Efficient Sequential Processing for Region Queries in Multidimensional Point Access Methods

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Abstract. The B^+-tree was proposed to support sequential processing for the B-tree. To the extent of authors' knowledge, however, there have been no studies supporting sequential processing in multidimensional point access methods(PAMs). To do this, the cells in a multilevel and multidimensional space managed by a multidimensional PAM must be linearly ordered systematically. In this paper, we present ideas for an approach that linearly orders cells in a multilevel and multidimensional space. Then, we present ideas for a novel sequential processing algorithm for region queries using this approach.

1 Introduction

After the B-tree was invented by multiple teams simultaneously, the B^+-tree was proposed for supporting sequential processing of range queries without tree traversal [1]. The B^+-tree not only has all of advantages of the B-tree but also supports efficient sequential processing of range queries in one dimensional space without costly tree traversal.

In the B^+-tree, the keys of all objects are stored in leaf nodes, and all the leaf nodes are linked in increasing order of the keys. These all linked leaf nodes are called the *sequence set* [1]. The SSS(*subset of the sequence set*) for a query is defined as a linked portion of the sequence set that starts from a leaf node containing the starting point of the query range and ends a node containing the ending point of the range. A range query in one dimensional space can be processed by scanning of an SSS for the query.

In the last two decades, various multidimensional point access methods(PAMs) were studied for handling of objects that can be represented as points in a multidimensional space. As results, robust PAMs that efficiently handle various distributions including skewed, such as KDB-tree [5], MLGF [7, 8, 6], and LSD-tree [2] were developed. These PAMs can be regarded as extensions of the B-tree or the B^+-tree for the multidimensional space in some aspects.

Also, a method using the space filling curve(SFC) for handling multidimensional objects without the extension of the B-tree was proposed [4]. The SFC is used for ordering of grid patterned square cells of a multidimensional space.

For processing of region queries, costly tree traversals are needed in a multidimensional PAM as in the B-tree. If sequential processing of region queries is supported as in the B^+-tree, we can expect great performance enhancements for region query processing. But, to the extent of the authors' knowledge, there have been no methods supporting sequential processing of region queries in multidimensional PAMs. In this paper, we propose an approach that extends multidimensional PAMs for supporting sequential processing of region queries. Using the approach, we can process a region query with efficient sequential accessing of multiple SSSs. An SSS is composed of linked disk blocks. The method does not lose all of current benefits of multidimensional PAMs.

This paper is organized as follows. The characteristics of SFC are explained in Section 2. In Section 3, we define extended grid pattern and SFC for handling of a multilevel and multidimensional space. Then, we show the splitting strategy of a multidimensional PAM must reflect the characteristics of the extended SFC. Next, we present ideas for a sequential processing algorithm of region queries in a multidimensional space using the extended SFC. In Section 4, we conclude the results and give further research directions of the proposed ideas.

2 Characteristics of Space Filling Curves

The SFC orders the square shaped cells with full grid patterns of n dimensional space that divide each axis with the number of 2^k ($k = 1, 2, ...$) with equal length. The total number of cells is 2^{kn}. The SFC can be used for mapping of cells in multidimensional space to linear order. Typical such SFCs are the Z-order and the Hilbert-order [4]. Figures 1 and 2 show how these two SFCs order cells in the two dimensional space when k = 1, 2, and 3.

Generally, the query region for a region query is determined as some prefixed range or a value for some of multiple attributes. We can regard that the entire range is selected for undetermined attributes. So, a query region is a hyper-rectangle. The cells intersected with the query region may be divided as m SSSs. In this case, the query may be processed m scans of SSSs.

Since the number of scans must be reduced to process queries efficiently, the SFC must have characteristics of making a small number of SSSs for various forms of a query region. This means that the SFC must have a property of **degree of locality** as high as possible. So, the SFC must locate cells in the linear order as adjacently as possible for adjacent cells in the multidimensional space to reduce the number of scans for query processing.

The reasoning behind the locality is that the cells must be located in the linear order as adjacently as possible for adjacent cells in the multidimensional space. For example, for the Hilbert-order of two-dimensional space, two cells among four adjacent cells are located at contiguous positions in the linear order and the average and variance of the distant value for the discontiguous two cells are relatively small. But, in the Z-order, only a cell of four adjacent cells is contiguous in some cases and the average and variance are relatively large. In [4], it was discussed that the Hilbert-order has the best property with theoretical and experimental investigation among the SFCs.

In the tree structured multidimensional PAMs, a region corresponding to a parent node of the tree includes all regions corresponding to all leaf nodes

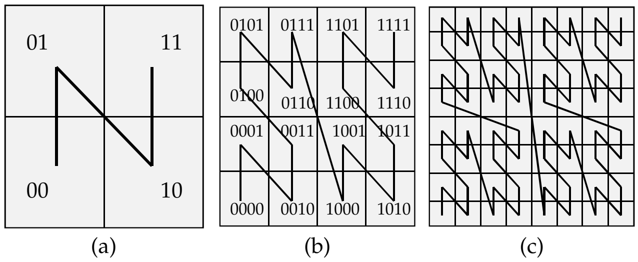

Fig. 1. The Z-order with $k = 1$, 2, and 3.

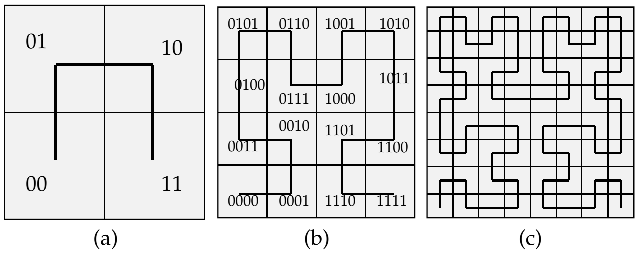

Fig. 2. The Hilbert-order with $k = 1$, 2, and 3.

pointed by the parent node. If an SFC is applied to a multidimensional space managed by a parent level of the tree, **the SFC used for the child level must be generated from the SFC used for the parent level recursively**. In other words, the SFCs must have a property of the fractal. If the granule of cells is reduced in one step (i.e., each cell is divided into 2^n cells), the order must be maintained recursively. Figures 1 and 2 also show this recursive property.

To handle the recursive property of the SFC efficiently, the order of each cell can be represented n bits for each level in n dimensional space. So, for l levels, nl bits are needed. For example, the binary value depicted in each cell of Figure 1(a) represents the cells' order value of the SFC for $k = 1$. Figure 1(b) represents the order value for $k = 2$ and the first two bits of all cells are same as the order value of a cell in $k = 1$ including each cell in $k = 2$. For the Hilbert-order, the characteristic is analogous. This method uses the lexicographical order for representing the order value of cells. In this method, the inclusion and adjacency relation among cells can be easily determined.

3 Multifarious Granuled Grid(MGG) and Multifarious Granuled SFC(MGSFC)

In Section 3.1, a new grid pattern usually used for managing the multidimensional space in multidimensional PAMs is defined. In Section 3.2, we define an extended SFC that can be applied to the new grid pattern and discuss that the splitting strategy used for PAMs must reflect the characteristics of this extended SFC. In Section 3.3, we propose a sequential processing algorithm using recursion for region query in multilevel and multidimensional space.

3.1 Multifarious Granuled Grid(MGG)

In Section 2, we only consider the full grid pattern that is constructed by the same sized square cells. The grid file [3] is an index structure managing objects in multidimensional space with a full grid pattern. If many cells point to the same disk block, then the cells may be unified to one cell. When multidimensional space is managed by a full grid pattern, the number of cells becomes too large for managing data with skewed distribution [7]. So, it is preferable to define the extended form of a grid pattern managing skewed distribution efficiently.

To reduce the complexity of managing the multidimensional space, we assume that the bounding lines of a cell are parallel to the axes, the shape of all cells is rectangle, and the start and ending position of all cells is $p/2^n$ (p=0,1,.., $2^n - 1$) and $q/2^n$ (q=1,2,.., 2^n), respectively: i.e., the shape of all cells is the hyper-rectangle. We define this type of a grid pattern as *Multifarious Granuled Grid(MGG)*. A representative multidimensional PAM handling the MGG is the MLGF [6–8].

Although cells in MGG may be splitted to be various styles, we only use a style. If the overflow for the disk block pointed by a cell occurs, we select an axis of the cell for splitting and divide the center of the range for the axis with a restriction of selecting the axis that is not selected yet from the square shaped cell for always maintaining square like shaped cells. This means that the length of an axis for a cell is equal or twice to the length of the other axes for the cell.

3.2 Multifarious Granuled Space Filling Curve(MGSFC)

As explained in Section 2, the SFC must be extended to order the cells of the MGG since the MGG does not have the grid pattern. Also, the extended SFC must have the properties explained in Section 2.

Since the SFC has the recursive property, the SFCs in multilevel space may be integrated according to the granules of the cells. So, we can order the cells of the MGG with the extended SFC. We call the extended SFC as *Multifarious Granuled Space Filling Curve(MGSFC)*. We also use the same method of representing the order value of the cells as explained in Section 2.

The MGSFC and the splitting strategy are very closely related. For example, Figure 3(a) shows an example of the MGG that uses the round-robin splitting strategy in two-dimensional space. The thin line in Figure 3(b) shows the Z-order of a grid pattern with $k = 2$, and the bold line shows the MG Z-order applied for the MGG of Figure 3(a). As we can see, the MG Z-order can be regarded as an integration of the Z-orders of multilevels. So, the MG Z-order is closely related with the round-robin splitting strategy.

In the MGG, the cells of a non-square shape can be divided into multiple sub-cells of a square shape and the order values of these sub-cells in the SFC must be consecutive. For example, the cell in the upper left corner in Figure 3(a) can be divided into two sub-cells of a square shape and the two sub-cells are consecutive

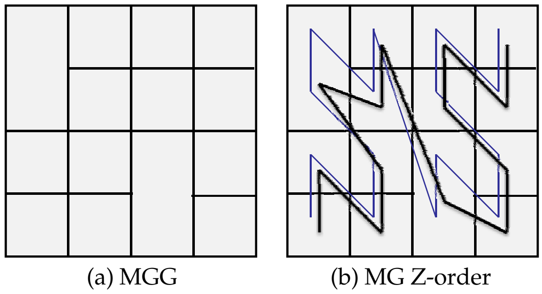

Fig. 3. An example of the MGG and the MG Z-order.

in the Z-order with $k = 2$. In such a case, we can regard the consecutive order values in the SFC as one order value in the MGSFC. For example, the order value of the cell in the upper left corner in Figure 3(a) can be represented as '010' that includes the prefix of the order values of the two sub-cells '0100' and '0101'.

The relation of the inclusion and adjacency of cells in the MGG can be easily determined using the method of representing the order values of the cells explained in Section 2. For the two cells with order values of the different length, the shorter cell is included in the longer cell if the prefix bits of longer cell is same as all the bits of the shorter cell. For example, the cells with order values '0100' and '0101' are included in the cell with an order value '010'. When the order value of the longer cell is increased or decreased by one, the two cells are adjacent if the prefix bits of the longer cell is same as all the bits of the shorter cell. For example, the cells '0011' and '01' are adjacent since the cells '0011' and '0100' are adjacent and the cell '0100' is included in the cell '01'. Also the cells '0100' and '00' are adjacent since the cells '0100' and '0011' are adjacent and the cell '0100' is included in the cell '01'.

The splitting strategy for the MGG must reflect the characteristics of the MGSFC. To use some MGSFC, we must use an adequate splitting strategy. Figure 4(a) shows an example of one to one correspondence with the MGG and the MG Hilbert-order.

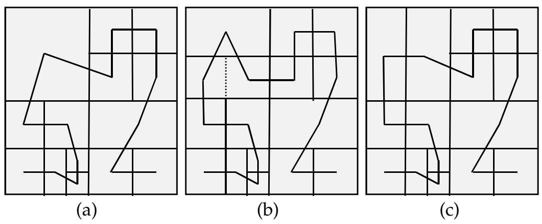

Fig. 4. One to one correspondence between the splitting strategy and the MGSFC: (a) MGG and MG Hilbert-order, (b) improperly split case, (c) properly split case.

To use some MGSFC, a proper axis must be selected as a splitting axis when a cell of the MGG is split. For example, we can't apply the MG Hilbert-order when the upper left cell of Figure 4(a) is splitting as the Figure 4(b). This is because the divided lower cell has two sub-cells (the dotted line divides the cell) with not adjacent values '0100' and '0111'. So, the cell can't have one order value in the MG Hilbert-order. As explained above, the cell in the MGG must include conservative ordered sub-cells in the SFC. On the other hand, the two upper and lower divided cells include two sub-cells with adjacent order values when the upper left cell of Figure 4(a) is split as in Figure 4(c). So, this split is adequate.

We must use the splitting strategy that has a property that the sub-cells of splitted cells must be adjacent in the SFC. We discussed it with the example of Figure 4. This principle also can be applied to the case of Figure 3 using the round robin splitting strategy and the MG Z-order.

Therefore, the splitting strategy must reflect the property of the adapted MGSFC. So, we can linearly order all the cells of multilevel and multidimensional space using the appropriate MGSFC and splitting strategy. Using the observations, we can process region queries sequentially.

3.3 Region Query Processing

We only consider the case that the multidimensional space is managed by multilevels and each level of multidimensional space can be represented by the MGG when the space is handled by a multidimensional PAM. In this case, the PAM should have a balanced tree structure and the database for an upper level is an abstract database of its lower level: i.e., a cell in an upper level may be divided into multiple cells in a lower level. It has one to n relationship. For example, in Figure 5 which represents MGGs of two levels, the cell 'x' of an upper level at Figure 5(a) is divided into cells 'A', 'B', and 'C' of its lower level at Figure 5(b).

When a query region is given for a multidimensional PAM, for each level, all pages corresponding to the cells intersected with the query region are accessed for query processing. For example, the pages corresponding to cells 'x' and 'z' are accessed for the query region 'Q' for the upper level. The sum of these cells is larger than or equal to the query region. We call the sum as *MGG corresponding region* of a query region.

However, the pages corresponding to the lower level cells that are divided from all the upper level cells within the MGG corresponding region for a query region may not be accessed for query processing. This is because that some cells in a lower level divided from its upper level cells within the MGG corresponding region may not be intersected with the query region: i.e., the MGG corresponding region of the upper level is larger than or equal to the region of its lower level. For example, in Figure 5, the page corresponding to the cell 'x' must be accessed for query processing but the page corresponding to the only cell 'C' (not all the cells 'A', 'B', and 'C' divided from the cell 'x') is accessed for query processing.

An MGG corresponding region for each level may be divided into multiple regions corresponding to multiple SSSs. This is because that the cells correspond-

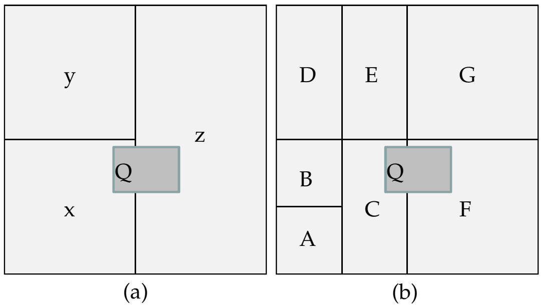

Fig. 5. The MGG of upper and lower levels.

ing to MGG corresponding region may be divided into regions corresponding to multiple SSSs.

An SSS of the upper level may be divided into multiple SSSs of the lower level. The MGG corresponding region of the lower level may be smaller than that of the upper level. So, the cells corresponding to the shrinked region of the lower level for query processing may not be adjacent in the MGSFC order of the lower level since the region of the lower level corresponding to the given upper level SSS may be smaller than the region of the upper level corresponding to a given SSS.

With above observations, we developed the depth-first processing algorithm. In this algorithm, an SSS is constructed in the root level at first and a lower level SSS is constructed. This process is done recursively. Then, leaf level pages having key values of objects are accessed for the lowest level. So, for each level, only an SSS is handled for query processing. We omit the pseudo code of the algorithm due to the space limitation.

4 Conclusions

The very famous one dimensional index structure B-tree was extended to the B^+-tree for supporting sequential processing [1]. But, there are no studies for supporting of sequential processing in multidimensional PAMs.

To support query processing for region queries sequentially in multidimensional space, the cells in multilevel and multidimensional space must be ordered linearly. To do this, we can use the SFC with some extension. We explain that the SFC has properties of locality and recursion basically. There are the Z-order and Hilbert order as the very famous SFC. These orders assume the full grid pattern for ordering of cells. But the full grid pattern yields a very large index structure.

To overcome the disadvantage of the full grid pattern, there are many studies of tree structured multidimensional PAMs. These PAMs integrate cells pointing to same disk blocks as one cell [2, 7]. We restrict the case that the PAMs use the half splitting strategy and have the square like cells having the length of an axis with equal or twice of the other axes. We define this type of a grid pattern as the MGG. We also discussed that an upper level MGG is an abstraction of a lower level MGG. We define the MGSFC, an extension of the SFC, for linear ordering of cells for the some case of the MGG. We also discussed that the splitting strategy used for a multidimensional PAM must reflects the characteristics of the MGSFC. This method is used for ordering of cells in a multilevel and multidimensional space to support sequential processing of region query. We present ideas for sequential query processing using this approach. The approach discussed here can be applied not only to the MLGF but also to all PAMs managing multilevel and multidimensional space with the MGG style.

We also designed the detailed method for applying this approach to the MLGF. After implementing this approach, we will study the advantages and disadvantage of our approach.

Acknowledgements

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