International Journal of Database Theory and Application
Vol.9, No.12 (2016), pp.173-184
http://dx.doi.org/10.14257/ijdta.2016.9.12.16

Improved PSO Research for Solving the Inverse Problem of Parabolic Equation

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Abstract

Parameter identification problem has important research background and research value, has become in recent years inverse problem of heat conduction of top priority. This paper studies the Parabolic Equation Inverse Problems of parameter identification problem, and applies PSO to solve research. Firstly, this paper establishes the model of the inverse problem of partial differential equations. The content and classification of the inverse problem of partial differential equations are explained. Frequently, the construction and solution of the finite difference method for parabolic equations are studied, and two stable schemes for one dimensional parabolic equation are given. And two numerical simulations were given. Partial differential equation discretization was with difference quotient instead of partial derivative. The partial differential equations with initial boundary value problem into algebraic equations, and then solving the resulting algebraic equations. Then, the basic principles of PSO and its improved algorithms are studied and compared. Particle swarm optimization algorithm program implementation. Finally, the Parabolic Equation Inverse Problems of particle swarm optimization algorithm performed three simulations. We use a set of basis functions gradually approaching the true solution, selection of initial value. The reaction is converted into direct problem question, then use difference method Solution of the direct problem. The solution of the problem with the additional conditions has being compared. The reaction optimization problem is transformed into the final particle swarm optimization algorithm to solve. Verify the Parabolic Equation Inverse Problems of particle swarm optimization algorithm correctness and applicability.

Keywords*: parabolic equation; finite difference method; parameter identification problem; Particle Swarm Optimization*

1. Introduction

In many fields (such as engineering, biological engineering, etc.), there are a lot of observations are not directly measured, then we need the help of some methods and some of the known conditions inverse true face of these observations, This is called Parameter Identification Inverse Problems in mathematics. In addition to the inverse operations we know, there are the points of the original proposition and the inverse proposition, the definition of function and inverse function, the meaning of integral and differential, etc. These are the positive and negative ideas that we have come into contact with. In the five categories of dialectics, the causes and results are dialectical and relative. Normal thinking is the result is obtained by reason. However, there are some things that do not

ISSN: 2005-4270 IJDTA

know the reason, but know the result. What is the reason for such a result, we need to go back to the reasons by the results. This kind of thinking is what we often say the reverse thinking.

The problem is the product of such a thought. If some of the known items are for some reason in the above problem. It is now the amount of unknown to be asked. But given a certain condition of the unknown function is $u(x,t)$. For example: a moment of distribution. As a supplement, to solve the unknown and unknown function $u(x,t)$ of the problem has become an inverse problem.

For example: parabolic equation set solution problem is as following.

$$\begin{cases} \dfrac{\partial u}{\partial t} = a^2 \dfrac{\partial^2 u}{\partial x^2}, 0 < x < 1, 0 < t < T \\ u(x,0) = \varphi(x), 0 \le x \le 1 \\ \dfrac{\partial u}{\partial x}(0,t) = \phi_1(t), 0 < t \le T \\ \dfrac{\partial u}{\partial x}(1,t) = \phi_2(t), 0 < t \le T \end{cases}$$

Which the parameter a is known and $\varphi(x), \phi_1(t), \phi_2(t)$ are known.

We often call the differential equation called the generalized equation, which can describe the law of some physical phenomena completely. The additional condition is the fixed solution condition, the condition of the fixed solution includes initial condition and boundary condition. The initial state of the physical phenomena and the constraints is on the boundary. By the generalized equation and the condition of the definite solution

The mathematical problem is called the fixed solution problem.

When the parameter a is no longer given and become unknown. But if you give an additional condition, such as the long-term observation value of space position 1 is as follows:

$$u(1,t) = g(t), 0 < t \le T$$

The inverse problem of parabolic equation is composed of the parameter a and the unknown function $u(x,t)$.

$$\begin{cases} \dfrac{\partial u}{\partial t} = a^2 \dfrac{\partial^2 u}{\partial x^2}, 0 < x < 1, 0 < t < T \\ u(x,0) = \varphi(x), 0 \le x \le 1 \\ \dfrac{\partial u}{\partial x}(0,t) = \phi_1(t), 0 < t \le T \\ \dfrac{\partial u}{\partial x}(1,t) = \phi_2(t), 0 < t \le T \\ u(1,t) = g(t), 0 < t \le T \end{cases}$$

Which the parameters a and $u(x,t)$ are unknown and $\varphi(x), \phi_1(t), \phi_2(t)$ are known.

The inverse problems of differential equations as a new role on the stage of history have a mark. The control of the mass transfer and the heat conduction process of the water flow and the accompanying water flow have been realized in the water environment system. In geophysical exploration, the structure of the earth's interior or the locations of the underground mineral deposits are determined by the seismic wave measurements.

Partial differential equation has a long history. In the course of its development, closely linked to the development of realistic features and production of science and technology, partial differential equations of the research content has been enriched and updated, many mathematical problems been solved, many new mathematical method was born, partial differential equation theory has been fully developed, but also to promote the partial differential equation and the mathematics of many branches and natural science departments between contact.

2. The Model and Classification of the Inverse Problem

The members of the inverse problem of the partial differential equations are partial differential equations, initial conditions, boundary conditions and an additional condition:

The differential equation is $Lu(x,y,t) = f(x,y,t),(x,y) \in \Omega, t \in (0,\infty)$

The initial condition is $Iu(x,y,t) = \psi(x,y),(x,y) \in \Omega$

The boundary condition is $Bu(x,y,t) = \varphi(x,y,t),(x,y) \in \partial\Omega_1$

The additional condition is $Au(x,y,t) = \kappa(x,y,t),(x,y) \in \partial\Omega_2$

The true solution of partial differential equation is $u(x,y,t)$, which $f(x,y,t)$ is the right end of partial differential equation. The function $\psi(x,y)$ is the initial condition. The function $\varphi(x,y,t)$ is the boundary condition and $\kappa(x,y,t)$ is the additional condition of partial differential equation. *L* is differential operator. *I* is initial operator. *B* is boundary operator and *A* is additional operator. When one of the $f(x,y,t),\psi(x,y),\varphi(x,y,t)$ variables is unknown, the inverse problem of partial differential equation is formed.

For the inverse problem of partial differential equations, mainly including the following three categories:

First kinds: solving the inverse problem of unknown parameters - the inverse problem of parameter identification. This kind of problem is the most common kind of inverse problem.Second categories: to solve the right side of the inverse problem - the source of the identification of the inverse problem.The third kind: solving the initial condition inverse problem - the inverse time problem.The fourth kind: solving the inverse problem of boundary condition - the inverse problem of boundary control.

In this paper, we mainly study the problem of parameter identification for the first class. For the inverse problem of parabolic equation, we care about the content is divided into 3 aspects:

(1) The proper characterization of solutions. A fixed solution problem, if the existence, uniqueness, stability, is known as well set; if one of the conditions is not satisfied, then it is called ill posed. Now we study the most is the existence and uniqueness of the solution, and sometimes will study stability.

(2)Numerical solution of the inverse problem of the parabolic equation is introduced now. There are some ways to study the problem up to now:

①Iterative method: the idea of this method is relatively simple, but the calculation is complex and the convergence is not good.

②The use of pulse technology (PST): this method is more widely used in reality, but this method will involve the Green function, to the calculation of a lot of trouble.

③Quantum scattering inversion method: Although this method has high precision, but the computation is very large, so it is not practical. D. monotone homotopy method:

this method has advantages, not only can solve nonlinear problems, but also in a wide range of convergence, but the structure of the homotopy mapping is not easy.

④The inverse finite element method: this method avoid the Green's function, is widely used in engineering mechanics, but can only get the discrete solution can not be expressed in a formula of approximate solutions.

(3) The research methods of the solution are mainly:

①Regularization method: the basic idea of this method is to approximate the solution of the original problem with the solution of a cluster and the adjacent problem of the original problem.

②This method is a traditional method to solve the inverse problem. The method has the advantages of high stability and good convergence, but it is a difficult task to determine the initial value of the iterative method.

③Genetic algorithm: this method uses the group search strategy. And use probability change rules to control the search direction. But it is not stable and operation is large, so it is usually used in combination with other methods.

3. Particle Swarm Optimization

Optimization problems reflect some of the very common phenomena that occur in practice. That is, in some ways, the best results are obtained as far as possible under the conditions permitted. In various fields, such as artificial intelligence, system control, production scheduling, pattern recognition and computer engineering, the optimization problem has been applied.The implication of the optimization problem is to find a set of numerical values under certain conditions, so that some optimality metrics can be satisfied. Mathematical symbols are described as:

$$\min \sigma = f(X)$$
$$s.t. X \in S = \{X | g_i(X) \le 0, i = 1, \cdots, m\}$$

Among them $\sigma = f(X)$ is the objective function and $g_i(X)$ are the constraint conditions.

Optimization algorithms for solving optimization problems can be divided into two categories. The classical optimization algorithms: simplex method, the ellipsoid algorithm, point method, the steepest decreased method, conjugate gradient method, Newton's method simplex method, the ellipsoid algorithm, point method, the steepest decreased method, conjugate gradient method, Newton's method. With the development of practical problems more and more complex, the classical algorithm is generally using local information, which can not avoid local minima.

Heuristic optimization algorithm is from some inspired by nature, people gradually in the law of nature itself runs found many ways to solve the practical problems, people will this method said is a heuristic algorithm, such as: Holland simulation earth biological evolution rule proposed genetic algorithm and metropolis simulation solid substance in the statistical physics of the crystallization process proposed simulated annealing algorithm, Eberhart and Kennedy group second foraging behavior of particle swarm optimization algorithm is proposed based on.

The research of particle swarm optimization algorithm is based on the background of bird flying foraging, and the best position is achieved through mutual help and collective cooperation among birds and birds. Although each individual's behavior is very simple, it is very complicated to be integrated into the whole group. Each particle in the particle swarm optimization algorithm with respect to the best particle search in the whole solution space. Particle swarm optimization algorithm has the advantage of a lot, compared with other algorithms in terms of particle swarm optimization algorithm continuity did not any special requirements, but also only slightly parameters need to set and adjust, with the exception of the algorithm of computing speed fast and simple to implement, and profound intelligence background, not only for scientific research, and particularly suitable for engineering application.

(1) the development of particle swarm optimization algorithm is as follows:

Basic particle swarm optimization algorithm is proposed for the first time by Eberhat & Kennedy.Den Bergh Van through the particle swarm in the best particle is always in a state of motion, to be guaranteed to converge to the local optimal algorithm.Kennedy studies the topological structure of the particle swarm, and analyzes the information flow of the ionic bond, and proposes a series of topological structure.Angeline will be introduced into the PSO operator, select the better after each iteration of the particle and copied to the next generation, to ensure that each iteration of the particle has a better performance.Higashi proposed mutation PSO algorithm, through the mutation operator to jump out of the attraction of the local extreme points, so as to improve the ability of the global search algorithm.Baskar proposed cooperative PSO algorithm, through the use of multi particle swarm optimization problem of different dimension and multi particle swarm optimization and other methods to improve.Quantum PSO, simulated annealing PSO, dissipative PSO, adaptive PSO and other hybrid improved algorithm.

Particle swarm algorithm is concise, easy to realize and less parameter adjustment, do not need gradient information advantages, so the PSO is continuous nonlinear optimization problem, combinatorial optimization problems and mixed integer nonlinear optimization problems like the optimization tool. At present have made a lot of achievements in the neural network training, function optimization, fuzzy system control, and other genetic algorithms applications, such as Eberhart et al. Success will particle swarm optimization algorithm is applied to Parkinson's syndrome and trembling disease analysis; he et al. The success of the particle swarm optimization algorithm is applied in training fuzzy neural network improved the speed of the update equation; et al Parsopoulos success of the particle swarm optimization algorithm is applied to the multi-objective optimization problem, maximum and minimum problem, integer programming problem and positioning all global extremum problem solving. In addition, the design of fuzzy controller, job shop scheduling, robot real time path planning, automatic target detection and time frequency analysis are the concrete application examples of PSO algorithm.

3.1. Basic Particle Swarm Optimization Algorithm

The basic particle swarm optimization algorithm is based on the behavior of birds, which was proposed by Eberhart and Kennedy in 1995. Background is that a flock of randomly distributed in a habitat, in this habitat is only a piece of food. Now the birds do not know where the food, but they know where they and food distance, so how can the fastest find food? The easy way is to follow the nearest bird, which is so far away from the food. So put the piece of food as particle swarm optimization algorithm of the utility model has the advantages that the birds and the distance between the piece of food as for the function of fitness value (judge the current position is not optimal), so the birds to find food is to seek the optimal process.

In space, each particle is treated as a point in the space. Assuming that the population is composed of a particle size m, if the size of the population is too large, and the speed and convergence of the algorithm is affected.

Four key assumptions are as follows:

①Based on the position of the $i(i=1,2,\cdots,m)$ particle in the D dimension space is $z_i=(z_{i1},z_{i2},\cdots,z_{iD})$. The fitness value of the current position z_i is calculated according to the fitness function to measure the current position of the particle.

②Suppose $v_i=(v_{i1},v_{i2},\cdots,v_{iD})$ is the particle velocity.

③Suppose $p_i=(p_{i1},p_{i2},\cdots,p_{iD})$ is the best position for particle search so far.

④Suppose $p_g=(p_{g1},p_{g2},\cdots,p_{gD})$ is for the best position of the particle swarm optimization

In the iterative process, the formula for the velocity and position of each particle is:

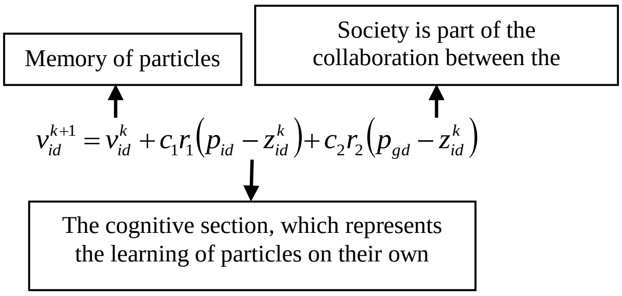


Figure 1. Velocity and Position Formula of Particle

Among $z_{id}^{k+1}=z_{id}^{k}+v_{id}^{k+1}$, $i=1,2,\cdots,m$, $d=1,2,\cdots,D$. k is the number of iterations. r_1 and r_2 is random numbers between $[0,1]$ and used to maintain the diversity of groups. c_1 and c_2 for the learning factor (acceleration factor), the particles have the ability to self sum up and learn from the excellent individuals in the group, so as to close to the best advantage.

Psychological explanation is in the exploration of cognitive process, everyone will think own understanding is correct and appropriate to retain their own understanding, when considering the cognitive around other people, if the recognition from others better than their own, people will dynamically update their cognition, when other people's cognition as their own good, people will retained their cognitive.

3.2. Algorithmic Processes

Begin

Select the threshold ε and the maximum number of iterations $N_{\max}$

Initial particle position is $z_i^{(0)}=(z_{i1},z_{i2},\cdots,z_{iD}),i=1,2,\cdots,m$

Initial velocity of each particle is $v_i^{(0)}=(v_{i1},v_{i2},\cdots,v_{iD})$

The fitness value $z_i^{(0)}$ of each particle is measured, which is expressed as $\overline{D_i^{(0)}}$, assume here is the objective function for the solution of the solution.

$p_i^{(0)}=z_i^{(0)}$

According $\overline{D_i^{(0)}}=\min\left\{\overline{D_1^{(0)}},\overline{D_2^{(0)}},\cdots,\overline{D_m^{(0)}}\right\}$ to find the global optimal $p_g^{(0)}$

$k=0$

$k\leftarrow k+1$

Update $v_i^{(k)}$

Update $z_i^{(k)}$

The fitness value z_i of the measurement is expressed as $\overline{D_i^{(k)}}$

$\overline{D^k} = \min\left\{\overline{D_1^{(k)}}, \overline{D_2^{(k)}}, \cdots, \overline{D_m^{(k)}}\right\}$

Update $p_i^{(k)}$ and $p_g^{(k)}$

If $\left(\overline{D^{(k-1)}} - \overline{D^{(k)}}\right) / \overline{D^{(k)}} > \varepsilon$ and $k < N_{\max}$, jump to step 1

End

The flow chart is as follows:

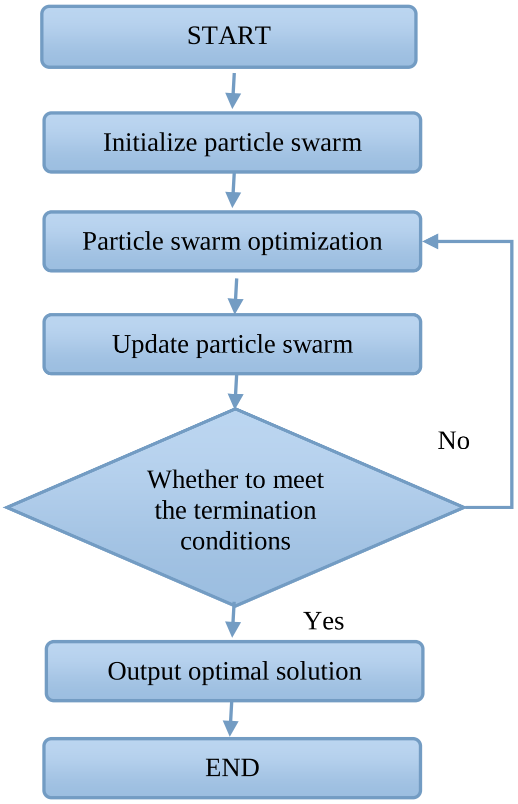


Figure 2. Flow Chart of Algorithm

4. Improved Particle Swarm Optimization Algorithm

Basic particle swarm optimization algorithm is as follows

$$v_{id}^{k+1} = v_{id}^{k} + c_1 r_1 \left(p_{id} - z_{id}^{k}\right) + c_2 r_2 \left(p_{gd} - z_{id}^{k}\right)$$

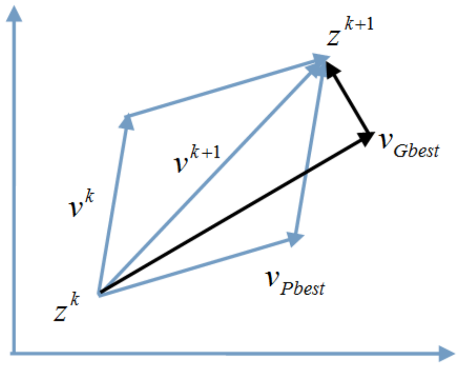
In the course of the study without a "cognitive" section and the "social" part, each particle will in accordance with the original speed to a direction to go flying, until it reaches the boundary, but if so that the particles will have great may fail to find the optimum solution of the, or let have is the optimal solution from the beginning has on the trajectory of the particle movement, but this is very few.

If there is no part of the memory of the particle, particle velocity will be determined by the current position and historical best position, because there is no memory particles, if the particle at the beginning in the superior position, then he would keep rest until the other particles find better position, so as to find the global optimum, so if no part of this, particle swarm optimization algorithm were will search scope with a little bit of evolution and gradually reduced, if it is so only is to get the optimal solution in the beginning of the search area, particles can be found to the solution.

Taking into account the above mentioned problems, Shi and Eberhart added an inertia weight to the velocity updating formula, i.e.:

$$v_{id}^{k+1} = \omega v_{id}^{k} + c_1 r_1 \left(p_{id} - z_{id}^{k}\right) + c_2 r_2 \left(p_{gd} - z_{id}^{k}\right)$$

The position update formula is invariant and the particle position adjustment process is shown in figure.:



z^k : Current search point, z^{k+1} ： Adjusted search point

v^k ： Current speed, v^{k+1} ： Adjusted speed

v_{Pbest} ： Speed based on pbest, v_{Gbest} ： Speed based on gbest

Figure 3. Particle Adjustment Position Processes

When given a smaller inertia weight (< 0.8), if PSO can find the global optimal solution, then its search time is very short; if an optimal solution to it in the search space, PSO is easy to can find the global optimal solution, otherwise it can not find the global optimal solution. When given a larger inertia weight (> 1.2), PSO is more like a global search method, it always to explore new search area, but it will take more iterations to reach the global optimal solution or, more likely, cannot find the global optimal solution. When given a moderate inertia weight, PSO will have a greater chance to find the global optimal, but the number of iterations will increase.

Chaos particle swarm optimization algorithm (CPSO) was proposed by Gao Ying et al in 2004 to introduce the idea of chaos optimization. Sometimes in order to prevent some of the particles in the iterative process of the stagnation phenomenon in the chaotic particle swarm optimization algorithm, the algorithm can use of the ergodicity of chaotic variables on particle swarm so far to search the global optimal position based on iteration will generate a chaotic sequence. The sequence is obtained in the optimal particle position to random to replace the current particle swarm in a particle position and iterative. This would solve the above appears as part of a particle stagnation caused the premature convergence problem.

The specific steps of the chaotic particle swarm optimization algorithm are as follows:

Step 1: to determine the parameters of the algorithm, random generation of N particle populations, initialization of the particle.

Step 2: update the particle velocity and position according to the basic particle swarm algorithm.

Step 3: the optimal position x_{gbest} of particle swarm optimization:

① x_{gbest} through equation as follows:

$$y_1^k = \frac{x_{gbest}^k - R_{\min}^k}{R_{\max}^k - R_{\min}^k}$$

It mapped to the domain of the $Logistic$ equation in interval $[0,1]$.

② y_1^k through $Logistic$ equation as follows:

$$y_{n+1}^k = \mu y_n^k \left(1 - y_n^k\right)$$

It iterated M times, then the chaotic sequence $y^k = \left(y_1^k, y_2^k, \cdots, y_M^k\right)$ is obtained.

③The chaotic sequence is mapped back to the original solution space by inverse of the equation:

$$x_{gbest,m}^{*k} = R_{\min}^k + (R_{\max}^k - R_{\min}^k) y_m^k, m = 1,2,\cdots,M$$

The feasible solution sequence of a chaotic variable is produced:

$$x_{gbest}^{*k} = (x_{gbest,1}^{*k}, x_{gbest,2}^{*k}, \cdots, x_{gbest,M}^{*k})$$

④The adaptive value of each feasible solution vector in the feasible solution sequence is calculated, and the feasible solution vector x_g^{*k} corresponding to the optimal value of the adaptive value is retained.

Step 4: randomly select a particle from the current particle swarm, and use the position vector x_g^{*k} instead of the position vector of the particle.

Step 5: jump to step (2) until the algorithm reaches the maximum number of iterations or the solution is satisfactory enough.

5. Numerical Simulation

Inverse problem is as follows:

Parabolic equation: $\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left[a(x,t)\frac{\partial u}{\partial x}\right] + f(x,t) \quad (0 < x < 1, 0 < t < 0.2)$

Initial condition: $u(x,0) = xe^x \quad (0 < x < 1)$

Boundary conditions: $u(0,t) = 0$, $u(1,t) = e^{1+t} \quad (0 < t < 0.2)$

Additional conditions: $u(x,0.2) = xe^{x+0.2} \quad (0 < x < 1)$

Among them $u(x,t)$ and $a(x,t)$ is unknown and $f(x,t)$ is as follows:

$$f(x,t) = -\left(3 + 3x + x^2 + 2t + xt\right)e^{x+t}$$

This case is a parameter inversion problem, which $a(x,t)$ needs to be obtained by additional conditions. The solution procedure is as follows:

Step 1: solving analysis

In this case, the real solution is $a(x,t) = 1 + x + t$. Now select a set of basis functions $\Phi(x,t) = \{1, x, t\}$. Used $k_1 + k_2 x + k_3 t$ to gradually approach $a(x,t)$.

Initial value is as follows:

$$a_0(x,t) = \frac{1}{2} + \frac{1}{2}x + \frac{1}{2}t.$$

So the inverse problem is transformed into a positive problem solving $u(x,t)$. Then the forward difference method is used to solve the forward problem $u_0(x,t)$. $u_0(x,0.2)$ can be obtained by $u_0(x,t)$. Compared $u_0(x,0.2)$ with the additional conditions $u(x,0.2)=(x+x^2)e^{x+0.2}$, this will turn the problem into an optimization problem:

$$\min \frac{\sum_{i=0}^{n}\left[u_m(x_i,0.2)-u(x_i,0.2)\right]^2}{n+1}$$

The function $u_m(x,0.2)$ meets the following questions:

$$\begin{cases} \dfrac{\partial u}{\partial t}=\dfrac{\partial}{\partial x}\left[a(x,t)\dfrac{\partial u}{\partial x}\right]+f(x,t) & (0<x<1,0<t<0.2) \\ u(x,0)=(x+x^2)e^x & (0<x<1) \\ u(0,t)=0,\ \ u(1,t)=2e^{1+t} & (0<t<0.2) \end{cases}$$

Solving this optimization problem, we use our particle swarm optimization algorithm to solve the problem.

Step 2: set the parameters

Using explicit forward difference method to solve the function $u_0(x,t)$. Step is $\Delta x=l=0.2$, $\Delta t=k=0.02$. Particle swarm optimization algorithm is used to solve k_1 and k_2 and k_3, and $N=60, c_1=c_2=2,\ w=0.8,\ M=200, D=3$.

Step 3: programming solution

Step 4: result

Table 1. Calculation Results

Iteration number	k_1	k_2	k_3	Objective function value
2000	0.8993147292	0.7908948369	1.2837422132	0.0212333425
3000	1.2210350846	1.1307238482	0.8927833841	0.0193246474
4000	1.0201035084	0.9923475837	0.9682451874	0.0131244532
true solution	1	1	1	

When the iteration number of times $n=4000$ the error is calculated as follows:

$$e=\frac{\|a-a_n\|_2}{n}=7.1797\times10^{-3}.$$

The curved surface of results $a(x,t)$ and $a_{4000}(x,t)$ are as follows:

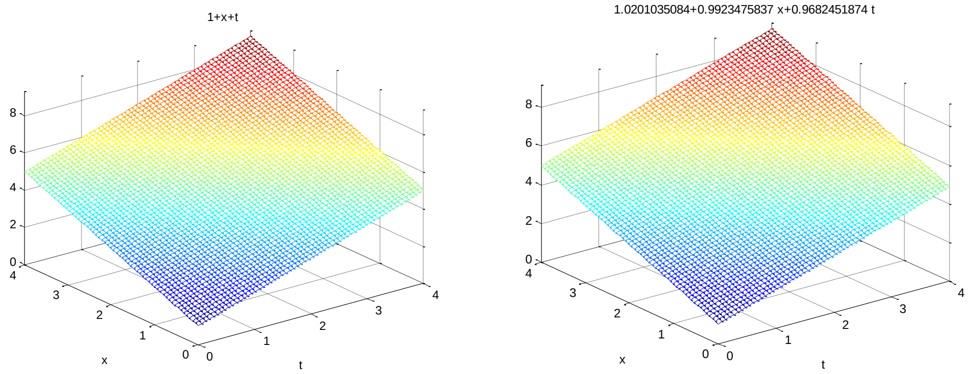


Figure 4. Curved Surface of $a(x,t)$ and $a_{4000}(x,t)$

In this section, we give a concrete example to solve the problem of the inverse problem of parabolic equation with the particle swarm optimization algorithm for solving parabolic equations. It is aimed at the numerical simulation of the inverse problem of the parameter, the real solution and the approximate solution are obtained, and the feasibility of the method is verified.

6. Summary

Inverse problem of parameter identification have important research background and research value, has become the most important inverse heat conduction problem research in recent years, but research on this problem is not very perfect, the research for makes a meaningful attempt to seek reasonable and effective inverse problem solving method. Particle swarm optimization algorithm of time is not very long, there are a lot of places need to be further explored. In this paper, the its application to the inverse problem of parabolic equation solving, to verify the feasibility of this method, the approximate solution and the true solution between the error is small and can therefore ask service. Secondly, the numerical simulation can be seen using particle group optimization algorithm is simple, is easy to realize and adjustable parameters are few, and do not need to the advantage of gradient information.

Acknowledgments

This paper is supported by the Hebei Science and technology project (15456252), Tangshan Science and Technology Project (14130271a).

References

[1] J. Zhen, L. Huilian and W. Qinghua, "Particle swarm optimization algorithm and application", Science Press, Beijing, **(2009)**.
[2] F. Hongming, "The numerical solution of partial differential equations", the machine industry press, Beijing, **(2014)**.
[3] Z. Suochun, "Finite difference numerical calculation for the parabolic problem of parabolic type", Science Press, Beijing, **(2010)**.
[4] W. Yanfei, "The calculation method and application of the inversion problem", Higher education press, Beijing, **(2007)**.
[5] L. Hui, "Particle swarm optimization for inverse problems of partial differential equations", Xi'an University of technology, Xi'an, **(2010)**.
[6] M. Tao, L. Hui, Y. Xiaoli and L. Hongpeng, "A hybrid particle swarm optimization algorithm for the inverse problem of parabolic equations", Xi'an University of technology, Xi'an, **(2010)**.
[7] X. Xiujuan, "The finite difference method for two kinds of parabolic equations", Harbin Institute of Technology, Harbin, **(2009)**.
[8] Y. Sufang, "A new method of parabolic partial differential equation inverse problem", Harbin Institute of Technology. Harbin, **(2006)**.

[9] L. Lin, L. Tao and J. Tao, "Research on Sample-level Parallel Simulation Based on Multi-node Multi-core Environment", Journal of System Simulation, vol. 11, no. 22, **(2011)**, pp. 115-119.
[10] Y. Wang, D. Wang, G. Liu and Z. Yang, "SVC Method for Textile Weave Recognizing", Journal of Digital Content Technology and its Applications, vol. 9, no. 5, **(2010)**, pp. 16-19.
[11] T. T. M. Onyango, D. B. Ingham, and D. Lesnic, "Reconstruction of heat transfer coefficients using the boundary element method", Journal of Computer Math Apply, vol. 56, no. 1, **(2008)**, pp. 114-126,
[12] F. Ternat, O. Orellana and P. Daripa, "Two stable methods with numerical experiments for solving the backward heat equation", Journal of Apply of Number Math, vol. 61, no. 2, **(2011)**, pp. 266-284.

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