T}(u_{22}) = (s_3,-0.25) = p_{22} \quad \tau'_{\tilde{S}_2\tilde{S}_T}(u_{32}) = (s_0,0) = p_{32}$$

$$\tau'_{\tilde{S}_2\tilde{S}_T}(u_{42}) = (s_5,0) = p_{42} \quad \tau'_{\tilde{S}_4\tilde{S}_T}(u_{14}) = (s_1,0.3333) = p_{14} \quad \tau'_{\tilde{S}_4\tilde{S}_T}(u_{24}) = (s_3,0.1667) = p_{24}$$

$$\tau'_{\tilde{S}_4\tilde{S}_T}(u_{34}) = (s_1,0.3333) = p_{34} \quad \tau'_{\tilde{S}_4\tilde{S}_T}(u_{44}) = (s_0,0) = p_{44}$$

The decision matrix in \tilde{S}_T' is

$$A' = \begin{pmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \\ p_{41} & p_{42} & p_{43} & p_{44} \end{pmatrix} = \begin{pmatrix} (s_0,0) & (s_0,0) & (s_2,0) & (s_1,0.3333) \\ (s_3,0.1667) & (s_3,-0.25) & (s_{-1},0) & (s_3,0.1667) \\ (s_{-3},-0.1667) & (s_0,0) & (s_5,0) & (s_1,0.3333) \\ (s_5,0) & (s_5,0) & (s_3,0) & (s_0,0) \end{pmatrix}$$

Step 4: Computing the collective performance values of alternatives. The collective performance values p_i ($i = 1, 2, \dots, n$) of alternatives x_i ($i = 1, 2, \dots, n$) can be obtained by Eq. (6).

$$p_1 = \tilde{\Delta}_T \left(\frac{\sum_{j=1}^4 w_j \tilde{\Delta}_T^{-1}(p_{1j})}{\sum_{j=1}^4 w_j} \right)$$

$$= \tilde{\Delta}_T \left(\frac{0.3512 \cdot \tilde{\Delta}_T^{-1}(s_0,0) + 0.1578 \tilde{\Delta}_T^{-1}(s_0,0) + 0.1811 \tilde{\Delta}_T^{-1}(s_2,0) + 0.3099 \tilde{\Delta}_T^{-1}(s_1,0.3333)}{0.3512 + 0.1578 + 0.1811 + 0.3099} \right)$$

$$= \tilde{\Delta}_T(0.8787) = (s_1, -0.1213)$$

$$p_2 = \tilde{\Delta}_T(1.6280) = (s_1, 0.3140) \quad p_3 = \tilde{\Delta}_T(0.2513) = (s_0, 0.2513)$$

$$p_4 = \tilde{\Delta}_T(3.0883) = (s_3, 0.0442)$$

Step 5: Choosing the best alternatives. According to the collective performance values

p_i ($i = 1, 2, \dots, n$), choose the best alternative(s) with the maximum collective performance value.

$$p_4 > p_2 > p_1 > p_3$$

The best choice is the detector production company.

6. Conclusions

To conclude, the proposed modified proportional 2-tuple, which had the form of 2-tuple has and the good qualities of proportional 2-tuple. The comparison rules and three aggregation operators of the modified proportional 2-tuple were also investigated to handle CW of sophisticated linguistic information.

Based on the modified proportional 2-tuple linguistic information, we provided a new method to address group decision making problem in which the experts use linguistic terms with different granularity and different scale at the same time. In the proposed method, BMTLS we set up by determining the granularity, scale and linguistic terms. A function was also reported which can transform the performance values to modified proportional 2-tuple in BMLTS to make the performance values uniform. Furthermore, the new method was fitted to a situation in which the attributes were evaluated by the respective experts of that field, According to the ranges and characteristics of attributes, experts choose freely the linguistic term sets according to their preferences.

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References

- [1] G. Bordogna, M. Fedrizzi and G. Passi, "A linguistic modeling of consensus in group decision making based on OWA Operator", *IEEE transactions on systems, Man, and Cybernetics-Part A, Systems and Humans*, vol. 27, no. 1, (1997), pp. 126-133.
- [2] Z. S. Xu, "A method for multiple attribute decision making with incomplete weight information in linguistic setting", *Knowledge-Based systems*, vol. 20, (2007), pp. 719-725.
- [3] P. P. Bonissone and K. S. Decker, "Selecting uncertainty calculi and granularity: an experiment in trading-off precision and Complexity in: L.H. Kanal, J.F. Lemmer (Eds.)", *Uncertainty in Artificial Intelligence*, North Holland, Amsterdam, (1986), pp. 217-247.
- [4] R. Delgado, J. L. Verdegay and M. A. Vila, "On aggregation operations of linguistic labels", *International Journal of Intelligent systems*, vol. 8, (1993), pp. 351-370.
- [5] F. Herrera and L. Martínez, "A 2-Tuple Fuzzy Linguistic Representation Model For Computing with Words", *IEEE transactions on fuzzy systems*, vol. 8, no. 6, (2000), pp. 746-752.
- [6] F. Herrera and L. Martínez, "A Model Based on Linguistic 2-Tuples for Dealing with Multi-granular Hierarchical Linguistic Contexts in Multi-Expert Decision-Making", *IEEE transactions and systems, man, cybernetics-part B: Cybernetics*, vol. 31, no. 2, (2001), pp. 227-234.
- [7] G. W. Wei, "Extensions of TOPSIS method for 2-tuple fuzzy linguistic multiple attribute group decision making with Incomplete weight information", *Knowledge Information System*, vol. 25, (2010), pp. 623-634.
- [8] Y. C. Dong, Y. F. Xu and S. Yu, "Computing the Numerical Scale of the Linguistic Term Set for the 2-Tuple Fuzzy Linguistic Representation Model", *IEEE transactions on fuzzy systems*, vol. 17, (2009), pp. 1366-1378.
- [9] Y. J. Xu and H. M. Wang, "Approaches based on 2-tuple linguistic power aggregation operators for multiple attribute group decision making under linguistic environment", *Applied soft computing*, vol. 11, (2011), pp. 3988-3997.
- [10] W. Yang and Z. P. Chen, "[New aggregation operators based on the Choquet integral and 2-tuple linguistic information", *Expert Systems with Applications*, vol. 39, (2012), pp. 2662-2668.
- [11] S. P. Wan, "2-Tuple linguistic arithmetic aggregation operators and application to multi-attribute group decision making", *Knowledge-Based Systems*, vol. 45, (2013), pp. 31-40.
- [12] J. H. Park, J. M. Park and Y. C. Kwun, "2-Tuple linguistic harmonic operators and their applications in group decision making", *Knowledge-Based Systems*, vol. 44, (2013), pp. 10-19.
- [13] J. M. Merigó and A. M. Gil-lafuente, "Induced 2-tuple linguistic generalized aggregation operators and their application in decision-making", *Information Sciences*, vol. 236, (2013), pp. 1-16.
- [14] G. W. Wei, "Grey relational analysis method for 2-tuple linguistic multiple attribute group decision making with incomplete weight information", *Expert Systems with Applications*, vol. 38, (2011), pp. 4824-4828.
- [15] S. Y. Wang, "Applying 2-Tuple Multi-granularity Linguistic Variables to Determine the Supply Performance in Dynamic Environment Based on Product-Oriented Strategy", *IEEE Transactions on fuzzy systems*, vol. 16, (2008), pp. 29-39.
- [16] S. L. Zhang, "A model for evaluating computer network security systems with 2-tuple linguistic information", *Computers and Mathematics with Applications*, vol. 62, (2011), pp. 1916-1922.
- [17] A. Fernández, M. J. D. Jesus and F. Herrera, "On the 2-tuples based genetic tuning performance for fuzzy rule based classification systems in imbalanced data-sets", *Information Science*, vol. 180, (2010), pp. 1268-1291.
- [18] K. H. Chang and T. C. Wen, "A novel efficient approach for DFMEA combining 2-tuple and the OWA operator", *Expert Systems with Applications*, vol. 37, (2010), pp. 2362-2370.
- [19] Y. B. Ju, A. H. Wang and X. Y. Liu, "Evaluating emergency response capacity by fuzzy AHP and 2-tuple fuzzy linguistic approach", *Expert Systems with Applications*, vol. 39, (2012), pp. 6972-6981.
- [20] J. H. Wang and J. H. Hao, "A New Version of 2-Tuple Fuzzy Linguistic Representation Model for Computing With Words", *IEEE transactions on fuzzy systems*, vol. 14, no. 3, (2006), pp. 435-445.
- [21] Y. Dong, Y. Xu and S. Yu, "Computing the Numerical Scale of the Linguistic Term Set for the 2-Tuple Fuzzy Linguistic Representation Model", *Transactions on fuzzy systems*, vol. 17, no. 6, (2009), pp. 1366-1378.
- [22] F. Herrera, E. H. Viedma and L. Martínez, "A fusion approach for managing multi-granularity linguistic term sets in decision making", *Fuzzy Sets and Systems*, vol. 114, (2000), pp. 43-58.
- [23] F. Herrera and L. Martínez, "A Model Based on Linguistic 2-Tuples for Dealing with Multi-granular Hierarchical Linguistic Contexts in Multi-Expert Decision-Making", *IEEE transactions on systems, man, and cybernetics-part B: Cybernetics*, vol. 31, no. 2, (2001), pp. 227-234.
- [24] S. L. Chang, R. C. Wang and S. Y. Wang, "Applying a direct multi-granularity linguistic and strategy-oriented aggregation approach on the assessment of supply performance", *European Journal of Operational Research*, vol. 177, no. 2, (2007), pp. 1013-1025.
- [25] Z. Chen and D. B. Arieh, "On the fusion of multi-granularity linguistic label sets in group decision making", *Computers and Industrial Engineering*, vol. 51, no. 3, (2006), pp. 526-541.

- [26] Z. P. Fan and Y. Liu. "A method for group decision-making based on multi-granularity uncertain linguistic information", *Expert Systems with Applications*, vol. 37, (2010), pp. 4000-4008.
- [27] Z. Zhang and C. Guo, "A method for multi-granularity uncertain linguistic group decision making with incomplete weight information", *Knowledge-Based Systems*, vol. 26, (2012), pp. 111-119.
- [28] J. J. Zhu and K. W. Hipel, "Multiple stages grey target decision making method with incomplete weight based on multi-granularity linguistic label", *Information Sciences*, vol. 212, (2012), pp. 15-32.
- [29] L. Martínez and F. Herrera, "An overview on the 2-tuple linguistic model for computing with words in decision making: Extensions, applications and challenges", *Information Sciences*, vol. 207, (2012), pp. 1-18.

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