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Two-sided Matching under the Conditions of Strict Order Relations and Threshold Orders

Qi Yue[1], Yongshan Peng[2] and Bingwen Yu[3]

[1]*School of Information Management, Jiangxi University of Finance and Economics, Nanchang, China*
[2]*School of Physical Education, Jiangxi University of Finance and Economics, Nanchang, China*
[3]*School of economics, Jiangxi University of Finance and Economics, Nanchang, China*
yueqichina@126.com, pengyongshanjxcd@sina.com, yubingwen2005@163.com

Abstract

A matching approach is proposed for solving the two-sided matching problem, where the preferences given by the two-sided agents are in the format of strict order relations and threshold orders. The two-sided matching problem under the conditions of strict order relations and threshold orders is firstly described. The related concepts of the two-sided matching are also introduced. In order to solve the considered two-sided matching problem, the strict order relations are converted into the Borda number matrix, and the threshold orders are transformed into the threshold Borda numbers. According to the Borda number matrix and the threshold Borda numbers of each side, the two Borda number cut matrix can be established, and then the two normalized Borda number cut matrixes can be set up. According to the two normalized Borda number cut matrixes, the synthetical normalized Borda number cut matrix can be established. Based on the synthetical normalized Borda number cut matrix, a matching model considering the two-sided matching constraint conditions can be developed. The matching alternative can be obtained by solving the matching model. Finally, a matching example between positions and staffs is given to illustrate the use of the proposed approach.

Keywords: *two-sided matching; strict order relation; threshold order; Borda number; optimization model.*

1. Introduction

The two-sided matching problems are the markets where the participants on each side give their preferences over the participants on the other side. The two-sided matching problems exist widely in many fields of real life, such as stable marriage assignment [1-4], college admission [5-7], employee selection [8-10], personnel assignment [11-13] and trading partner selection [14]. Therefore two-sided matching is a research topic with extensive application backgrounds.

Gale and Shapley initially investigate the concept, existence, optimality and algorithm of stable assignment [15]. A fundamental concept in the two-sided matching markets is stability. A two-sided matching is stable if there is no pair of an agent on the one side and another agent on the other side who likes each other better than their current cooperators. Since then, various approaches, techniques and solution algorithms have been proposed for solving the two-sided matching problem with different formats of information. For example, Hurwitz examine the impact of legacy status on admissions decisions at 30 highly selective colleges and universities by using conditional logistic regression with fixed effects for colleges to draw conclusions about the impact of legacy status on

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admissions odds [16]. Boon and Sierksma match position with player in soccer team formation using linear optimization models [17]. Ehlers uses the deferred acceptance algorithm to study the truncation strategies in matching markets, and show that truncation strategies are also applicable to all priority mechanisms and all linear programming mechanisms [18]. Uetake and Watanabe present a method for estimating a non-transferable utility in two-sided matching models [19]. Azevedo gives a simple equilibrium model of an imperfectly competitive matching market, in which an infinite number of firms is matched to a continuum of workers [20]. Utku Ünver [21] uses a genetic algorithm to study the strategic behavior in a two-sided matching game with incomplete information, and also demonstrate that the stability needn't be required for the success of a matching mechanism under incomplete information. Fleiner [22] gives a linear characterization of the bipartite stable b-matching polytope, and displays that the stable b-matching polytope is the convex hull of the characteristic vectors of stable b-matchings. Sethuraman et al. [23] focus on the geometric structure of fractional stable matchings in the stable admission problem. Teo and Sethuraman [24] study the stable marriage and stable roommate problems using a polyhedral approach. For the stable roommate problem, a new LP formulation is given. For the stable marriage problem, a related geometry is used to express any fractional solution in the stable marriage polytope as a convex combination of stable marriage solutions. Navarro [25] investigates the efficiency of two-sided investments in a matching model with high- and low-productivity jobs.

The existing researches enrich the two-sided matching theories, and develop the solution algorithms for the two-sided matching problems with different formats of information, and expand the practical application background. However, in some practical problem, the two-sided agents are in the environment of order relations. The preferences provided by two-sided agents are in the format of strict order relations and threshold orders, and the existing studies seldom consider solving this kind of problem. Hence, how to study the two-sided matching problem under the conditions of strict order relations and threshold orders is a valuable research topic. In this paper, a novel approach is presented for solving the two-sided matching problem with strict order relations and threshold orders.

The remainder of this paper is provided as follows: Section 2 formulates the two-sided matching problem under the conditions of strict order relations and threshold orders. Section 3 presents a matching approach using the generalized Borda numbers transformation. Section 4 uses a numerical example to illustrate the proposed approach. Section 5 concludes the main features of this paper.

2. Presentation of the Problem

The two-sided matching problem, where the preferences provided by the two-sided agents are strict order relations and threshold orders, is considered in this paper. The notation of the considered two-sided problem is displayed as follows.

$P=\{P_1,P_2,\ldots,P_m\}$: the set of agents of side P, $m\ge 2$;

P_i: the ith agent of side P, $i\in I=\{1,2,\ldots,m\}$;

$Q=\{Q_1,Q_2,\ldots,Q_n\}$: the set of agents of side Q, $n\ge m$;

Q_j: the jth agent of side Q, $j\in J=\{1,2,\ldots,n\}$;

$Q_{i_1}\succ Q_{i_2}\succ\cdots\succ Q_{i_n}$: the strict order relation provided by agent P_i;

$\{i_1,i_2,\cdots,i_n\}$: a permutation of $\{1,2,\ldots,n\}$;

o_i^P : the threshold order provided by agent P_i, $o_i^P \in J$;

$P_{j_1} \succ P_{j_2} \succ \cdots \succ P_{j_m}$: the strict order relation provided by agent Q_j ;

$\{j_1, j_2, \cdots, j_m\}$: a permutation of $\{1,2,\ldots,m\}$;

o_j^Q: the threshold order provided by agent Q_j , $o_j^Q \in I$;

"$\succ$": "superior to".

Remark 1. The meanings of threshold orders o_i^P and o_j^Q may be different in various situations. For convenience, take the threshold order o_j^Q as an example. The first one is that agent Q_j prefers the agents of side P whose orders are in front of o_j^Q, and prefers to remain single than match with the opposite agents whose orders are behind o_j^Q. The second one is that agent Q_j prefers the agents of side P whose orders are in front of o_j^Q, and prefers to match with the opposite agents whose orders are behind o_j^Q than remain single. The first one is considered in this paper.

The concept of two-sided matching is demonstrated as follows [26-28].

Definition 1. A two-sided matching is a one-to-one mapping $\mu: P\bigcup Q \to P\bigcup Q$ such that for all $P_i \in P$ and $Q_j \in Q$, (i) $\mu(P_i) \in Q\bigcup\{P_i\}$, (ii) $\mu(Q_j) \in P\bigcup\{Q_j\}$, (iii) $\mu(P_i) = Q_k$ if and only if $\mu(Q_k) = P_i$.

Remark 2. In Definition 1, $\mu(P_i) = Q_k$ denotes that P_i (or Q_k) is matched with Q_k (or P_i) in two-sided matching μ, and $\mu(P_i) = P_i$ (or $\mu(Q_j) = Q_j$) denotes that P_i (or Q_j) is unmatched in two-sided matching μ. Furthermore, if $\mu(P_i) = Q_k$, then pair (P_i, Q_k) is called as a matching pair. For convenience, note (P_i, P_i) (or (Q_j, Q_j)) as the matching pair in the case of $\mu(P_i) = P_i$ (or $\mu(Q_j) = Q_j$). Therefore, a two-sided matching (or a matching alternative) can be expressed by the set of n matching pairs, such as $\mu = \{(A_1, B_3), (A_2, B_5), \ldots, (B_n, B_2), \ldots\}$.

In conclusion, the problem involved in this paper is how to determine the reasonable matching alternative based on strict order relations $Q_{i_1} \succ Q_{i_2} \succ \cdots \succ Q_{i_n}$ and $P_{j_1} \succ P_{j_2} \succ \cdots \succ P_{j_m}$, and threshold orders o_i^P and o_j^Q, $i \in I$, $j \in J$.

3. Presentation of the Method

3.1. Building of the Synthetical Normalized Borda Number Cut Matrix

In order to deal with the strict order relations and threshold orders, the generalized Borda numbers are used.

Definition 2. Let $bn_{ij}^{P\to Q}$ be the generalized Borda number of agent P_i over agent Q_j, then the generalized Borda number $bn_{ij}^{P\to Q}$ can be provided by

$$bn_{ij}^{P\to Q} = n + 1 - k\,,\ j = i_k\,,\ k \in J \tag{1}$$

Thus, by Eq. (1), the strict order relations $Q_{i_1} \succ Q_{i_2} \succ \cdots \succ Q_{i_n}$ ($i \in I$) can be changed into the Borda number matrix $B_{P\to Q} = [bn_{ij}^{P\to Q}]_{m\times n}$.

Definition 3. Let tn_i^P be the threshold Borda number corresponding to threshold order o_i^P, then the threshold Borda number tn_i^P can be expressed by

$$tn_i^P = n + 1 - o_i^P,\ i \in I \tag{2}$$

Definition 4. Let $bn_{ij}^{Q\to P}$ be the generalized Borda number of agent Q_j over agent P_i, then the generalized Borda number $bn_{ij}^{Q\to P}$ can be expressed by

$$bn_{ij}^{Q\to P} = m + 1 - l,\ i = j_l,\ l \in I \tag{3}$$

Thus, by Eq. (3), the strict order relations $P_{j_1} \succ P_{j_2} \succ \cdots \succ P_{j_m}$ ($j \in J$) can be changed into the Borda number matrix $B_{Q\to P} = [bn_{ij}^{Q\to P}]_{m\times n}$.

Definition 5. Let tn_j^Q be the threshold Borda number corresponding to threshold order o_j^Q, then the threshold Borda number tn_j^Q can be expressed by

$$tn_j^Q = m + 1 - o_j^Q,\ j \in J \tag{4}$$

According to the Borda number matrix $B_{P\to Q} = [bn_{ij}^{P\to Q}]_{m\times n}$ and the threshold Borda number tn_i^P, the Borda number cut matrix $C_{P\to Q} = [c_{ij}^{P\to Q}]_{m\times n}$ is established, where

$$c_{ij}^{P\to Q} = \begin{cases} bn_{ij}^{P\to Q} - t_i^P, & bn_{ij}^{P\to Q} \ge tn_i^P, \\ -K, & bn_{ij}^{P\to Q} < tn_i^P, \end{cases} \quad i \in I,\ j \in J \tag{5}$$

Here, K is a large enough positive number. Then, based on the Borda number cut matrix $C_{P\to Q} = [c_{ij}^{P\to Q}]_{m\times n}$, the normalized Borda number cut matrix $Cn_{P\to Q} = [cn_{ij}^{P\to Q}]_{m\times n}$ is established, where

$$cn_{ij}^{P\to Q} = \begin{cases} \dfrac{c_{ij}^{P\to Q}}{\max\max\{c_{ij}^{P\to Q}\}}, & bn_{ij}^{P\to Q} \ge tn_i^P, \\ -K, & bn_{ij}^{P\to Q} < tn_i^P, \end{cases} \quad i \in I,\ j \in J \tag{6}$$

Similarly, according to the Borda number matrix $B_{Q\to P} = [bn_{ij}^{Q\to P}]_{m\times n}$ and the threshold Borda number tn_j^Q, the Borda number cut matrix $C_{Q\to P} = [c_{ij}^{Q\to P}]_{m\times n}$ is established, where

$$c_{ij}^{Q\to P} = \begin{cases} bn_{ij}^{Q\to P} - t_i^Q, & bn_{ij}^{Q\to P} \ge tn_i^Q, \\ -K, & bn_{ij}^{Q\to P} < tn_i^Q, \end{cases} \quad i \in I,\ j \in J \tag{7}$$

Then, based on the Borda number cut matrix $C_{Q\to P} = [c_{ij}^{Q\to P}]_{m\times n}$, the normalized Borda number cut matrix $Cn_{Q\to P} = [cn_{ij}^{Q\to P}]_{m\times n}$ is established, where

$$cn_{ij}^{Q\to P} = \begin{cases} \dfrac{c_{ij}^{Q\to P}}{\max\max\{c_{ij}^{Q\to P}\}}, & bn_{ij}^{Q\to P} \geq tn_i^Q, \\ -K, & bn_{ij}^{Q\to P} < tn_i^Q, \end{cases} \quad i \in I\,,\ j \in J \tag{8}$$

According to normalized Borda number cut matrixes $Cn_{P\to Q} = [cn_{ij}^{P\to Q}]_{m\times n}$ and $Cn_{Q\to P} = [cn_{ij}^{Q\to P}]_{m\times n}$, the synthetical normalized Borda number cut matrix $Cn = [cn_{ij}]_{m\times n}$ can be built, where

$$cn_{ij} = w_p cn_{ij}^{P\to Q} + w_Q cn_{ij}^{Q\to P}\,,\ i \in I\,,\ j \in J \tag{9}$$

Here, w_P (or w_Q) is the weight of agents of side P (or Q), and satisfies $w_P, w_Q \in [0,1]$, and $w_P + w_Q = 1$. If the statuses of two-sided agents are considered identical, then we obtain $w_P = w_Q = 0.5$. If not, then we obtain $w_P \neq w_Q$.

3.2 Construction of the Matching Model

Based on the synthetical normalized Borda number cut matrix $Cn = [cn_{ij}]_{m\times n}$, we consider constructing a matching model for obtaining the matching alternative in the following.

According to the meaning of Borda number, we know that the greater $cn_{ij}^{P\to Q}$ (or $cn_{ij}^{Q\to P}$) is, the higher the satisfaction degree of agent P_i over Q_j (or the satisfaction degree of agent Q_j over P_i) is. Hence, the normalized Borda numbers $cn_{ij}^{P\to Q}$ (or $cn_{ij}^{Q\to P}$) can be regarded as the objectives. Furthermore, we can maximize the synthetical normalized Borda number cn_{ij} in order to maximize the synthetical satisfaction degrees of agent P_i over Q_j and agent Q_j over P_i. Moreover, the 0-1 variable x_{ij} is also introduced, where $x_{ij} = \begin{cases} 1, & \mu(P_i) = Q_j \\ 0, & \mu(P_i) \neq Q_j \end{cases}$, then the following matching model (10) can be constructed:

$$\max \quad Z = \sum_{i=1}^{m}\sum_{j=1}^{n} cn_{ij} x_{ij} \tag{10a}$$

$$\text{s.t.} \quad \sum_{j=1}^{n} x_{ij} \leq 1\,,\ i \in I \tag{10b}$$

$$\sum_{i=1}^{m} x_{ij} \leq 1\,,\ j \in J \tag{10c}$$

$$x_{ij} \in \{0,1\}\,,\ i \in I\,,\ j \in J \tag{10d}$$

In model (10), Eq. (10a) is the objective function. The meaning of Eq. (10a) is to maximize the sum of synthetical normalized Borda numbers (i.e., the synthetical satisfaction degree of two-sided agents). The meaning of (10b) is that agent P_i matches at most an agent of side Q. The meaning of (10c) is that agent Q_j matches at most one agent of side P.

By solving model (10), the optimal solution or the optimal matching matrix $X^{*}=[x_{ij}^{*}]_{m\times n}$, can be determined. Based on the matching matrix $X^{*}=[x_{ij}^{*}]_{m\times n}$, the matching alternative can be obtained.

3.3 Determination of the Matching Process

Based on the above analysis, an algorithm for solving the two-sided matching problem under the conditions of strict order relations and threshold orders is developed. The steps of the algorithm are provided as follows:

Step 1. Convert the strict order relations $Q_{i_1}\succ Q_{i_2}\succ\cdots\succ Q_{i_n}$ ($i\in I$) into the Borda number matrixes $B_{P\to Q}=[bn_{ij}^{P\to Q}]_{m\times n}$ by Eq. (1); Convert the threshold order o_i^P into the threshold Borda number tn_i^P by Eq. (2). Convert the strict order relations $P_{j_1}\succ P_{j_2}\succ\cdots\succ P_{j_m}$ ($j\in J$) into the Borda number matrixes $B_{Q\to P}=[bn_{ij}^{Q\to P}]_{m\times n}$ by Eq. (3); Convert the threshold order o_j^Q into the threshold Borda number tn_j^Q by Eq. (4).

Step 2. Establish the Borda number cut matrix $C_{P\to Q}=[c_{ij}^{P\to Q}]_{m\times n}$ according to the Borda number matrix $B_{P\to Q}=[bn_{ij}^{P\to Q}]_{m\times n}$ and the threshold Borda number tn_i^P by Eq. (5); Establish the normalized Borda number cut matrix $Cn_{P\to Q}=[cn_{ij}^{P\to Q}]_{m\times n}$ based on the Borda number cut matrix $C_{P\to Q}=[c_{ij}^{P\to Q}]_{m\times n}$ by Eq. (6).

Step 3. Establish the Borda number cut matrix $C_{Q\to P}=[c_{ij}^{Q\to P}]_{m\times n}$ according to the Borda number matrix $B_{Q\to P}=[bn_{ij}^{Q\to P}]_{m\times n}$ and the threshold Borda number tn_j^Q by Eq. (7); Establish the normalized Borda number cut matrix $Cn_{Q\to P}=[cn_{ij}^{Q\to P}]_{m\times n}$ based on the Borda number cut matrix $C_{Q\to P}=[c_{ij}^{Q\to P}]_{m\times n}$ by Eq. (8).

Step 4. Establish the synthetical normalized Borda number cut matrix $Cn=[cn_{ij}]_{m\times n}$ according to the normalized Borda number cut matrixes $C_{P\to Q}=[c_{ij}^{P\to Q}]_{m\times n}$ and $Cn_{Q\to P}=[cn_{ij}^{Q\to P}]_{m\times n}$ by Eq. (9).

Step 5. Construct the matching model (10) based on the synthetical normalized Borda number cut matrix $Cn=[cn_{ij}]_{m\times n}$ considering the matching constraint conditions.

Step 6. Determine the matching alternative by solving model (10).

4. Analysis of the Example

In this section, an example is used to illustrate the potential application of the proposed method.

Suppose an oversea venture-capital company plans to invest a shoe store in Nan Ning of China. In order to enable the new shoe store to run smoothly, the manager intends to assign experienced staffs to vacant positions in the new factory according to the suggestion provided by the intermediary. Each position in the new shoe store is held by a staff, and each staff is assigned to only a position. There are six vacant positions, which consist of a purchaser (P_1), a material handler (P_2), a production planner (P_3), a technician (P_4), a quality inspector (P_5), and a general staff member (P_6). Now eight experienced staffs who have multiple skills apply to six vacant positions. Eight experienced staffs consist of Q_1, Q_2, …, and Q_8. The decision-makers from six position departments evaluate the staffs from five perspectives: personality characteristics, problem-solving skill, technical skill, previous experience, and human relationship skill. Eight staffs evaluate positions from four perspectives: salary and welfare, development space, work environment, and market prospect. The strict order relations $Q_{i_1} \succ Q_{i_2} \succ \cdots \succ Q_{i_n}$ and $P_{j_1} \succ P_{j_2} \succ \cdots \succ P_{j_m}$, and the threshold orders o_i^P ($i \in I = \{1, 2, \cdots, 6\}$) and o_j^Q ($j \in J = \{1, 2, \cdots, 8\}$) are provided as follows.

$$P_1 : Q_4 \succ Q_3 \succ Q_7 \succ Q_6 \succ Q_2 \succ Q_8 \succ Q_5 \succ Q_1,$$

$$P_2 : Q_7 \succ Q_6 \succ Q_3 \succ Q_4 \succ Q_8 \succ Q_5 \succ Q_1 \succ Q_2,$$

$$P_3 : Q_3 \succ Q_8 \succ Q_4 \succ Q_5 \succ Q_1 \succ Q_7 \succ Q_6 \succ Q_2,$$

$$P_4 : Q_2 \succ Q_4 \succ Q_7 \succ Q_5 \succ Q_8 \succ Q_6 \succ Q_3 \succ Q_1,$$

$$P_5 : Q_8 \succ Q_7 \succ Q_1 \succ Q_5 \succ Q_6 \succ Q_4 \succ Q_2 \succ Q_3,$$

$$P_6 : Q_1 \succ Q_5 \succ Q_2 \succ Q_6 \succ Q_3 \succ Q_8 \succ Q_4 \succ Q_7,$$

$$o_1^P = o_2^P = o_3^P = 6,\ o_4^P = o_5^P = o_6^P = 7;$$

$$Q_1 : P_5 \succ P_2 \succ P_6 \succ P_4 \succ P_3 \succ P_1,\ Q_2 : P_2 \succ P_4 \succ P_3 \succ P_6 \succ P_1 \succ P_5,$$

$$Q_3 : P_2 \succ P_3 \succ P_5 \succ P_1 \succ P_6 \succ P_4,\ Q_4 : P_5 \succ P_1 \succ P_6 \succ P_4 \succ P_3 \succ P_2,$$

$$Q_5 : P_6 \succ P_1 \succ P_4 \succ P_2 \succ P_3 \succ P_5,\ Q_6 : P_4 \succ P_2 \succ P_5 \succ P_6 \succ P_1 \succ P_3,$$

$$Q_7 : P_5 \succ P_1 \succ P_3 \succ P_4 \succ P_6 \succ P_2,\ Q_8 : P_3 \succ P_5 \succ P_2 \succ P_1 \succ P_4 \succ P_6\ ,$$

$$o_1^Q = o_2^Q = o_3^Q = o_4^Q = 4,\ o_5^Q = o_6^Q = o_7^Q = o_8^Q = 5.$$

In order to solve the above two-sided matching problem, the proposed approach is used and the matching process is displayed as follows.

Step 1. According to the strict order relations $Q_{i_1} \succ Q_{i_2} \succ \cdots \succ Q_{i_n}$ ($i \in I = \{1, 2, \ldots, 6\}$), the Borda number matrixes $B_{P \to Q} = [bn_{ij}^{P \to Q}]_{6 \times 8}$ can be established by Eq. (1), which is shown in Table 1. By Eq. (2), the threshold order o_i^P is converted into the threshold Borda number tn_i^P, i.e,, $tn_i^P = \begin{cases} 3, & i = 1, 2, 3, \\ 2, & i = 4, 5, 6 \end{cases}$.

Table 1. The Borda Number Matrixes $B_{P \to Q} = [bn_{ij}^{P \to Q}]_{6 \times 8}$

$bn_{ij}^{P \to Q}$	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8
P_1	1	4	7	8	2	5	6	3
P_2	2	1	6	5	3	7	8	4
P_3	4	1	8	6	5	2	3	7
P_4	1	8	2	7	5	3	6	4
P_5	6	2	1	3	5	4	7	8
P_6	8	6	4	2	7	5	1	3

According to the strict order relations $P_{j_1} \succ P_{j_2} \succ \cdots \succ P_{j_m}$ ($j \in J = \{1, 2, \ldots, 8\}$), the Borda number matrixes $B_{Q \to P} = [bn_{ij}^{Q \to P}]_{6 \times 8}$ can be established by Eq. (3), which is shown in Table 2. By Eq. (4), the threshold order o_j^Q is converted into the threshold Borda number tn_j^Q, i.e., $tn_i^Q = \begin{cases} 3, & i = 1,2,3,4, \\ 2, & i = 5,6,7,8 \end{cases}$.

Table 2. The Borda Number Matrixes $B_{Q \to P} = [bn_{ij}^{Q \to P}]_{6 \times 8}$

$bn_{ij}^{Q \to P}$	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	$c_{ij}^{P \to Q}$
Q_1	1	2	3	5	5	2	5	3
Q_2	5	6	6	1	3	5	1	4
Q_3	2	4	5	2	2	1	4	6
Q_4	3	5	1	3	4	6	3	2
P_5	6	1	4	6	1	4	6	5
P_6	4	3	2	4	6	3	2	1

Step 2. According to the Borda number matrix $B_{P \to Q} = [bn_{ij}^{P \to Q}]_{6 \times 8}$ and the threshold Borda number tn_i^P, the Borda number cut matrix $C_{P \to Q} = [c_{ij}^{P \to Q}]_{6 \times 8}$ can be established by Eq. (5), which is shown in Table 3; According to the Borda number matrix $C_{P \to Q} = [c_{ij}^{P \to Q}]_{6 \times 8}$, the normalized Borda number cut matrix $Cn_{P \to Q} = [cn_{ij}^{P \to Q}]_{6 \times 8}$ can be established by Eq. (6), which is shown in Table 4.

Table 3. The Borda Number Cut Matrix $C_{P \to Q} = [c_{ij}^{P \to Q}]_{6 \times 8}$

$c_{ij}^{P \to Q}$	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8
P_1	$-K$	1	4	5	$-K$	2	3	0
P_2	$-K$	$-K$	3	2	0	4	5	1
P_3	1	$-K$	5	3	2	$-K$	0	4
P_4	$-K$	6	0	5	3	1	4	2
P_5	4	0	$-K$	1	3	2	5	6
P_6	6	4	2	0	5	3	$-K$	1

Table 4. The Normalized Borda Number Cut Matrix $Cn_{P \to Q} = [cn_{ij}^{P \to Q}]_{6 \times 8}$

$cn_{ij}^{P \to Q}$	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8
P_1	$-K$	0.1667	0.6667	0.8333	$-K$	0.3333	0.5	0
P_2	$-K$	$-K$	0.5	0.3333	0	0.6667	0.8333	0.1667
P_3	0.1667	$-K$	0.8333	0.5	0.3333	$-K$	0	0.6667
P_4	$-K$	1	0	0.8333	0.5	0.1667	0.6667	0.3333
P_5	0.6667	0	$-K$	0.1667	0.5	0.3333	0.8333	1
P_6	1	0.6667	0.3333	0	0.8333	0.5	$-K$	0.1667

Step 3. According to the Borda number matrix $B_{Q \to P} = [bn_{ij}^{Q \to P}]_{6 \times 8}$ and the threshold Borda number tn_j^Q, the Borda number cut matrix $C_{Q \to P} = [c_{ij}^{Q \to P}]_{6 \times 8}$ can be established by Eq. (7), which is shown in Table 5; According to the the Borda number cut matrix

$C_{Q\to P}=[c_{ij}^{Q\to P}]_{6\times 8}$, the normalized Borda number cut matrix $Cn_{Q\to P}=[cn_{ij}^{Q\to P}]_{6\times 8}$ can be established by Eq. (8), which is shown in Table 6.

Table 5. The Borda Number cut matrix $C_{Q\to P}=[c_{ij}^{Q\to P}]_{6\times 8}$

$c_{ij}^{Q\to P}$	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8
P_1	$-K$	$-K$	0	2	3	0	3	1
P_2	2	3	3	$-K$	1	3	$-K$	2
P_3	$-K$	1	2	$-K$	0	$-K$	2	4
P_4	0	2	$-K$	0	2	4	1	0
P_5	3	$-K$	1	3	$-K$	2	4	3
P_6	1	0	$-K$	1	4	1	0	$-K$

Table 6. The Normalized Borda Number Cut Matrix $Cn_{Q\to P}=[cn_{ij}^{Q\to P}]_{6\times 8}$

$cn_{ij}^{Q\to P}$	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8
P_1	$-K$	$-K$	0	0.5	0.75	0	0.75	0.25
P_2	0.5	0.75	0.75	$-K$	0.25	0.75	$-K$	0.5
P_3	$-K$	0.25	0.5	$-K$	0	$-K$	0.5	1
P_4	0	0.5	$-K$	0	0.5	1	0.25	0
P_5	0.75	$-K$	0.25	0.75	$-K$	0.5	1	0.75
P_6	0.25	0	$-K$	0.25	1	0.25	0	$-K$

Step 4. We might as well assume $w_P=0.55$, $w_Q=0.45$. Then, according to the normalized Borda number cut matrixes $Cn_{P\to Q}=[cn_{ij}^{P\to Q}]_{6\times 8}$ and $Cn_{Q\to P}=[cn_{ij}^{Q\to P}]_{6\times 8}$, the synthetical normalized Borda number cut matrix $Cn=[cn_{ij}]_{6\times 8}$ can be established by Eq. (9), which is shown in Table 7

Table 7. The Synthetical Normalized Borda Number Cut Matrix $Cn=[cn_{ij}]_{6\times 8}$

cn_{ij}	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8
P_1	$-K$	$-K$	0.3667	0.6833	$-K$	0.1833	0.6125	0.1125
P_2	$-K$	$-K$	0.6125	$-K$	0.1125	0.7042	$-K$	0.3167
P_3	$-K$	$-K$	0.6833	$-K$	0.1833	$-K$	0.225	0.8167
P_4	$-K$	0.775	$-K$	0.4583	0.5	0.5417	0.4792	0.1833
P_5	0.7042	$-K$	$-K$	0.4292	$-K$	0.4083	0.9083	0.8875
P_6	0.6625	0.3667	$-K$	0.1125	0.9083	0.3875	$-K$	$-K$

Step 5. Based on the synthetical normalized Borda number cut matrix $Cn=[cn_{ij}]_{6\times 8}$, the matching model (10) considering the matching constraint conditions can be constructed, i.e.,

$$\max \quad Z=\sum_{i=1}^{6}\sum_{j=1}^{8}cn_{ij}x_{ij} \tag{10a}$$

$$\text{s.t.} \quad \sum_{j=1}^{8} x_{ij} \le 1,\ i \in I \tag{10b}$$

$$\sum_{i=1}^{6} x_{ij} \le 1,\ j \in J \tag{10c}$$

$$x_{ij} \in \{0,1\},\ i \in I,\ j \in J \tag{10d}$$

Step 6. By solving the matching model (10), the matching matrix $X^* = [x_{ij}^*]_{6\times 8}$ can be determined, which is shown in Table 8.

Table 8. The Matching Matrix $X^* = [x_{ij}^*]_{6\times 8}$

x_{ij}^*	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8
P_1	0	0	0	1	0	0	0	0
P_2	0	0	0	0	0	1	0	0
P_3	0	0	0	0	0	0	0	1
P_4	0	1	0	0	0	0	0	0
P_5	0	0	0	0	0	0	1	0
P_6	0	0	0	0	1	0	0	0

Based on the matching matrix $X^* = [x_{ij}^*]_{6\times 8}$, the matching alternative μ^* can be determined, i.e., $\mu^* = \{(P_1,Q_4),(P_2,Q_6),(P_3,Q_8),(P_4,Q_2),(P_5,Q_7),(P_6,Q_5)$, $(Q_1,Q_1),(Q_3,Q_3)\}$, which is vividly displayed in Figure 1.

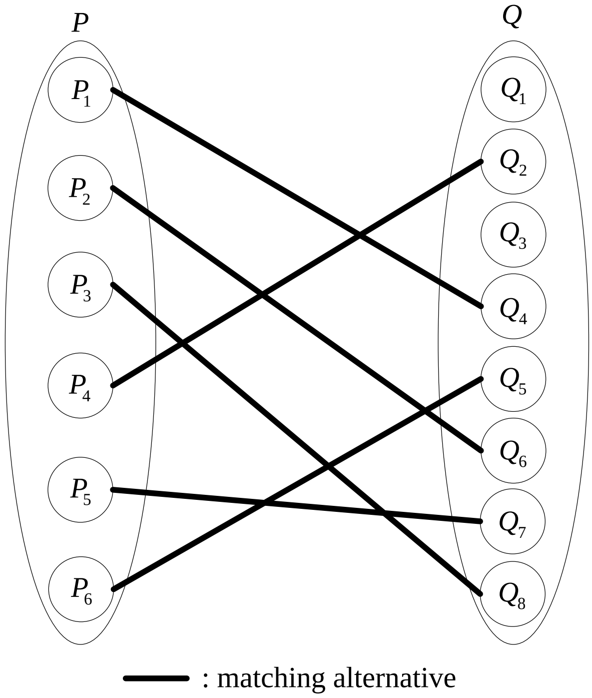

Figure 1. The Staffs-positions Matching Alternative

In other words, position P_1 matches with staff Q_4, position P_2 matches with staff Q_6, position P_3 matches with staff Q_8, position P_4 matches with staff Q_2, P_5 matches with staff Q_7, position P_6 matches with staff Q_5, staffs Q_1 and Q_3 are unmatched.

5. Conclusion

A novel approach for solving the two-sided matching problem with strict order relations and threshold orders is proposed. In this approach, the strict order relations are changed into the Borda number matrixes, and the threshold orders are changed into the threshold Borda numbers. According to the Borda number matrixes and threshold Borda numbers, the synthetical normalized Borda number cut matrix can be established. Moreover, a matching model can be built based on the synthetical normalized Borda number cut matrix. By solving the matching model, the matching alternative can be determined. Comparing with the existing approaches, the proposed approach has some distinct characteristics.

Firstly, in this approach, the generalized Borda numbers are used to deal with strict order relations and threshold orders, and then the synthetical normalized Borda number cut matrix are built, which are new ideas.

Secondly, by solving a developed matching model, it could be easier to obtain the reasonable matching alternative.

Additionally, the proposed approach is theoretically sound and computationally simple which provides a new pathway to solve the two-sided matching problem under the conditions of strict order relations and threshold orders and can be adopted for practical use.

In terms of future research, the proposed approach can be extended to support the situations in which the preference information are in other formats, such as incomplete order relations or uncertain order relations, etc.

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Authors

Qi Yue, he was born in 1983. He received the Ph.D. degree in school of business administration from northeastern university, shenyang, China, in 2011. He is currently a lecturer in the school of information management at jiangxi university of finance and economics. He is the author or coauthor of fifty papers published in international and local journals. His current interests include information science, management science, decision analysis and operations research.

E-mail: yueqichina@126.com.

YongShan Peng, he was born in 1976. He received the Ph.D. degree in school of physical education from south China normal university, guangzhou, China, in 2012. He is currently a lecturer in the school of physical education at jiangxi university of finance and economics. He is the author or coauthor of thirty papers published in international and local journals. His current interests include sports culture, physical education and school physical education.

E-mail: pengyongsh@sohu.com.

Bingwen Yu, he was born in 1974. He received the Ph.D. degree in school of business administration from Zhongnan University of Economics and Law, wuhan, China, in 2011. He is currently a lecturer in the school of economics at jiangxi university of finance and economics. He is the author or coauthor of fifteen papers published in international and local journals. His current interests include industrial economy, appraisal evaluation and real estate research.

E-mail: yubingwen2005@163.com.