

## Learning Efficiency Evaluation on the College Students via Cross Data Envelopment Analysis

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### Abstract

*This study aims to evaluate the comprehensive learning efficiency of college students via Cross DEA. Firstly the study introduces DEA model and analyzes its weakness. Based on it, Cross DEA is introduced in the paper. Secondly, taking eight college students majoring in international trade as an example, the study establishes an evaluation indicator system of learning efficiency. Based on relevant collected data, the study makes the corresponding data processing via MATLAB and EXCEL software, analyzes the learning efficiency of the students according to the processing result. Finally the study evaluates the performance of DEA efficiency and resort them, and effectively distinguish individual student learning. The use and analysis of Cross DEA can provide some important reference and help to propose improvements for students and education authorities.*

**Keywords:** College students; Cross; Data Envelopment Analysis (DEA); Evaluation; Learning efficiency

### 1. Introduction

With the increasing popularity and flourish of higher education, college students will face increasing pressure on employment. In order to better adapt to this trend of the society, college students have to learn more knowledge and master more skills. But learning time of students in school is very limited, they must consider good learning methods, and higher learning efficiency. China has placed extra emphasis on quality education and made the comprehensive quality evaluation of students [1]. At present, the evaluation methods of the students' learning efficiency in the colleges and universities are more concerned, and some more mature evaluation methods are also introduced in the schools. At present, the main practice in many colleges and universities are indicators for the evaluation of linear weighted to get the final results [2], some scholars have introduced the analytic hierarchy process [3-4], fuzzy analytic hierarchy weighting method [5-6], neural network method [7-9], and these methods depend on experts to make the different indicators. Because each expert is different and their opinions are subjective, the given weights are different, so the evaluation score is very random, it will directly have an effect on the stability of the evaluation results. And DEA has been introduced to solve the above problems [10]. But DEA model has some blind sides in the DEA efficiency and ranking. In order to solve above problems, this paper tries to establish Cross DEA model to study the learning efficiency of College students.

## 2. DEA Model and Cross DEA Model

### 2.1. DEA Model

DEA is a popular and non-parametric method to measure the relative efficiency of a group of decision making units (DMUs) (such as the universities, hospitals, banks *etc.*), which is developed by Charnes, Cooper and Rhodes. Since then, the method has spread rapidly and is used widely, especially in the field of efficiency evaluation, and the basis of this method is linear programming theory and Western econometric [11].

Based on the "relative efficiency" concept of development, efficiency is measured by using multiple outputs and multiple inputs without the need to predetermine their weights. And weights are determined by DEA models rather than by a decision maker. So the evaluation results are objective.

For each decision making unit (DMU), research is conducted related to multiple input indicators and output indicators. Supposing  $n$  DMUs will be evaluated, each DMU with  $m$  different inputs and  $s$  different outputs,  $x_{ij}$  represents amount of the NO.  $i$  input consumed by  $DMU_j$ ,  $y_{rj}$  represents amount of the NO.  $r$  output produced by  $DMU_j$ .

In 1978, Charnes A, Cooper WW and Rhodes E put forward the DEA model *CCR*, which includes two forms of a fractional programming model and linear programming [12]. For the convenience of calculation, people usually use the latter. The *CCR* model is as follows in Equation (1):

$$\text{Maximize } \theta_0 = \frac{\sum_{r=1}^s u_r y_{r0}}{\sum_{i=1}^m v_i x_{i0}}$$

$$s.t. \begin{cases} \theta_j = \frac{\sum_{r=1}^s u_r y_{rj}}{\sum_{i=1}^m v_i x_{ij}} \leq 1, j = 1, K, n \\ u_r, v_i \geq \varepsilon, r = 1, K, s; i = 1, K, m \end{cases} \quad (1)$$

Where  $u_r (r = 1, \dots, s)$  and  $v_i (i = 1, \dots, m)$  are respectively the output and input weights to be determined,  $DMU_0$  represents the DMU under evaluation and  $\varepsilon$  is a very small positive number called non-Archimedean infinitesimal.

Efficiency is a relative value, which is measured relative to the other *DMU* and is therefore referred to as relative efficiency. *CCR* model is to seek a set of input and output weights that are the most favorable to  $DMU_0$  to maximize its efficiency under the constraints that all the efficiency of  $n$  DMUs are less than or equal to one. As the weights are the most favorable to  $DMU_0$ , its efficiency is referred to the best relative efficiency. And solution of *CCR* Model by Linear Programming is as follows:

$$\text{Maximize } \theta_0 = \sum_{r=1}^s u_r y_{r0}$$

$$s.t. \begin{cases} \sum_{i=1}^m v_i x_{i0} = 1 \\ \sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} \leq 0, \\ j = 1, K, n \\ u_r, v_i \geq \varepsilon, r = 1, K, s; i = 1, K, m \end{cases} \quad (2)$$

In above Equation (2), if  $\theta_0^* = 1$ , then  $DMU_0$  is referred to DEA efficiency, otherwise it is referred to non-DEA efficiency. And the slack variables  $S^-$  and the remaining variables  $S^+$  are further introduced, the above inequality constraints are converted into equality constraints, and its dual programming model  $CCR - D$  is as follows in Equation (3):

$$\begin{aligned}
 & \text{Minimize } \theta \\
 & s.t. \begin{cases} \sum_{j=1}^n \lambda_j X_j + S^- = \theta X_0 \\ \sum_{j=1}^n \lambda_j Y_j - S^+ = Y_0 \\ \lambda_j \geq 0 (j=1, 2, \dots, n) \\ S^- \geq 0, S^+ \geq 0 \end{cases} \quad (3)
 \end{aligned}$$

If  $\theta_0^* = 1$ ,  $S^{-*}$  and  $S^{+*}$  is not all zero,  $DMU_0$  is weak efficiency. Weak DEA units cannot make full use of all the input and output information to generate the best relative efficiency of one. So they are not real DEA efficient units.

### 2.2. Cross DEA Model

In order to overcome the weakness of the DEA model  $CCR$ , this study introduces the method of Cross DEA [13-14]. Its main idea is to use optimal weight of else DMU to calculate the evaluation value of present DMU, and cross evaluation values are obtained, finally the average value of all cross evaluation values is regarded as the evaluation value of present DMU. And the calculation matrix is shown as Table 1, in which numerical values on the main diagonal are DEA evaluation value, and others are cross evaluation values.

**Table 1. The Calculation Matrix of Cross DEA Model**

DMU	Cross efficiency				Average
	DMU <sub>1</sub>	DMU <sub>2</sub>	...	DMU <sub>n</sub>	
DMU <sub>1</sub>	$\theta_{11}$	$\theta_{12}$	...	$\theta_{1n}$	$\sum \theta_{1j}/n$
DMU <sub>2</sub>	$\theta_{21}$	$\theta_{22}$	...	$\theta_{2n}$	$\sum \theta_{2j}/n$
...	...	...	...	...	...
DMU <sub>n</sub>	$\theta_{n1}$	$\theta_{n2}$	...	$\theta_{nn}$	$\sum \theta_{nj}/n$

### 3. The Comprehensive Learning Efficiency Evaluation Via Cross DEA

The comprehensive learning efficiency evaluation of college students is an overall assessment for the development situation of individual students. In order to analyze the advantages and disadvantages of college students, the school need to make corresponding contrast of college students between the input indicators and output indicators, thus has a comprehensive efficient evaluation results for the college students.

#### 3.1. The Selection of the Evaluation Indicators

The evaluation basis of DEA is input indicators and output indicators of DMU. The input indicators refer to the something that need to be consumed, the output indicators refer to some information or results through the inputs of the system. The students' comprehensive evaluation is divided into two levels: one is the subject, the other is social

practice. Then the DEA method is used to implement the analysis from the angle of input and output.

### 3.2. The Evaluation Indicator System

Through the above analysis, this study establishes the evaluation indicator system of college students' learning efficiency based on DEA, which is as shown in Table 2:

**Table 2. The Evaluation Indicator System of College Students' Learning Efficiency**

First-level indicators	Second-level indicators	NO	Weight	Connotation of indicators
Input indicators	The class situation	A <sub>1</sub>	v <sub>1</sub>	It refers to the time spent by the students in class every month;
	The completing of the schoolwork	A <sub>2</sub>	v <sub>2</sub>	It refers to the time spent by the students to complete the schoolwork every month;
	The learning situation after school	A <sub>3</sub>	v <sub>3</sub>	It refers to the time spent by the students to study in addition to finish the homework every month;
	The social practice	A <sub>4</sub>	v <sub>4</sub>	It refers to the time spent by the students to participate in social practice every month;
	The physical exercise	A <sub>5</sub>	v <sub>5</sub>	It refers to the time that the student spent to participate in physical exercise every month;
Output indicators	The Foundation course grades	B <sub>1</sub>	u <sub>1</sub>	It refers to the average grade of seven foundation courses , including Politics, English, <i>etc.</i> ;
	The professional course score	B <sub>2</sub>	u <sub>2</sub>	It refers to the average grade of seven professional courses , including Discrete Mathematics, <i>etc.</i> ;
	The Physical curriculum grade	B <sub>3</sub>	u <sub>3</sub>	It refers to the average grade of the physical curriculum selected by the student.

### 3.3. Case Study

Through the above analysis, this study establishes the evaluation indicator system of college students' comprehensive evaluation of learning efficiency based on DEA. In the major in international trade of our college, eight professional sophomores are taken as an example to analyze comprehensive evaluation of the performance. The eight students are regarded respectively as  $DMU_1$ ,  $DMU_2$ ,  $DMU_3$ ,  $DMU_4$ ,  $DMU_5$ ,  $DMU_6$ ,  $DMU_7$ ,  $DMU_8$ . Relevant statistical data are shown in the Table 1 below:

**Table 3. The Original Data Table of Eight Dmus**

Unit	A <sub>1</sub>	A <sub>2</sub>	A <sub>3</sub>	A <sub>4</sub>	A <sub>5</sub>	B <sub>1</sub>	B <sub>2</sub>	B <sub>3</sub>
DMU <sub>1</sub>	125	42	26	4	3	76	84	89
DMU <sub>2</sub>	122	45	25	18	15	75	85	81
DMU <sub>3</sub>	116	48	32	20	27	76	78	46
DMU <sub>4</sub>	118	42	30	2	5	76	77	86
DMU <sub>5</sub>	130	43	27	14	5	80	78	80
DMU <sub>6</sub>	122	46	27	4	20	75	83	83
DMU <sub>7</sub>	112	50	35	10	0	72	74	83
DMU <sub>8</sub>	117	45	26	8	30	79	83	53

By means of the Linear programming of CCR model and Microsoft Excel 2003, the original data in Table 3 are calculated, and the optimal weight of input and output is shown in Table 4 as follows:

**Table 4. The Optimal Weight of Input and Output**

Unit	u1	u2	u3	v1	v2	v3	v4	v5
DMU <sub>1</sub>	0.0046	0.0062	0.0014	0.0068	0.0035	0.0000	0.0000	0.0000
DMU <sub>2</sub>	0.0103	0.0005	0.0023	0.0064	0.0000	0.0086	0.0000	0.0000
DMU <sub>3</sub>	0.0128	0.0000	0.0000	0.0086	0.0000	0.0000	0.0000	0.0000
DMU <sub>4</sub>	0.0101	0.0005	0.0022	0.0063	0.0000	0.0085	0.0000	0.0000
DMU <sub>5</sub>	0.0112	0.0000	0.0013	0.0041	0.0070	0.0062	0.0000	0.0000
DMU <sub>6</sub>	0.0033	0.0056	0.0034	0.0079	0.0000	0.0014	0.0000	0.0000
DMU <sub>7</sub>	0.0052	0.0064	0.0018	0.0086	0.0008	0.0000	0.0000	0.0000
DMU <sub>8</sub>	0.0105	0.0005	0.0023	0.0066	0.0000	0.0088	0.0000	0.0000

The efficiency values of each student are calculated respectively as follow in Table 5:

**Table 5. The Efficiency Values of Each Unit**

Unit	1	2	3	4	5	6	7	8
Efficient values	1.0000	1.0000	0.9703	1.0000	1.0000	0.9928	1.0000	1.0000

In order to further analyze above outcome, DEA model responding to Equation (3) and MATLAB software are used to further processing of data in Table 3, its outcome is as follows on Table 6.

As can be seen from the above Table 4, the unit of  $\theta_i = 1$  is relative to DMU<sub>1</sub>, DMU<sub>2</sub>, DMU<sub>4</sub>, DMU<sub>5</sub>, DMU<sub>7</sub> and DMU<sub>8</sub>, and  $S_i^- = S_i^+ = (0, 0, 0, 0, 0)$ , ( $i = 1, 2, 4, 5, 6, 7, 8$ ), some conclusions are drawn that DMU<sub>1</sub>, DMU<sub>2</sub>, DMU<sub>4</sub>, DMU<sub>5</sub>, DMU<sub>7</sub> and DMU<sub>8</sub> refer to DEA efficiency; but DMU<sub>3</sub> and DMU<sub>6</sub> is non-DEA efficiency.

Based on the Cross DEA model, all DMUs that refer to DEA efficiency can be analyzed, evaluated and sorted. According to the average value of Cross efficiency, eight units that represent eight student are sorted. So the order level of the learning efficiency, respectively, is DMU<sub>4</sub>, DMU<sub>8</sub>, DMU<sub>1</sub>, DMU<sub>2</sub>, DMU<sub>6</sub>, DMU<sub>7</sub>, DMU<sub>5</sub>, DMU<sub>3</sub>. As can be seen, the use of cross-evaluation method overcomes the disadvantages of CCR model, which can evaluate the performance of DMU whose efficient value is 1 and resort them, and effectively distinguish individual student learning.

**Table 6. The Outcome of Data Processing**

Variable	DMU <sub>1</sub>	DMU <sub>2</sub>	DMU <sub>3</sub>	DMU <sub>4</sub>	DMU <sub>5</sub>	DMU <sub>6</sub>	DMU <sub>7</sub>	DMU <sub>8</sub>
$\lambda$	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000
	0.0000	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000
	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000
	0.0000	0.0000	0.0931	0.0000	0.0000	0.0000	1.0000	0.0000
	0.0000	0.0000	0.8772	0.0000	0.0000	0.0000	0.0000	1.0000
$S^-$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	0.0000	0.0000	2.6541	0.0000	0.0000	0.0000	0.0000	0.0000
	0.0000	0.0000	5.1229	0.0000	0.0000	0.0000	0.0000	0.0000

	0.0000	0.0000	11.5441	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$S^+$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	0.0000	0.0000	1.6949	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	0.0000	0.0000	8.2180	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

**Table 7. The Calculation Matrix of Cross DEA and Rank**

DMU	Cross efficiency								Average	Rank
	DMU <sub>1</sub>	DMU <sub>2</sub>	DMU <sub>3</sub>	DMU <sub>4</sub>	DMU <sub>5</sub>	DMU <sub>6</sub>	DMU <sub>7</sub>	DMU <sub>8</sub>		
DMU <sub>1</sub>	1.0000	1.0000	0.9005	1.0000	1.0000	1.0000	0.9909	1.0000	0.9864	3
DMU <sub>2</sub>	1.0000	1.0000	0.9105	1.0000	0.9753	1.0000	1.0000	1.0000	0.9857	4
DMU <sub>3</sub>	0.9389	0.9073	0.9703	0.9073	0.9041	0.8765	0.9477	0.9073	0.9199	8
DMU <sub>4</sub>	1.0000	1.0000	0.9539	1.0000	1.0000	1.0000	1.0000	1.0000	0.9942	1
DMU <sub>5</sub>	0.9333	0.9782	0.9114	0.9782	1.0000	0.9133	0.9239	0.9782	0.9521	7
DMU <sub>6</sub>	0.9869	0.9865	0.9105	0.9865	0.9865	0.9587	0.9928	0.9909	0.9749	5
DMU <sub>7</sub>	0.9695	0.9471	0.9521	0.9471	0.8913	1.0000	1.0000	0.9471	0.9568	6
DMU <sub>8</sub>	1.0000	1.0000	1.0000	1.0000	1.0000	0.9415	1.0000	1.0000	0.9927	2

#### 4. Conclusion

In this study, DEA and Cross DEA model is applied to evaluate the college students' comprehensive learning efficiency, and main conclusions are described as follows:

1) DEA is a good means to develop the efficient evaluation. It has the two obvious advantages. One is objectivity; DEA in the process of evaluation does not need present weight vector. In other words, all the results are directly determined by the data itself, and evaluation process is not influenced by subjective factors during the evaluation process. The other is feedback, as is seen from the example analysis, the DEA method is not in the end after the DEA efficiency or non-DEA efficiency about the evaluation. But for the invalid results, some discussions have been made to raise the direction of the object of evaluation efficiency, which has more development and further research, and one of the essential factors about this point is the difference among DEA and other methods.

2) Cross DEA model is introduced to overcome some limitations in the rankings for DEA. As a result of the evaluation is efficiency indicator, it is easy to conduct the rank for a non-DEA efficiency evaluation object. But for ranking issues of DEA efficiency evaluation object, DEA evaluation method is not fit to be operated. So the average value of efficiency via Cross DEA model is applied to sort the all DMUs and puts forward some thoughts to improve the learning efficiency.

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