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Output Single Phase Variable Structure Control for Mismatched Uncertain Systems

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Abstract

This paper proposes a new single phase variable structure control (SPVSC) for mismatched uncertain systems. A new sliding mode without reaching phase is designed to address two most important problems in the variable structure control area: 1) the system states are in sliding mode at the initial time movement; and 2) using only output variables directly. First, a new sufficient condition in terms of linear matrix inequality is derived such that the existence of a sliding surface guarantees asymptotic stability of the sliding mode dynamics from the initial time instant. Second, a static output feedback SPVSC law is designed to force the system states to stay in the sliding surface from beginning to end. Final, the lateral motion of a B-26 aircraft is simulated to demonstrate the advantages and effectiveness of the proposed SPVSC scheme.

Keywords*: linear matrix inequalities; mismatched uncertain systems; variable structure control; output single phase*

1. Introduction

Over the past three decades, there has been an increasing research interest in the variable structure control (VSC) theory and application. The main advantages of VSC are fast response and strong robustness with respect to uncertainties and external disturbances [1-4]. Thanks to these advantages, the VSC theory has been successfully applied to a wide variety of practical engineering systems such as robot manipulators, aircrafts, underwater vehicles, spacecraft, flexible space structures, electrical motors, power systems, automotive engines [5]. The conventional VSC is attained by applying a discontinuous control law to drive state trajectories onto a sliding surface and force them to remain on it thereafter (this process is called reaching phase), and then to keep the state trajectories moving along the surface towards the origin with the desired performance (such motion is called sliding phase) [6].

Generally speaking, the conventional VSC design can be divided into two phases: the reaching phase and the sliding mode. Firstly, in the reaching phase, give a switching control law such that the system trajectory can be trapped on a switching surface and remain on it thereafter. Secondly, in the sliding mode, determine the switching surface such that the system dynamics in the sliding mode have good performance. Obviously, all the robust properties of VSC are valid during the sliding regime [6-8]. In addition, the state variables of many practical systems are not always accessible or expensive to measure all of them [9]. Then, it is necessary that the sliding mode predominates over the reaching phase without the measurement of all state variables. With this purpose, it is

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essential to design VSC without reaching phase and the reaching time is equal to zero. This approach belongs to the class of VSC without reaching phase.

Many authors have applied various techniques to solve the above problems. Recently, some good results have been published in high quality journal such as [5-6], [8], [10-16]. The authors of [10-12] have presented a new integral sliding mode control (SMC). The feature of the integral SMC law assures the robustness of the system entire response of the system starting from the initial time instance. However, those approaches given in [10-12] failed when facing the systems with mismatched uncertainties in the state matrix. The authors of [6] proposed new totally invariant conditions for systems with mismatched uncertainties. The system with mismatched in both the state matrix and input matrix was solved in [5] by applying the concept of integral SMC. By using the conventional and elastic acceleration constraints, the authors of [8] presented a new SMC without reaching phase for a third-order system. The work in [13] defined a new integral sliding manifold for reducing the effect of the disturbance terms and uncertainty. In [14], the authors presented a novel sliding mode without reaching phase for a class of nonlinear systems. More recently, the study in [15] proposed a universal fuzzy integral SMC for mismatched uncertain systems. The authors of [16] developed a new integral SMC for handling a larger class of mismatched uncertainties. Using observer-based integral SMC in [17], a new approach was proposed to approximate the system states and disturbance vectors. In addition, the stability of the sliding mode in the term of linear matrix inequalities (LMI) has some benefits over conventional approach methods and offers additional design flexibility. Since LMI problems can be quite easily and efficiently solved by LMI Toolbox in Matlab software. The robustness of the integral SMC via LMI technique can be guaranteed throughout its entire trajectories starting from the initial time.

However, it should be mentioned that most of the previous results have been developed under the assumption that all the system states are available, including the initial conditions. Thus, those approaches could not be applied directly for mismatched uncertain systems when only the output information is available. Therefore, it is quite important to establish a new single phase VSC to control mismatched uncertain systems via output feedback.

Motivated by the aforementioned analysis, this paper proposes a new single phase VSC for mismatched uncertain systems where only output information is available. First, a sliding surface is designed such that the reaching phase required in the conventional VSC scheme is eliminated since the system trajectories always start from the sliding surface. Therefore, the desired dynamic behaviour of the system is obtained from the beginning of its motion. Second, appropriate LMI stability conditions by the Lyapunov method are derived to guarantee the stability of the system for all time. Final, a static output feedback single phase variable structure controller is designed to force the system states stay in the sliding surface for all time.

Notation: The notation used throughout this paper is fairly standard. X^T denotes the transpose of matrix X. $I_{n\times m}$ and $0_{n\times m}$ are used to denote the $n\times m$ identity matrix and the $n\times m$ zero matrix, respectively. The subscripts n and $n\times m$ are omitted where the dimension is irrelevant or can be determined from the context. $\|x\|$ stands for the Euclidean norm of vector x and $\|A\|$ stands for the matrix induced norm of the matrix A. The expression $A>0$ means that A is symmetric positive definite. R^n denotes the n-dimensional Euclidean space.

2. Statement of the Problem

Consider the following mismatched uncertain systems:

$$\begin{aligned} \dot{x} &= [A+\Delta A(x,t)]x + B[u+\xi(x,t)] \\ y &= Cx. \end{aligned} \tag{1}$$

where $x \in R^n$ is the state, $u \in R^m$ is the control input, $y \in R^p$ is the output and $m < p < n$. The matrices A, B and C are constant matrices with appropriate dimensions. The term ΔA represents the mismatched uncertainties of the plant, which the matching condition is not satisfied and $\xi(x,t)$ is the disturbance input.

In order to design a new output SPVSC for the system (1), we assume the following to be valid:

Assumption 1. $\Delta A(x,t)$ is of the form

$\Delta A(x,t) = DF(x,t)E$

where $F(x,t)$ is unknown but bounded as $\|F(x,t)\| \le 1$ for all $(x,t) \in R^n \times R$, and D and E are any non-zero matrices of appropriate dimensions.

Assumption 2. $rank(CB) = m$.

Assumption 3. There exist known non-negative constants k_ξ and k_m such that $\|\xi(x,t)\| \le k_\xi + k_m \|x(t)\|$.

3. Output Single Phase Variable Structure Control for Mismatched Uncertain Systems

In this section, we design a novel single phase variable structure control for the system (1). There are three steps involved in the design of our output single phase VSC scheme. In the first step, a proper sliding surface is designed such that the desired dynamic behaviour of the system is obtained from the beginning of its motion and the sliding surface is dependent on only output variables. In the second step, appropriate linear matrix inequalities (LMI) stability conditions by the Lyapunov method are derived to guarantee the stability of the system for all time. In the final step, we design an output feedback single phase variable structure controller to keep the system states stay in the sliding mode from beginning to end.

3.1. Output Single Phase Sliding Surface Design

Let us first design a sliding surface by using the concept of output SPVSC. Under Assumption 2, it follows from equations (11), (12) and (13) of paper [18] that there exists a coordinate transformation $z = Tx$ such that the system (1) has following regular form.

$$\dot{z} = \left(\begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix} + \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} F \begin{bmatrix} E_1 & E_2 \end{bmatrix}\right) z + \begin{bmatrix} 0 \\ B_2 \end{bmatrix} [u + \xi(T^{-1}z, t)] \tag{2}$$

$$y = \begin{bmatrix} 0 & C_2 \end{bmatrix} z \tag{3}$$

where $TAT^{-1} = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix}$, $TDFET^{-1} = \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} F \begin{bmatrix} E_1 & E_2 \end{bmatrix}$, $TB = \begin{bmatrix} 0 \\ B_2 \end{bmatrix}$ and $CT^{-1} = \begin{bmatrix} 0 & C_2 \end{bmatrix}$. The matrices $B_2 \in R^{m \times m}$, $C_2 \in R^{p \times p}$ are non-singular. Then, the output sliding surface can be defined as follows:

$$\sigma(y(t),t) = \bar{\sigma}(y(t)) - \bar{\sigma}(y(0))\exp(-\beta t) = 0 \tag{4}$$

where $\bar{\sigma}(y,t)$ is given by

$$\bar{\sigma}(y(t)) = KC_2^{-1}y = K\begin{bmatrix} N & 0_{(p-m)\times m} \\ 0_{m\times(n-m)} & I_{m\times m} \end{bmatrix} z$$
$$= \begin{bmatrix} K_1 N & K_2 \end{bmatrix} z \tag{5}$$
$$= K_2 z_2$$

in which $z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$ with $z_1 \in R^{n-m}$, $z_2 \in R^m$, $N = \begin{bmatrix} 0_{(p-m)\times(n-p)} & I_{(p-m)\times(p-m)} \end{bmatrix}$, and $K = \begin{bmatrix} 0_{m\times(p\text{-}m)} & K_2 \end{bmatrix}$. The matrix $K_2 \in R^{m\times m}$ is of the form

$$K_2 = \Psi P \Psi^T \tag{6}$$

where $P \in R^{(n-m)\times(n-m)}$ is defined later and the matrix $\Psi \in R^{m\times(n-m)}$ is selected such that the matrix $K_2 \in R^{m\times m}$ is non-singular. From equation (5), the output sliding surface (4) can be rewritten as

$$\sigma(y(t),t) = \bar{\sigma}(y(t)) - \bar{\sigma}(y(0))\exp(-\beta t) = K_2 z_2(t) - K_2 z_2(0)\exp(-\beta t) = 0. \tag{7}$$

According to equation (7) and since $K_2 \in R^{m\times m}$ is non-singular, in the sliding mode $\sigma(y(t),t) = \dot{\sigma}(y(t),t) = 0$, it is obvious that

$$z_2 = z_2(0)\exp(-\beta t). \tag{8}$$

Then, from the structure of system (2)-(3) and equation (8), the sliding mode dynamics of system (1) associated with the output sliding surface (4) is described by

$$\dot{z}_1 = \bar{A}_1 z_1 + \bar{A}_2 z_2(0)\exp(-\beta t) \tag{9}$$

in which $\bar{A}_1 = A_1 + D_1 F E_1$, $\bar{A}_2 = A_2 + D_1 F E_2$.

Remark 1: It is obvious that $\sigma(y(0),0) = 0$ and the desired dynamic behaviour of the system are obtained from the beginning of its motion, which makes the system more robust against perturbations than the VSC with reaching phase.

3.2. Stability Analysis of Sliding Motion

In the last section, we have designed the output sliding surface, which the desired dynamic behaviour of the system is obtained from the beginning of its motion. There are still two important tasks that should be done. The first task is to derive appropriate LMI stability conditions by the Lyapunov method to guarantee the stability of the sliding mode dynamics (9). The second task is to design an output feedback single phase variable structure controller to keep the system states stay in the sliding surface (4) from beginning to end.

This section focuses on the former task. We begin by considering the following two LMIs:

$$A_1^T P + PA_1 + \hat{\varepsilon}^{-1} E_1^T E_1 + \hat{\varepsilon} P D_1 D_1^T P < 0 \tag{10}$$

and

$$\tilde{\varepsilon} P A_2 A_2^T P + \varepsilon P D_1 D_1^T P < 0 \tag{11}$$

where $P \in R^{(n-m)\times(n-m)}$ is any positive definite and the scalars $\hat{\varepsilon} > 0$, $\tilde{\varepsilon} > 0$ and $\varepsilon > 0$. Then, we can establish the following theorem.

Theorem 1: Suppose that LMIs (10) and (11) have a solution $P > 0$ and the scalars $\hat{\varepsilon} > 0$, $\tilde{\varepsilon} > 0$ and $\varepsilon > 0$. The sliding surface is given by equation (4). Then, the sliding motion described in (9) is asymptotically stable.

Before proving the theorem 1, we recall the following Lemmas.

Lemma 1 [19]: Let X, Y and F are real matrices of suitable dimension with $F^T F \le I$ then, for any scalar φ>0, the following matrix inequality holds:

$$XFY + Y^T F^T X^T \le \varphi^{-1} XX^T + \varphi Y^T Y.$$

Lemma 2 [20]: Let X and Y are real matrices of suitable dimension then, for any scalar $\mu > 0$, the following matrix inequality holds:

$$X^T Y + Y^T X \le \mu X^T X + \mu^{-1} Y^T Y.$$

Proof of Theorem 1: First, Let us define a Lyapunov function candidate as

$$V = z_1^T P z_1 \tag{12}$$

where the positive-definite matrix P is defined in (10) and (11). If we differentiate V with respect to time combined with (9) then

$$\begin{aligned} \dot{V} &= z_1^T [(A_1 + D_1 F E_1)^T P + P(A_1 + D_1 F E_1)] z_1 \\ &\quad + [z_2^T(0)(A_2 + D_1 F E_2)^T P z_1 + z_1^T P (A_2 + D_1 F E_2) z_2(0)] \exp(-\beta t). \end{aligned} \tag{13}$$

Applying Lemma 1, we have

$$\begin{aligned} \dot{V} &\le z_1^T (A_1^T P + P A_1 + \hat{\varepsilon}^{-1} E_1^T E_1 + \hat{\varepsilon} P D_1 D_1^T P) z_1 \\ &\quad + [z_2^T(0) A_2^T P z_1 + z_1^T P A_2 z_2(0) + \varepsilon^{-1} z_2^T(0) E_2^T E_2 z_2(0) + \varepsilon z_1^T P D_1 D_1^T P z_1] \exp(-\beta t). \end{aligned} \tag{14}$$

where the scalars $\hat{\varepsilon} > 0$ and $\varepsilon > 0$. According to equation (14) and Lemma 2, it can be seen that

$$\begin{aligned} \dot{V} &< z_1^T (A_1^T P + P A_1 + \hat{\varepsilon}^{-1} E_1^T E_1 + \hat{\varepsilon} P D_1 D_1^T P) z_1 + z_1^T (\tilde{\varepsilon} P A_2 A_2^T P \\ &\quad + \varepsilon P D_1 D_1^T P) z_1 \exp(-\beta t) + [\tilde{\varepsilon}^{-1} z_2^T(0) z_2(0) + \varepsilon^{-1} z_2^T(0) E_2^T E_2 z_2(0)] \exp(-\beta t). \end{aligned} \tag{15}$$

where the scalar $\tilde{\varepsilon} > 0$. In addition

$$\lim_{t \to \infty} \{[\tilde{\varepsilon}^{-1} z_2^T(0) z_2(0) + \varepsilon^{-1} z_2^T(0) E_2^T E_2 z_2(0)] \exp(-\beta t)\} = 0. \tag{16}$$

Using equations (10) and (11), one can get that

$$\begin{aligned} & z_1^T (A_1^T P + P A_1 + \hat{\varepsilon}^{-1} E_1^T E_1 + \hat{\varepsilon} P D_1 D_1^T P) z_1 \\ &\quad + z_1^T (\tilde{\varepsilon} P A_2 A_2^T P + \varepsilon P D_1 D_1^T P) z_1 \exp(-\beta t) < 0 \end{aligned} \tag{17}$$

The equations (15), (16) and (17) imply that the sliding motion described in (9) is asymptotically stable. □

3.3. Output Feedback Single phase Variable Structure Controller Design

In the last section, we dealt with the first and second elements of the design process. In this section, we design an output feedback single phase variable structure controller to keep the system states stay on the sliding surface from beginning to end. It is enough if and only if $\sigma(y(t),t) = \dot{\sigma}(y(t),t) = 0,\ \forall t \ge 0$. In order to satisfy the above conditions, the modified output feedback single phase variable structure controller is selected to be

$$u(t) = -(K_2 B_2)^{-1} [\beta \left\| K C_2^{-1} \right\| \|y(0)\| \exp(-\beta t) + k_\xi \|K_2\| \|B_2\| + \bar{\rho} \|y\| + \rho + \alpha] \frac{\sigma}{\|\sigma\|} \tag{18}$$

where $\bar{\rho} = \|K_2\| (\|A_4\| + \|D_2\| \|E_2\| + k_m \|B_2\| \|H_2\|) \left\| K_2^{-1} \right\| \left\| K C_2^{-1} \right\|$,

$\rho = \|K_2\| (\|A_3\| + \|D_2\| \|E_1\| + k_m \|B_2\| \|H_1\|) \eta$, $[H_1 \quad H_2] = T^{-1}$, the scalars $\alpha > 0$ and $\eta > 0$. It should be pointed out that the controller (18) uses only output variables.

Then we can establish the following theorem.

Theorem 2: Suppose that LMIs (10) and (11) have a solution $P>0$ and the scalars $\hat{\varepsilon}>0$, $\tilde{\varepsilon}>0$ and $\varepsilon>0$. Consider the closed loop of system (1) with the above output feedback single phase variable structure controller (18) where the sliding surface is given by equation (4). Then the system states stay on the sliding surface (4) from beginning to end for all $\|z_1\|\le\eta$.

Proof of Theorem 2: Let us consider the following Lyapunov function

$$V(\sigma(y(t),t))=\|\sigma(y(t),t)\|. \tag{19}$$

By differentiating V with regard to time using (4) yields that

$$\dot{V}=\frac{\sigma^T}{\|\sigma\|}\dot{\sigma}=\frac{\sigma^T}{\|\sigma\|}(K_2\dot{z}_2+\beta KC_2^{-1}y(0)\exp(-\beta t)). \tag{20}$$

From equation (2), it is clearly that

$$\dot{z}_2=(A_3+D_2FE_1)z_1+(A_4+D_2FE_2)z_2+B_2[u+\xi(z,t)]. \tag{21}$$

Substituting equation (21) into equation (20), one can get

$$\begin{aligned}\dot{V}\le{}&\|K_2\|(\|A_3\|+\|D_2\|\|E_1\|)\|z_1\|+\|K_2\|(\|A_4\|+\|D_2\|\|E_2\|)\|z_2\|\\&+\beta\left\|KC_2^{-1}\right\|\|y(0)\|\exp(-\beta t)+\|B_2\|\|K_2\|\|\xi\|+\frac{\sigma^T}{\|\sigma\|}K_2B_2u.\end{aligned} \tag{22}$$

Since $x=H_1z_1+H_2z_2$ where $[H_1 \quad H_2]=T^{-1}$ and from Assumption 3, we have

$$\|\xi(x,t)\|\le k_\xi+k_m\|x(t)\|\le k_\xi+k_m(\|H_1\|\|z_1\|+\|H_2\|\|z_2\|). \tag{23}$$

According to equations (22) and (23), we obtain

$$\begin{aligned}\dot{V}\le{}&\|K_2\|(\|A_3\|+\|D_2\|\|E_1\|)\|z_1\|+\|K_2\|(\|A_4\|+\|D_2\|\|E_2\|)\|z_2\|+\frac{\sigma^T}{\|\sigma\|}K_2B_2u\\&+\beta\left\|KC_2^{-1}\right\|\|y(0)\|\exp(-\beta t)+k_\xi\|K_2\|\|B_2\|+k_m\|K_2\|\|B_2\|(\|H_1\|\|z_1\|+\|H_2\|\|z_2\|).\end{aligned} \tag{24}$$

The equation (5) implies that

$$\|z_2\|\le\left\|K_2^{-1}\right\|\left\|KC_2^{-1}\right\|\|y\|. \tag{25}$$

According to equation (25), the inequality (24) can be rewritten as

$$\begin{aligned}\dot{V}\le{}&\|K_2\|(\|A_3\|+\|D_2\|\|E_1\|)\eta+\|K_2\|(\|A_4\|+\|D_2\|\|E_2\|)\left\|K_2^{-1}\right\|\left\|KC_2^{-1}\right\|\|y\|+\frac{\sigma^T}{\|\sigma\|}K_2B_2u(t)\\&+\beta\left\|KC_2^{-1}\right\|\|y(0)\|\exp(-\beta t)+[k_\xi+k_m(\|H_1\|\eta+\|H_2\|\left\|K_2^{-1}\right\|\left\|KC_2^{-1}\right\|\|y\|)]\|K_2\|\|B_2\|.\end{aligned} \tag{26}$$

By substituting the controller (18) into equation (26), we have

$$\begin{aligned}\dot{V}\le{}&\|K_2\|(\|A_3\|+\|D_2\|\|E_1\|)\eta+\|K_2\|(\|A_4\|+\|D_2\|\|E_2\|)\left\|K_2^{-1}\right\|\left\|KC_2^{-1}\right\|\|y\|\\&+\beta\left\|KC_2^{-1}\right\|\|y(0)\|\exp(-\beta t)+[k_\xi+k_m(\|H_1\|\eta+\|H_2\|\left\|K_2^{-1}\right\|\left\|KC_2^{-1}\right\|\|y\|)]\|K_2\|\|B_2\|\\&-\frac{\sigma^T}{\|\sigma\|}K_2B_2(K_2B_2)^{-1}[\beta\left\|KC_2^{-1}\right\|\|y(0)\|\exp(-\beta t)+k_\xi\|K_2\|\|B_2\|+\bar{\rho}\|y\|+\rho+\alpha]\frac{\sigma}{\|\sigma\|}\\&=-\alpha<0.\end{aligned} \tag{27}$$

So, it is obvious that $\dot{V}(\sigma(y(t),t))<0$. Since $\sigma(y(0),0)=0$, suppose that there exists a time $t>0$ such that $V(\sigma(y(t),t))=\|\sigma(y(t),t)\|\le V(\sigma(y(0),0))=\|\sigma(y(0),0)\|=0$. Thus, the identities

$$\sigma(y(t),t) = \dot{\sigma}(y(t),t) = 0 \tag{28}$$

holds for all t≥0, *i.e.*, there is no reaching phase to the sliding mode and the system states remain on the sliding mode for all time t≥0. Thus, the proof is completed

Remark 2: From VSC theory, Theorems 1 and 2 together have shown that the sliding surface (4) with the output feedback single phase variable structure controller (18) guarantee that: 1) at any initial value the system states remain on the sliding mode from beginning to end; and 2) the mismatched uncertain system (1) is asymptotically stable.

Remark 3: A key feature of using output SPVSC is that the matched uncertainty of the system not only compensates right after the beginning of the process but also guarantees the robustness of the system throughout its entire trajectories starting from the initial time instant.

4. Numerical Example

In order to demonstrate the validity and effectiveness of the proposed approach, in this section, we are going to apply the output single phase VSC given in previous sections for the lateral motion of a B-26 aircraft, which is modified from [9].

$$\dot{x} = (\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -2.93 & -4.75 & -0.78 \\ 0.086 & 0 & -0.11 & -1 \\ 0 & -0.042 & 2.59 & -0.39 \end{bmatrix} + \Delta A)x + \begin{bmatrix} 0 & 0 \\ 0 & -3.91 \\ 0.035 & 0 \\ -2.53 & 31 \end{bmatrix}(u+\xi) \tag{29}$$

$$y = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} x \tag{30}$$

where $x = \begin{bmatrix} \phi & \dot{\phi} & \omega & \gamma \end{bmatrix}^T$, $u = \begin{bmatrix} \delta_\gamma & \delta_a \end{bmatrix}^T$ with ϕ is the bank angle, ω is the sideslip angle, γ is the yaw rate, δ_γ is the rudder deflection and δ_a is the aileron deflection. It is assumed that $\dot{\phi}$, ω and γ are the output signals. The mismatched uncertainties are given as $\Delta A = DFE$ with $D = \begin{bmatrix} 1 & 1 & 1 & 0 \end{bmatrix}^T$, $E = \begin{bmatrix} 0 & 1 & 1 & 1 \end{bmatrix}$ and

$$F = 0.9\sin(\phi \times \dot{\phi} \times \omega \times \gamma \times t + \dot{\phi} \times t + 12.56\phi \times t + \dot{\phi} \times \omega + \omega \times \phi + \omega \times t) \tag{31}$$

The disturbance is assumed to satisfy as $\|\xi(x,t))\| \le 1 + 1.01\|x\|$. For this work, the following parameters are selected as follows: $\alpha = 0.6$, $\beta = 1$, $\varphi = 10.0109$, $k_\xi = 0.01$, $k = 1.01$ and $k_m = 1.01$. The initial conditions for the above system are selected to be $x(0) = [1 \;\; \text{-}1 \;\; 0 \;\; \text{-}0.15]^T$.

According to the algorithm given in [9], the coordinate transformation is given as $T = \begin{bmatrix} -1 & -0.2527 & 0.0277 & 0.0004 \\ 0 & 0.1090 & 0.9939 & 0.0138 \\ 0 & 0.994 & -0.1090 & -0.0015 \\ 0 & 0 & -0.0138 & 0.999 \end{bmatrix}$. The matrices $B_2 = \begin{bmatrix} -0.0000 & -3.9330 \\ -2.5280 & 30.9690 \end{bmatrix}$, $C_2 = \begin{bmatrix} 0.1090 & 0.9941 & -0.0000 \\ 0.9940 & -0.1090 & -0.0139 \\ 0.0137 & -0.0015 & 1.0008 \end{bmatrix}$ are non-singular. Then, by solving the LMIs (10) and

(11), we have a feasible solution $P=\begin{bmatrix}0.0098 & 0.0031\\ 0.0031 & 0.1087\end{bmatrix}\times 10^{-3}$. The matrix $K_2=\Psi P\Psi^T=\begin{bmatrix}0.0027 & 0.0017\\ 0.0017 & 0.0200\end{bmatrix}$ is non-singular. According to equation (4), the output sliding surface for the system (29)-(30) is designed as

$$\sigma(y,t)=\begin{bmatrix}0.0027 & -0.0003 & 0.0017\\ 0.0017 & -0.0005 & 0.0199\end{bmatrix}y(t)-\begin{bmatrix}0.0027 & -0.0003 & 0.0017\\ 0.0017 & -0.0005 & 0.0199\end{bmatrix}y(0)\exp(-t)=0$$

(32)

From Theorem 1 and Figures 1, 2 and 3, the system (29)-(30) in sliding surface (4) is asymptotically stable.

From equation (18), the output feedback single phase variable structure controller for the system (29)-(30) is designed as

$$u(t)=-\begin{bmatrix}-1195.5 & 82.9\\ -98.7 & 8.5\end{bmatrix}[\,0.0056\exp(-t)+5.5636\|y\|+0.75\eta+0.6063]\frac{\sigma}{\|\sigma\|} \quad (33)$$

In order to eliminate chattering phenomenon, the discontinuous controller (33) is replaced by the following continuous approximation:

$$u(t)=-\begin{bmatrix}-1195.5 & 82.9\\ -98.7 & 8.5\end{bmatrix}[0.0056\exp(-t)+5.5636\|y\|+0.75+0.6063]\frac{\sigma}{\|\sigma\|+0.00001}. \quad (34)$$

By checking the sliding surface (32) and the controller (33), it is easy to know that this approach uses only output information completely in sliding surface and controller design. Therefore, conservatism is reduced and robustness is enhanced.

Remark 4: From equation (31), the system (29)-(30) is non-linear and time-varying. Thus, the approach given in [9] cannot be applied for the system (29)-(30). From Figure 1 to Figure 5, it is obvious that the system has a good performance and is effective in dealing with matched and mismatched uncertainties.

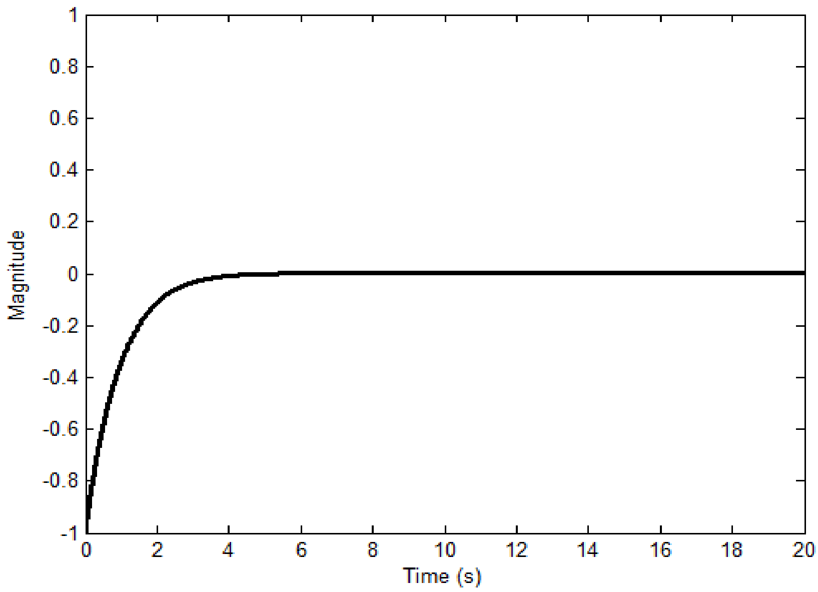

Figure 1. The Time Histories of the State $\dot{\phi}$

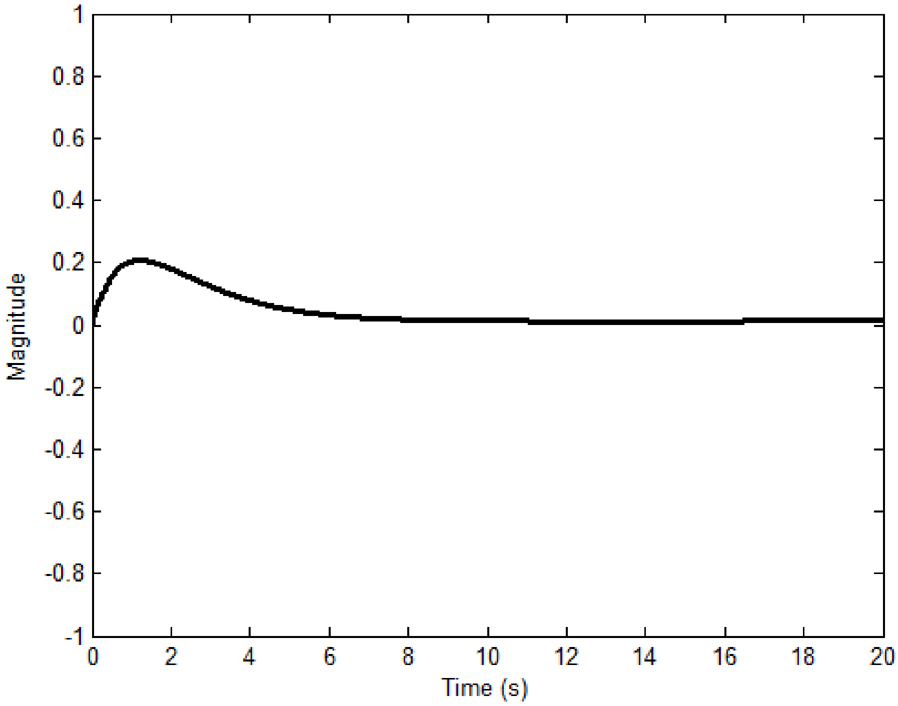

Figure 2. The Time Histories of the State ω

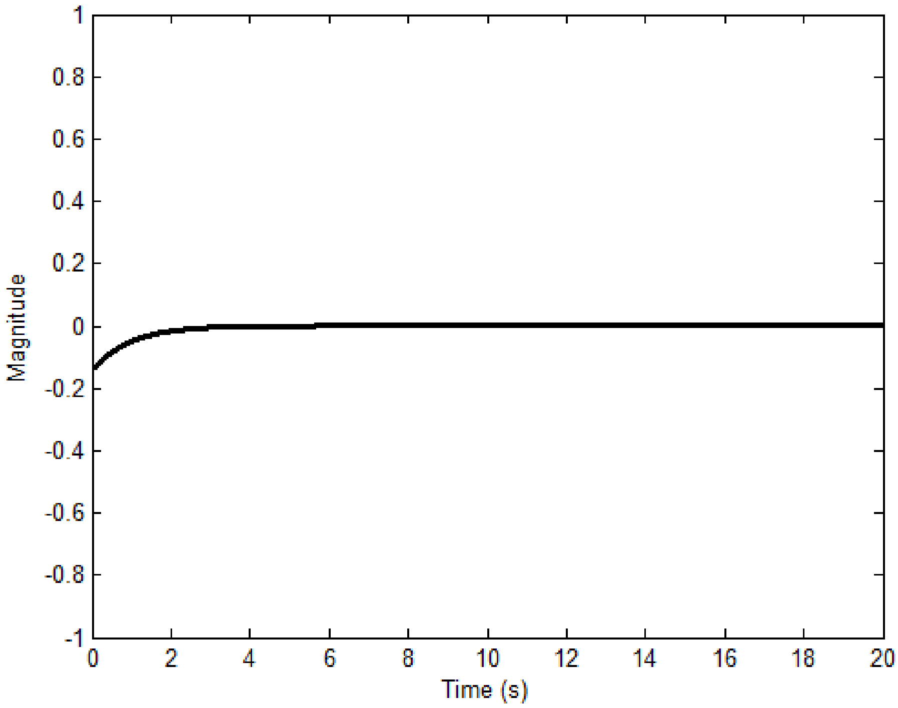

Figure 3. The Time Histories of the State γ

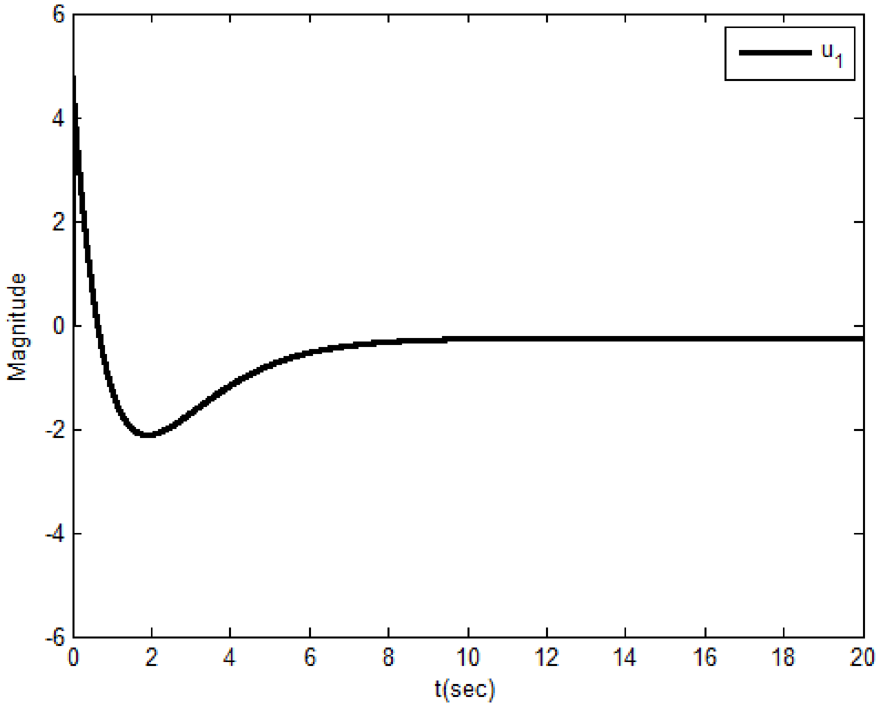

Figure 4. Control Input u_1

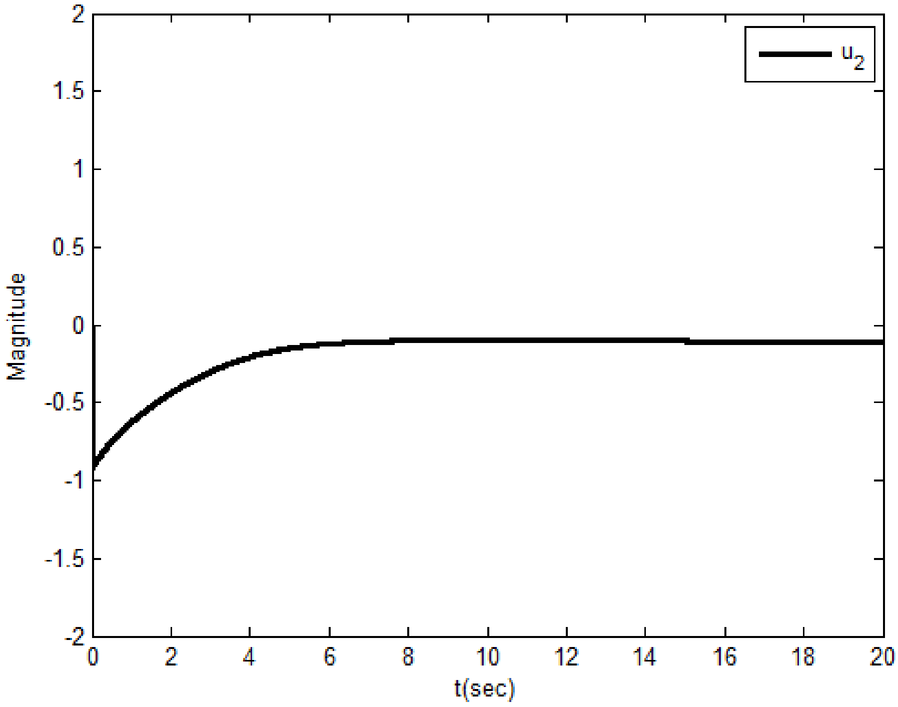

Figure 5. Control Input u_2

5. Conclusion

This paper has presented a new single phase variable structure control (SPVSC) for mismatched uncertain systems where only output information is available. By applying the SPVSC, the reaching time is eliminated and the system is more robust. Especially, the sliding function and the sliding mode controller in this study are designed based on only the output variables directly. The use of the SPVSC not only compensate the matched uncertainties right after the beginning of the process but also be enable to improve in the performance of the system without causing a large increase in the control, especially at t=0. Our approach does not need the availability of the state variables so that our method is very useful and more realistic since it can be easily implemented in many practical systems. In future work, we will extend the proposed approach for mismatched uncertain time delayed systems.

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