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Performance Improvement of the OFDM System Corrupted by Phase Noise

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Abstract

Phase noise is produced in local oscillator or PLL(phase locked loop), which affects seriously BER performance of the OFDM (orthogonal frequency division multiplexing) system. ICI (inter-sub-carrier-interference) caused by the phase noise is a serious problem. In this paper, two modified ICI self-cancellation methods are proposed and compared for the performance evaluation. In the original OFDM system, the proposed symmetric data-conjugate method has the best ICI minimization capability, which becomes gradually degraded according to the order of symmetric data-conjugate method, data-conjugate method, symmetric data-conversion method and data-conversion method. When phase noise dominates in the OFDM system, symmetric data-conjugate method can result in the best BER performance, when HPA nonlinearity cannot be neglected, data-conjugate method achieves the best BER performance.

Keywords: *Phase noise, ICI, ICI self-cancellation, Symmetric data-conjugate, Symmetric data-conversion*

1. Introduction

OFDM has been adopted as the standards for Europe DAB/DVB (digital audio and video broadcasting) system, the high-rate WLAN (wireless local area networks) such as IEEE802.11x, HIPERLAN II and MMAC (multimedia mobile access communications). Additionally, OFDM is used for the Korean terrestrial DMB (digital multimedia broadcasting) system and WiBro(wireless broadband internet).

However, OFDM has the disadvantage of high PAPR (peak to average power ratio) and serious ICI (inter sub-carrier interference) unlike single carrier system. Especially, the ICI caused by phase noise or frequency offset seriously degrades system performance because it may break down the orthogonality between sub-carriers which is the basis of OFDM transmission.

have been many previous studies on the phase noise and ICI [2-5]. There, the phase rotation common phase error (CPE) and ICI caused by phase noise were analyzed in detail. To estimate and compensate ICI influence, ICI self-cancellation using the data-conversion method or data-conjugate method etc was proposed to minimize the ICI effect on the system performance [6-11].

In this paper, two kinds of modified ICI self-cancellation methods are newly proposed and compared for the performance evaluation. CPE (common phase error), ICI and CIR (carrier to interference power ratio) are derived and discussed by the linear approximation of the phase noise.

2. Phase Noise in the OFDM System

2.1. OFDM System

Through fading channel, the received signal in presence of phase noise can be expressed as

$$r(n) = [x(n) \otimes h(n) + v(n)] \cdot e^{j\varphi(n)}, \tag{1}$$

where $\varphi(n)$ is phase noise process generated in the transceiver.

After FFT, the recovered output for the kth sub-carrier is as follows:

$$\begin{aligned} Y_k &= \frac{1}{N}\sum_{n=0}^{N-1} r[n] \cdot e^{-j\frac{2\pi}{N}nk} \\ &= \frac{1}{N}\sum_{i=0}^{N-1} X_i \cdot H_i \sum_{n=0}^{N-1} e^{j\left(\frac{2\pi}{N}(i-k)n+\varphi(n)\right)} + N_k \end{aligned}, \tag{2}$$

where N_k is the FFT version of AWGN influenced by phase noise, that is $N_k = \frac{1}{N}\sum_{n=0}^{N-1} v(n) \cdot e^{j\varphi[n]} \cdot e^{-j\frac{2\pi}{N}nk}$.

To analyze phase noise affect independently, suppose $H_i = 1$, and define Q_L as follows:

$$Q_L = \frac{1}{N}\sum_{n \in S} e^{j\left[\frac{2\pi}{N}Ln+\varphi[n]\right]}, \tag{3}$$

then,

$$\begin{aligned} Y_k &= \sum_{i=0}^{N-1} X_i \cdot H_i \cdot Q_{i-k} + N_k \\ &= X_k \cdot Q_0 + \sum_{i=0, i\neq k}^{N-1} X_i \cdot Q_{i-k} + N_k \end{aligned}. \tag{4}$$

Generally, after all OFDM sub-carriers are corrupted by the same phase noise. The corrupted OFDM signal involves two kinds of components. One component is its own sub-carrier signal corrupted by CPE (common phase error) and the other is ICI (inter-sub-carrier interference) from the adjacent sub-carrier signals. ICI is the summation of the signal of the other sub-carriers multiplied by some complex number resulted from an average of phase noise with spectral shift. CPE component rotates the signal constellation and ICI component breaks down the orthogonality between the sub-carriers of OFDM system.

3. Modified ICI Self-Cancellation Methods

3.1. Proposed Symmetric Data-conjugate Method

Symmetric data-conjugate method can be derived from the conventional data-conjugate method. In the symmetric data-conjugate ICI self-cancellation method, the high-speed information data pass through the serial to parallel converter and become parallel data streams of $N/2$ branch. Then, they are converted into N branch parallel data by the symmetric data-conjugate method. The conversion process is as follows. After serial to parallel converter, the parallel data streams are remapped as the form of $X_k^{'} = X_k, X_{N-1-k}^{'} = -X_k^{*}$. Here, X_k is the information data in the kth branch before symmetric data-conjugate method mapping, and $X_k^{'}$ is the information data in the kth carrier after mapping. Likewise, every information data is mapped onto a pair of symmetrically allocated sub-carriers by symmetric data-conjugate method, so the N/2 branch data are extended to map onto the N sub-carriers.

Here, Y_k is the kth sub-carrier data, $Z_k^{'}$ is the kth branch information data after symmetric data-conjugate method de-mapping. Finally, the information data can be found through the detection process.

The complex base-band OFDM signal after symmetric data-conjugate method mapping is as follows.

$$\begin{aligned} x(n) &= \sum_{i=0}^{N-1} X_i^{'} \cdot e^{j\frac{2\pi}{N}in} \\ &= \sum_{k=0}^{N/2-1} \left[X_k \cdot e^{j\frac{2\pi}{N}kn} - X_k^{*} \cdot e^{j\frac{2\pi}{N}(N-1-k)n} \right] \end{aligned} \quad \text{for } 0 \le n < N. \tag{5}$$

where X_k is data symbol for the kth parallel branch and $X_i^{'}$ is the ith sub-carrier data symbol after symmetric data-conjugate mapping. $x(n)$ is corrupted by the phase noise, so, it is expressed as

$$r(n) = [x(n) \otimes h(n) + v(n)] \cdot e^{j\varphi(n)} \tag{6}$$

In the symmetric data-conjugate method, the sub-carrier data is mapped in the form of $X_k^{'} = X_k, X_{N-1-k}^{'} = -X_k^{*}$. Therefore, the kth sub-carrier data after FFT in the receiver is arranged as

$$Y_k = \sum_{l=0}^{N/2-1} \left[X_l H_l Q_{l-k} - X_l^{*} H_{N-1-l} Q_{N-1-l-k} \right] + N_k . \tag{7}$$

Similarly, the $N-1-k$ th sub-carrier signal is expressed as

$$Y_{N-1-k} = \sum_{l=0}^{N/2-1} \left[X_l H_l Q_{l-N+1+k} - X_l^{*} H_{N-1-l} Q_{k-l} \right] + N_{N-1-k} . \tag{8}$$

In the receiver, the decision variable $Z_k^{'}$ of the kth symbol is found from the difference of the symmetrically allocated sub-carrier signals affected by phase noise. That is,

$$\begin{aligned} Z_k^{'} &= (Y_k - Y_{N-1-k}{}^{*})/2 \\ &= \frac{1}{2} X_k \left(H_k Q_0 + H_{N-1-k}^{*} Q_0^{*} \right) - \frac{1}{2} X_k^{*} \left(H_{N-1-k} Q_{N-1-2k} + H_k^{*} Q_{-N+1+2k}^{*} \right) \\ &+ \frac{1}{2} \sum_{\substack{l=0 \\ l \neq k}}^{N/2-1} \left\{ X_l \left[H_l Q_{l-k} + H_{N-1-l}^{*} Q_{k-l}^{*} \right] - X_l^{*} \left[H_{N-1-l} Q_{N-1-l-k} + H_l^{*} Q_{l-N+1+k}^{*} \right] \right\} + N_k^{'} \end{aligned} \quad ,(9)$$

where $N_k^{'} = \frac{1}{2}\left(N_k - N_{N-1-k}^{*} \right)$ is the AWGN of the kth parallel branch data in the receiver.

When channel is flat, frequency response of channel equals 1. $Z_k^{'}$ is as follows.

$$Z_k^{'} = X_k - \frac{1}{2} \sum_{\substack{l=0 \\ l \neq k}}^{N/2-1} X_l^{*} \left[Q_{N-1-l-k} + Q_{l-N+1+k}^{*} \right] + N_k^{'} . \quad (10)$$

3.2. Proposed Symmetric Data-conversion Method

Symmetric data-conversion method can be derived from the conventional data-conversion method. In the symmetric data-conversion ICI self-cancellation method, the sub-carrier signal after serial to parallel converter is mapped in the form of $X_k^{'} = X_k , X_{N-1-k}^{'} = -X_k$. Every information signal is mapped onto pairs of symmetric sub-carriers by symmetric data-conversion method, so the $N/2$ branches data are arranged onto the N sub-carriers. In the receiver, after serial to parallel converter and FFT, the signal with N carriers is converted back into the signal with $N/2$ branches by de-mapping of the symmetric data-conversion method. So, using $Z_k^{'} = (Y_k - Y_{N-1-k})/2$, the original signal can be recovered from two symmetrical sub-carrier signals.

The complex base-band OFDM signal after symmetric data-conversion mapping is as follows.

$$\begin{aligned} x(n) &= \sum_{i=0}^{N-1} X_i^{'} \cdot e^{j\frac{2\pi}{N} in} \\ &= \sum_{k=0}^{N/2-1} \left[X_k \cdot e^{j\frac{2\pi}{N} kn} - X_k \cdot e^{j\frac{2\pi}{N}(N-1-k)n} \right] \end{aligned} \quad \text{for } 0 \le n < N . \quad (11)$$

In the symmetric data-conversion method, the kth sub-carrier data after FFT in the receiver is arranged as

$$Y_k = \sum_{l=0}^{N/2-1} \left[X_l H_l Q_{l-k} - X_l H_{N-1-l} Q_{N-1-l-k} \right] + N_k . \quad (12)$$

Similarly, the $N-1-k$ th sub-carrier signal is expressed as

$$Y_{N-1-k} = \sum_{l=0}^{N/2-1} \left[X_l H_l Q_{l-N+1+k} - X_l H_{N-1-l} Q_{k-l} \right] + N_{N-1-k} . \quad (13)$$

In the receiver, the decision variable $Z_k^{'}$ of the kth symbol is found from the difference of the symmetrically allocated sub-carrier signals affected by phase noise. That is,

$$\begin{aligned} Z_k^{'} &= (Y_k - Y_{N-1-k})/2 \\ &= \frac{1}{2} X_k \left(H_k Q_0 + H_{N-1-k} Q_0 - H_{N-1-k} Q_{N-1-2k} + H_k Q_{-N+1+2k} \right) \\ &+ \frac{1}{2} \sum_{\substack{l=0 \\ l \neq k}}^{N/2-1} X_l \left[H_l Q_{l-k} + H_{N-1-l} Q_{k-l} - H_{N-1-l} Q_{N-1-l-k} - H_l Q_{l-N+1+k} \right] + N_k^{'} \end{aligned} , \quad (14)$$

where $N_k^{'} = \frac{1}{2}\left(N_k - N_{N-1-k} \right)$ is the AWGN of the kth parallel branch data in the receiver.

When channel is flat, frequency response of channel equals 1. $Z_k^{'}$ is as follows.

$$\begin{aligned} Z_k^{'} &= \frac{1}{2} X_k \left(2Q_0 - Q_{N-1-2k} + Q_{-N+1+2k} \right) \\ &+ \frac{1}{2} \sum_{\substack{l=0 \\ l \neq k}}^{N/2-1} X_l \left[Q_{l-k} + Q_{k-l} - Q_{N-1-l-k} - Q_{l-N+1+k} \right] + N_k^{'} \end{aligned} . \quad (15)$$

4. Simulation Results and Discussion

Simulation parameters are as follows: (1) Modulation method: QPSK modulation; (2) OFDM sub-carrier number: 64; (3) HPA: SSPA; (4) backoff: 0dB; (5) Channel: AWGN; (6) Phase noise model: the generalized model of PLL (ref [4,5]); (7) Phase noise parameter: σ_φ^2 =0.06 rad^2 ; (8) Coding efficiency of each method: C_r ; (9) required-transmitted-signal-to-noise-ratio: $E_b / C_r N_0$. Here, BER versus the required-transmitted-signal-to-noise-ratio $E_b / C_r N_0$, C_r is the coding efficiency ; N_0 is the spectral density coefficient for the white noise) is considered [10].

CIR means carrier to interference ratio and is defined as follows:

$$CIR = \frac{\sigma_c^2}{\sigma_I^2} = \frac{E[|C_k|^2]}{E[|ICI_k|^2]} . \quad (16)$$

PAPR means peak to average power ratio and is defined as follows:

$$PAPR = \frac{\max\limits_{0 \le n \le N-1} [|x_n|^2]}{E[|x_n|^2]} . \quad (17)$$

where x_n is time domain OFDM signal.

The CCDF is defined as

$$\begin{aligned} CCDF(PAPR_0) &= \Pr(PAPR(x) > PAPR_0) \\ &= 1-\left(1-\exp\left(-PAPR_o\right)\right)^{\alpha N}, \end{aligned} \qquad (18)$$

where α commonly is 2.8 in most cases.

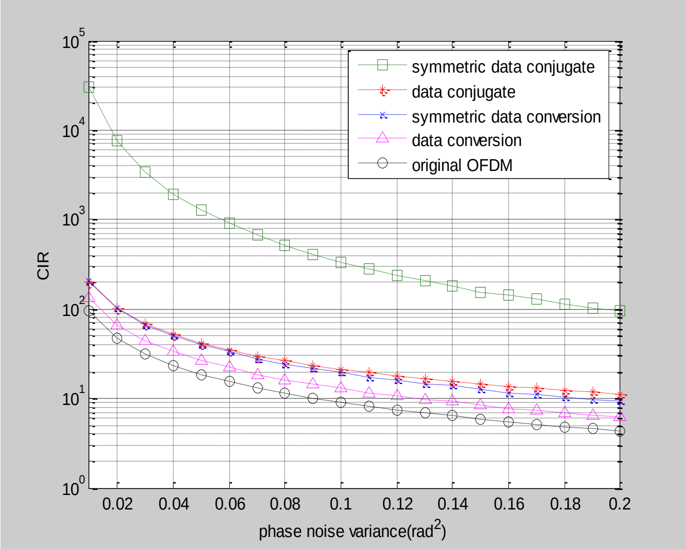

Figure 1. CIR Comparison by Semi-analytical Method

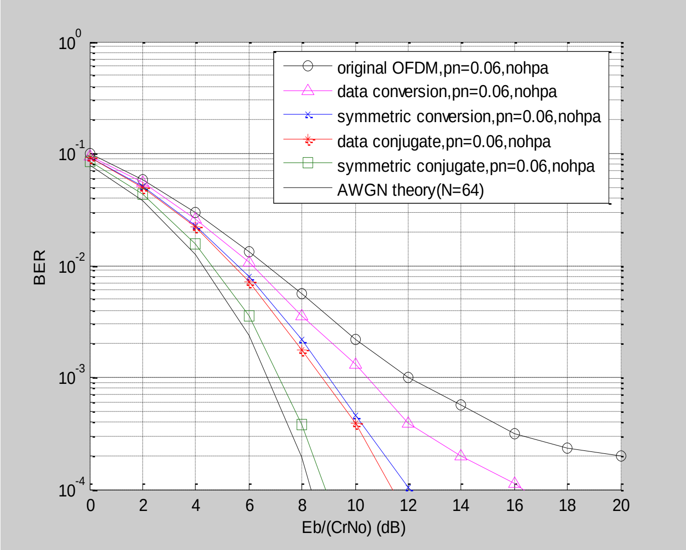

Figure 2. BER in AWGN Channel with Phase Noise

As seen from the Figure 1, CIR becomes greater according to the order of original OFDM, OFDM with data-conversion method, OFDM with symmetric data-conversion method, OFDM with data-conjugate method and OFDM with symmetric data-conjugate method. The symmetric data-conjugate method has significantly larger CIR compared with other methods. The CIR of the data-conjugate method is nearly similar with symmetric data-conversion method in the low phase noise range, a bit larger than symmetric data-conversion method in the high phase noise range. The data-conversion method has the least CIR improvement among the four ICI self-cancellation methods, but also has better CIR property than original OFDM system.

Figure 2 shows BERs in AWGN channel without HPA when phase noise variance is 0.06 rad^2 . As seen in the Figure 2, without considering HPA nonlinearity, BER performance increases in the order of original OFDM, data-conversion method, symmetric data-conversion method, data-conjugate method and symmetric data-conjugate method. At $BER = 10^{-3}$, 0.4dB, 2dB, 2.2dB, 3.8dB, 5.3dB SNR penalties appear respectively in the symmetric data-conjugate method, data-conjugate method, symmetric data-conversion method, data-conversion method and original OFDM compared with AWGN theory.

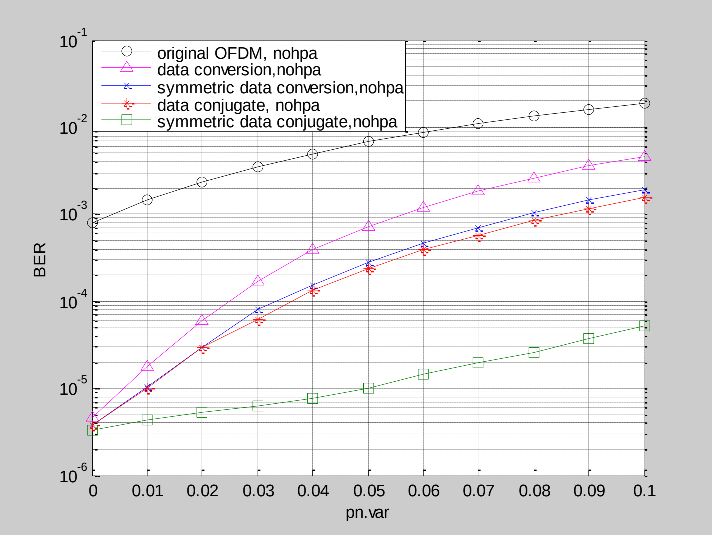

Figure 3. Phase Noise Variance vs BER ($E_b / C_r N_0 = 10\text{dB}$)

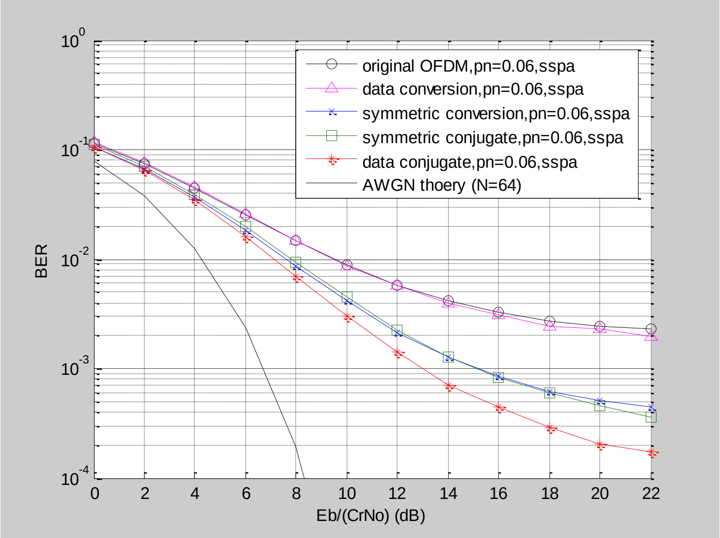

Figure 4. BER in AWGN Channel with Phase Noise and SSPA (backoff=0dB)

Figure 3 shows the BER according to the phase noise variance in AWGN channel when $E_b / C_r N_0$ is fixed to 10dB. As seen in the figure, performance can be considerably improved in the order of data-conversion method, symmetric data-conversion method, data-conjugate method and symmetric data-conjugate method compared with original OFDM when phase noise is considered only. To reach $BER = 10^{-3}$, about $0.05\,rad^2$, $0.075\,rad^2$ and $0.082\,rad^2$ more phase noise variances can be tolerated respectively in the data-conversion method, symmetric data-conversion method and data-conjugate method compared with original OFDM, and symmetric data-conjugate method can achieve BER lower than 10^{-4}.

Figure 4 shows the BERs in AWGN channel when phase noise variance $0.06\,rad^2$ and SSPA (solid state power amplifier) with backoff=0dB are considered. As seen in the Fig.4, when phase noise and HPA nonlinearity are considered together, at $BER = 10^{-3}$, 6.2dB, 8.3dB, 8.3dB SNR penalties appear respectively in the data-conjugate method, symmetric data-conjugate method and symmetric data-conversion method compared with AWGN theory, but error floor occur in the data-conversion method and original OFDM. So, the data-conjugate method has the best performance improvement. The symmetric data-conjugate method and the symmetric data-conversion method have the similar performance improvement. The data-conversion method hardly has performance improvement compared with original OFDM.

5. Conclusions

In this paper, two kinds of modified ICI self-cancellation methods are newly proposed and compared for the performance evaluation. CPE (common phase error), ICI and CIR (carrier to interference power ratio) are derived and discussed by the linear approximation of the phase noise. As results, this proposed symmetric data-conjugate method has the best ICI minimization efficiency, ICI minimization performance gradually degrades according to the order of symmetric data-conjugate method, data-conjugate method, symmetric data-conversion method and data-conversion method. However, the symmetric data-conjugate method provokes PAPR problem unlike the proposed symmetric data-conversion method. PAPR property becomes worse according to the order of data-conjugate method, symmetric data-conversion method, symmetric data-conjugate method and data-conversion method. Therefore, symmetric data-conjugate method can result in the best BER performance, when HPA nonlinearity cannot be neglected, data-conjugate method achieves the best BER performance.

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