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Stackelberg Game Led by Manufacturers in Fuzzy Supply Chain

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Abstract

This paper considers a two-echelon supply chain including one manufacturer and one retailer will be considered. In the supply chain, the retailer plays the dominant role. In consideration of various factors of uncertainty in the real economy, the market demand function, the manufacturer's manufacturing cost, and retailer's operating costs are considered as fuzzy variables. Stackelberg model is adopted to solve the game problem between retailers and manufacturers. The expected value model and the chance constrained model are introduced to solve the optimal decision problem. The optimal wholesale price and marginal profit per unit of each model that are at equilibrium are provided to obtain the maximum profit of retailers and manufacturers. Finally, a numerical example illustrates the effectiveness of the supply chain game model.

Keywords*: Supply chain models, Stackelberg game, Fuzzy supply chain, Optimal strategy*

1. Introduction

The improvement of productivity, the variety and quality of goods has been greatly improved with the development of global science and technology. Consumers expect retailers to provide more kinds of goods and adequate supply. Such a change in market demand results in gradual increase in large-scale retailers, the retailer's power is growing, and their position in the industrial chain is improved. At present, the supply chain model led by retailers has attracted more attention than ever and relevant scholars have made a lot of research.

Choi, S (1996) made an in-depth analysis of the price competition under retailer duopoly, concluding that the equilibrium price under Stackelberg game is higher than the equilibrium price under Nash game, and finding that the product differentiation benefits of the manufacturer but not the retailer and shop differentiation benefits the retailer but not the manufacturer [1]. Pan, Kewen, Lai, K *et al*. (2009) analyzed the pricing and ordering strategy of the two-stage supply chain led by the retailer in the case of demand uncertainty, proving the existence and uniqueness of the optimal strategy[2]. Pan, Kewen, Lai, K *et al*. (2010) considered a supply chain which contains two manufacturers, one retailer and a supply chain which contains a manufacturer, two retailers, analyzed the different output under the wholesale price contract and revenue sharing contract, in order to determine the advantage of revenue sharing contract [3].

Yao H, Yan W L. (2009) summed up the study on the supply chain led by the retailer, summarized and concluded the research on the formation, market efficiency, information sharing and coordination of all aspects of the supply chain led by the retailer [4]. Song J, Li B Y. (2006) established the Stackelberg game model led by the retailer for the supply chain with a manufacturer and a retailer, and presented the equilibrium solution when members in the supply chain grasp the different market information, showing that asymmetric information leads to low efficiency of supply chain [5]. Zhang G L, Liu Z X. (2006) also considered such a supply chain. They analyzed the supply chain profit

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distribution equilibrium led by the manufacturer and led by the retailer respectively, finding that the subordinate enterprises receive only retained profits, while the leading enterprises always receive all the remaining profits [6]. Ren F X, Zhang R J. (2009) chose a two-level supply chain led by the retailer that consists of two retailers and one manufacturer and retail market for analysis, and made a comparative analysis of the equilibrium result of Cournot competition and Stark M Berg (Stackelberg) competition between retailers, finding that the differences between retailers are beneficial to its own and manufacturers [7].

In the Stackelberg model, the data and parameters used must be determined value [8]. In the real decision-making process, fluctuations in market demand and manufacturer cost often leads to uncertain information; or people's perceive uncertainty on the uncertainty factors greatly limits the application range of determined value model. Therefore, in order to enhance the ability of classical game theory model to explain the realistic problems, it is necessary to extend the deterministic model to the non deterministic cases.

Proposition of fuzzy sets and fuzzy logic concept [9], provides an effective measure method to understand the things whose boundaries are not clear enough. For example, Yao J S, Wu K. (1999), by converting fuzzy number into a certain value and under the condition that the demand and supply are fuzzy linear function, analyzed the consumer surplus and producer surplus at market equilibrium [10]; Dang J F. Hong L H. (2010) studied Cournot model under fuzzy environment, obtained optimal production of Cournot manufacturers under fuzzy environment by using triangular fuzzy number, and analyzed the impact of fuzzy parameter of inverse demand function and the cost function on manufacturers' profits [11]; Sang S J.(2014) considered supply chain models with two competitive manufacturers acting as the leaders and a retailer acting as follower under a fuzzy decision environment, and the two manufacturers were assumed to pursue Cournot competitive behavior and the optimum policy of the expected value and chance-constrained programming models were derived, and gave a conclusion that the confidence level of the profits for supply chain members affects the final optimal solutions[12].

Based on the above literature research, this paper puts forward the equilibrium solving method and the corresponding equilibrium result of Stackelberg game under fuzzy demand and fuzzy cost. Arcelus and Srinivasan (1987), Ertek and Grffin (2002), Lau and Lau (2005) had used the fuzzy method in their articles to study supply chain game problems. The fuzzy object is mainly the consumer demand function and the manufacturer cost. In order to find manufacturers and retailers' optimal price strategy and realize maximized profit, this paper makes use of the supply chain expected value model and the chance constrained mechanism model [13] for solution. Based on the above literature research, this paper not only considers the manufacturer's manufacturing cost, also considers the retailer's operating cost, further expands the scope of fuzzy variables, thus the research is more close to the economic reality.

2. Preliminaries

Fuzzy set theory develops very quickly since its inception. The corresponding fuzzy techniques penetrate into almost all areas of economic research. Fuzzy theory uses $Pos\{A\}$ to describe the probability of event occurrence of A. In order to guarantee the rationality of $Pos\{A\}$ in the practice, it needs to meet some mathematical properties (Zhou [13]).

Provided that Θ is nonempty set, $P(\Theta)$ is the power set of Θ, then:

Axiom 1. $P\{\Theta\}=1$;

Axiom 2. $P\{\Phi\}=1$;

Axiom 3. For any set $\{A_i\}$ in $P(\Theta)$, $Pos\{U_i A_i\}=\sup_i Pos\{A_i\}$;

If the above 3 axioms are met, it can be referred to as possibility measure and the triple (Θ，$P(\Theta), Pos$) is a possibility space.

In the followed analysis process in this paper, it will take the following definitions and properties as the premise and foundation of the research:

Definition 1. (Nahmias[14]) Provided that fuzzy variable ξ is a function from a possibility space(Θ，$P(\Theta), Pos$)to a real line R , then ξ can be said to be a fuzzy variable defined on a possibility space(Θ，$P(\Theta), Pos$).

Definition 2. (Liu[15]) Fuzzy variable ξ is non-negative (or positive) variable, if and only if $Pos\{\xi < 0\} = 0$ (or $Pos\{\xi \le 0\} = 0$).

Proposition 1. (Liu[15]) Provided that ξ_i is a mutually independent fuzzy variable, function $f_i : R \to R, i = 1,2,...m$, then $f_1(\xi_1)$，$f_2(\xi_2)$，…，$f_m(\xi_m)$ are also mutually independent fuzzy variables.

Definition 3. (Liu[15]) Provided that ξ is a fuzzy variable defined in the possibility space (Θ，$P(\Theta), Pos$) and $\alpha \in (0,1]$, then in the following formula,

$\xi_\alpha^L = \inf\{r \mid Pos\{\xi \le r\} \ge \alpha\}$ and $\xi_\alpha^U = \sup\{r \mid Pos\{\xi \ge r\} \ge \alpha\}$

are respectively referred to as α pessimistic value and α optimistic value of fuzzy variable ξ.

Here, r is the value that fuzzy variable ξ achieves with possibility α .The α-pessimistic value ξ_α^L is the infimum value that ξ achieves with possibility α, and α-optimistic value ξ_α^U is the supremum value that ξ achieves with possibility α.

Example 1. The triangular fuzzy variable $\xi = (a, b, c)$ has its α-pessimistic value and α-optimistic value

$\xi_\alpha^L = a + (b - a)\alpha$ and $\xi_\alpha^U = c - (c - b)\alpha$

Proposition 2. (Liu and Liu[16], Zhou *et al.*[17]) There are two mutually independent fuzzy variables, which are respectively expressed as ξ and η, then we have the following conclusions:

(1)For any $\alpha \in (0,1]$，$(\xi + \eta)_\alpha^L = \xi_\alpha^L + \eta_\alpha^L$;

(2)For any $\alpha \in (0,1]$，$(\xi + \eta)_\alpha^U = \xi_\alpha^U + \eta_\alpha^U$.

(3) For any $\alpha \in (0,1]$，$(\xi \cdot \eta)_\alpha^L = \xi_\alpha^L \cdot \eta_\alpha^L$,

(4) For any $\alpha \in (0,1]$，$(\xi \cdot \eta)_\alpha^U = \xi_\alpha^U \cdot \eta_\alpha^U$.

Definition 4. (Liu and Liu[16]). Let ξ be a fuzzy variable.And r_0 is a real number defined from $-\infty$ to ∞.The expected value of ξ is defined by

$$E[\xi] = \int_0^{+\infty} C_r\{\xi \ge r_0\} dr_0 - \int_{-\infty}^0 C_r\{\xi \le r_0\} dr_0$$

Provided that at least one of the two integrals is finite. Especially, if ξ is a nonnegative fuzzy variable, then

$$E[\xi] = \int_0^{+\infty} C_r\{\xi \ge r_0\} dr_0$$

Example 2. The triangular fuzzy variable $\xi = (a, b, c)$ has an expected value

$$E[\xi] = \frac{a + 2b + c}{4}$$

Proposition 3. (Liu 和 Liu[16]) Provided that ξ is fuzzy variable with limited expectations, then

$$E[\xi] = \frac{1}{2}\int_0^1 (\xi_\alpha^L + \xi_\alpha^U) d\alpha$$

Proposition 4. (Liu 和 Liu[16]) Provided that ξ and η and are mutually independent fuzzy variables with limited expectations, then for any number a and b, there is following formula

$$E[a\xi + \mathrm{b}\eta] = aE[\xi] + bE[\eta]$$

3. Model Descriptions

This paper takes a two-stage supply chain as the research object, namely the supply chain consists of a manufacturer and a retailer. The manufacturer wholesales goods to the retailer, and then the retailer sells the ordered goods to the customer. In order to maximize their profits, manufacturers and retailers formulate the optimal wholesale price and the marginal profit per unit. When the retailer is dominant in the supply chain, it becomes the core enterprise in supply chain. In Stackelberg model, the dominant player will be the first to make decisions, and then the follower makes his own decisions according to the dominant player's decisions. Therefore, in the two-stage supply chain Stackelberg game led by retailers, the decision order is that the retailer first decides the marginal profit per unit of products, then after observing the retailer's unit profit, the manufacturer decides the wholesale price hereby; retailers and manufacturers maximize their profits respectively. In this model, the retailer operating costs will be considered to make the model closer to the reality. In order to construct a two-stage supply chain model under fuzzy environment, the following basic symbols will be used.

Notation

w The wholesale price per unit product;

c_m The manufacturing cost per unit product;

c_r The retailer's operating costs per unit product;

v_m The manufacturer's single product profit $v_m = w - c_m$

m The retailer's product purchase and sale price differential, referred to as marginal profit per unit;

Π_{M} The profits of the manufacturer, and the function of w and m;

Π_R The profits of the retailer, and the function of w and m;

Provided that the customer demand function is a linear decreasing function on the wholesale prices and the marginal profit per unit that is denoted as $D = a - b(w+m)$. In it, a and b are two mutually independent non-negative fuzzy variables. Parameter a represents the maximum market capacity and parameter b represents the demand to price change rate; the customer demand D is also a fuzzy variable. As the demand in the realistic economic life is positive, $Pos\{\mathrm{a} - b(w+m) \le 0\} = 0$.

The profit function of the manufacturer and the retailer can be respectively expressed as:

$$\Pi_{\mathrm{R}}(w,m) = (m - c_r)D = (m - c_r)(a - b(m+w)), \quad (1)$$

$$\Pi_{\mathrm{M}}(w,m) = (w - c_m)D = (w - c_m)(a - b(m+w)) \quad (2)$$

4. Fuzzy Two-Echelon Supply Chain Models

Here we analyzed the situation where a manufacturer is the dominant role. When the manufacturer is the dominant role, it becomes the key enterprise in the supply chain, and therefore the retailer becomes the follower. Suppose that the information between manufacturer and retailer is symmetric, according to the Stackelberg game model, the manufacturer will first make decisions, and since in this paper the production cost of manufacturer is considered, the decision variable of manufacturer is the profit of unit product; Whereafter, the retailer will formulate the product price according to the observed profit of unit product; In addition, both can realize their biggest profit. Based on the previous basic assumptions, we can build the expected value model of supply chain when manufacturers are the dominant role.

$$\begin{cases} \max\limits_{w} E[\Pi_m(w,m)] = \max\limits_{w} E\{(w-c_m)(a-b(w+m))\} \\ s.t. \\ \quad w-c_m > 0 \\ \quad m^* \text{ is the optimal solution of model in the lower level} \\ \quad \begin{cases} \max\limits_{m} E[(m-c_r)(a-b(w+m))] \\ s.t. \\ \quad Pos\{a-b(w+m) \le 0\} = 0 \\ \quad Pos\{m-c_r \le 0\} = \end{cases} \end{cases} \tag{3}$$

Suppose the $E[\Pi_r(m)]$ is the expected profit of retailer. Regarding the up-mentioned two-echelon planning model, there will be following tenable conclusion:

Theorem 1. Suppose the unit product profit w is constant, if $Pos\{a-b\frac{E[a]+E[bc_r]-E[b]w}{2E[b]} \le 0\} = 0$ and $Pos\{c_r \ge \frac{E[a]+E[bc_r]\text{-}E[b]w}{2E[b]}\} = 0$, then

The best reaction function of retailer upon the unit profit margins is

$$m^* = \frac{E[a]+E[bc_r]-E[b]w}{2E[b]} \tag{4}$$

Proposition 5. The best reaction function of retailer m^* decreases strictly with w.

Proof.

$$\begin{aligned} & E[\Pi_r(m)] \\ & = \frac{1}{2}\int_0^1 \{[(m-c_r)(a-b(w+m))]_\alpha^L + [(m-c_r)(a-b(w+m))]_\alpha^U\} d\alpha \\ & = \frac{1}{2}\int_0^1 \{[(m-c_r)_\alpha^L (a-b(w+m))_\alpha^L] + [(m-c_r)_\alpha^U (a-b(w+m))_\alpha^U]\} d\alpha \\ & = \frac{1}{2}\int_0^1 [(m-c_{r\alpha}^U)(a_\alpha^L - b_\alpha^U(w+m)) + (m-c_{r\alpha}^L)(a_\alpha^U - b_\alpha^L(w+m))] d\alpha \\ & = \text{-E[b]m}^2 + (E[a]+E[bc_r]-wE[b])m + wE[bc_r] - \frac{1}{2}\int_0^1 (a_\alpha^L c_{r\alpha}^U + a_\alpha^U c_{r\alpha}^L) d\alpha \end{aligned} \tag{5}$$

Regarding above equations, the first and second order derivatives of m are

$$\frac{dE[\Pi_r(m)]}{dm} = (E[a]+E[bc_r]-wE[b]) - 2E[b]m,$$

$$\frac{d^2E[\Pi_r(m)]}{dm^2} = -2E[b] < 0$$

Therefore $E[\Pi_r(m)]$ is concave function, which can obtained the max value in equation

$$m^*(w) = -\frac{1}{2}w + \frac{E[a]+E[bc_r]}{2E[b]} \tag{6}$$

Apparently, m^* is a strict decreasing function related with w

Suppose $E[\Pi_m(w, m^*(w))]$ is the expected profit of manufacturer, regarding the up-mentioned two-echelon planning model, there will be following tenable conclusion:

Theorem 2.

If $Pos\{c_r \ge \frac{E[a]-E(bc_m)+3E(bc_r)}{4E[b]}\} = 0$ and $Pos\{a-b(\frac{E[a]+E[bc_r]+E[bc_m]}{4E[b]} \le 0\} = 0$, then

The optimal wholesale price and the optimal unit marginal profit are respectively:

$$w^* = \frac{E[a]+E[bc_m]-E[bc_r]}{2E[b]},\quad m^* = \frac{E[a]-E[bc_m]+3E[bc_r]}{4E[b]}$$

Proposition 6. In $(w^*, m^*(w^*))$, the retailer and manufacturer achieve their max expected profit, respectively:

$$E[\Pi_r(w^*, m^*(w^*))] = \frac{(E[a]-E[bc_m]+3E[bc_r])^2}{16E[b]} + \frac{E[bc_r](E[a]+E[bc_m]-E[bc_r])}{2E[b]} - \frac{1}{2}\int_0^1 (a_\alpha^L c_{r\alpha}^U + a_\alpha^U c_{r\alpha}^L) d\alpha$$

and

$$E[\Pi_m(w^*, m^*(w^*))] = \frac{(E[a]+E[bc_m]-E[bc_r])^2}{8E[b]} + \frac{E[a]+E[bc_r]}{2E[b]} E[bc_m] - \frac{1}{2}\int_0^1 (a_\alpha^L c_{m\alpha}^U + a_\alpha^U c_{m\alpha}^L) d\alpha$$

Proof. The process is the same as Proposition 5. Substitute m^* into the above equations, we get

$$\begin{aligned} &E[\Pi_m(w, m^*(w))] \\ &= -E[b]w^2 + (E[a]+E[bc_m]-mE[b])w + mE[bc_m] - \frac{1}{2}\int_0^1 (a_\alpha^L c_{m\alpha}^U + a_\alpha^U c_{m\alpha}^L) d\alpha \\ &= -\frac{E[b]}{2}w^2 + \frac{E[a]+E[bc_m]-E[bc_r]}{2}w + \frac{E[a]+E[bc_r]}{2E[b]}E[bc_m] - \frac{1}{2}\int_0^1 (a_\alpha^L c_{m\alpha}^U + a_\alpha^U c_{m\alpha}^L) d\alpha \end{aligned} \quad (7)$$

Respectively calculate the first and second order derivatives of above equations regarding w

$$\frac{dE[\Pi_m(w, m^*(w))]}{dw} = -E[b]w + \frac{E[a]+E[bc_m]-E[bc_r]}{2},$$

$$\frac{dE[\Pi_m(w, m^*(w))]}{dw} = -E[b] < 0$$

Therefore $E[\Pi_m(w, m^*(w))$ is a concave function，which realized its max value in

$$w^* = \frac{E[a]+E[bc_m]-E[bc_r]}{2E[b]} \quad (8)$$

The max profit of manufacturers is

$$E[\Pi_m(w^*, m^*(w^*))] = \frac{(E[a]+E[bc_m]-E[bc_r])^2}{8E[b]} + \frac{E[a]+E[bc_r]}{2E[b]}E[bc_m] - \frac{1}{2}\int_0^1 (a_\alpha^L c_{m\alpha}^U + a_\alpha^U c_{m\alpha}^L) d\alpha$$

The max profit of retailer is

$$E[\Pi_r(w^*, m^*(w^*))] = \frac{(E[a]-E[bc_m]+3E[bc_r])^2}{16E[b]} + \frac{E[bc_r](E[a]+E[bc_m]-E[bc_r])}{2E[b]} - \frac{1}{2}\int_0^1 (a_\alpha^L c_{r\alpha}^U + a_\alpha^U c_{r\alpha}^L) d\alpha$$

Strategy $(w^*, m^*(w^*))$ is the Stackelberg-Nash equilibrium solution for expected value model of supply chain.

In addition，it can also build the $\max i \max$ chance constrained model and $\min i \max$ chance constrained model.

Firstly build the $\max i \max$ chance constrained model as follow：

$$
\begin{cases}
\max\limits_{w} \Pi_m \\
s.t. \\
\quad Pos\{(w-c_m)(a-b(w+m^*(w))) \geq \Pi_m\} \geq \alpha \\
\quad w-c_m > 0 \\
\quad m^* \text{ is the optimal solution for lower level plan} \\
\quad \begin{cases}
\max\limits_{r} \Pi_r \\
s.t. \\
\quad Pos\{(m-c_r)(a-b(w+m)) \geq \Pi_r\} \geq \alpha \\
\quad Pos\{a-b(w+m) \leq 0\} = 0 \\
\quad Pos\{m-c_r \leq 0\} =
\end{cases}
\end{cases} \tag{9}
$$

Wherein, α is the predefined confidence level for the retailer and the manufacturer, for all provided available (w,m) strategy, $\max\limits_{r} \Pi_{\mathrm{m}}$ and $\max\limits_{m} \Pi_{\mathrm{r}}$ are the α optimistic value of profit for respectively the retailer and manufacturer, therefore the model (9) is equivalent to the model below:

$$
\begin{cases}
\max\limits_{m}((w-c_m)(a-b(w+m^*(w))))_{\alpha}^{\mathrm{U}} \\
s.t. \\
\quad w-c_m > 0 \\
\quad m^* \text{ is the optimal solution for the lower level plan} \\
\quad \begin{cases}
\max\limits_{m}((m-c_r)(a-b(w+m)))_{\alpha}^{\mathrm{U}} \\
s.t. \\
\quad Pos\{a-b(w+m) \leq 0\} = 0 \\
\quad Pos\{m-c_r \leq 0\} =
\end{cases}
\end{cases} \tag{10}
$$

Wherein, $(\Pi_{\mathrm{m}}(w,m^*(w)))_{\alpha}^{\mathrm{U}}$, $(\Pi_{\mathrm{r}}(m))_{\alpha}^{\mathrm{U}}$ are the α optimistic value of profit for respectively the manufacturer and retailer.

Proposition 7.

If $Pos\{c_r \geq \frac{a_{\alpha}^{\mathrm{U}} - b_{\alpha}^{L} c_{m\alpha}^{\ L} + b_{\alpha}^{L} c_{r\alpha}^{\ L}}{4b_{\alpha}^{\mathrm{L}}}\} = 0$, $Pos\{a - b\frac{3a_{\alpha}^{\mathrm{U}} + c_{m\alpha}^{\ L} b_{\alpha}^{L} + 3b_{\alpha}^{\mathrm{L}} c_{R\alpha}^{\ \mathrm{L}}}{4b_{\alpha}^{\mathrm{L}}} \leq 0\} = 0$, the model (10) has the one and only α optimistic value, Stackelberg-Nash equilibrium solution $(\frac{a_{\alpha}^{\mathrm{U}} + b_{\alpha}^{L} c_{m\alpha}^{\ L} + b_{\alpha}^{L} c_{r\alpha}^{\ L}}{2b_{\alpha}^{\mathrm{L}}}, \frac{a_{\alpha}^{\mathrm{U}} - b_{\alpha}^{L} c_{m\alpha}^{\ L} + b_{\alpha}^{L} c_{r\alpha}^{\ L}}{4b_{\alpha}^{\mathrm{L}}})$.

Proof. The optimistic value of retailer's profit

$$
\begin{aligned}
&\max_{m}(\Pi_{\mathrm{r}}(m))_{\alpha}^{\mathrm{U}} = ((m-c_r)(a-b(m+w)))_{\alpha}^{\mathrm{U}} \\
&= (m - c_{r\alpha}^{\ \mathrm{L}})(a_{\alpha}^{\mathrm{U}} - b_{\alpha}^{\mathrm{L}}(m+w)) \\
&= -b_{\alpha}^{\mathrm{L}} m^2 + (a_{\alpha}^{\mathrm{U}} + b_{\alpha}^{\mathrm{L}} c_{r\alpha}^{\ \mathrm{L}} - b_{\alpha}^{\mathrm{L}} w) m + b_{\alpha}^{\mathrm{L}} c_{r\alpha}^{\ \mathrm{L}} w - a_{\alpha}^{\mathrm{U}} c_{r\alpha}^{\ \mathrm{L}}
\end{aligned} \tag{11}
$$

Calculate the first and second order derivatives of above equations regarding m

$$\frac{d \max\limits_{m}(\Pi_{\mathrm{r}}(m))_{\alpha}^{\mathrm{U}}}{dm} = -2b_{\alpha}^{\mathrm{L}} m + a_{\alpha}^{\mathrm{U}} + b_{\alpha}^{\mathrm{L}} c_{r\alpha}^{\ \mathrm{L}} - b_{\alpha}^{\mathrm{L}} w,$$

$$\frac{d^2 \max\limits_{m}(\Pi_{\mathrm{r}}(m))_{\alpha}^{\mathrm{U}}}{dm^2} = -2b_{\alpha}^{\mathrm{L}} < 0$$

So, $\max\limits_{m}(\Pi_{\mathrm{r}}(m))_{\alpha}^{\mathrm{U}}$ is a concave function, and realize it max value in

$$m^*(w) = \frac{a_{\alpha}^{\mathrm{U}} + b_{\alpha}^{\ \mathrm{L}} c_{r\alpha}^{\ \mathrm{L}} - b_{\alpha}^{\ \mathrm{L}} w}{2b_{\alpha}^{\mathrm{L}}} \tag{12}$$

Apparently, m^* is a strict decreasing function regarding w

The optimistic value of retailer's profit, put m^* into equation (12), it can obtain

$$\begin{aligned}&\max_w(\Pi_m(w,m^*(w)))^{\mathrm{U}}_\alpha=((w-c_m)(a-b(m+w)))^{\mathrm{U}}_\alpha\\&=(m-c^{\mathrm{L}}_{m\alpha})(a^{\mathrm{U}}_\alpha-b^{\mathrm{L}}_\alpha(m+w))\\&=-\frac{b^{\mathrm{L}}_\alpha}{2}w^2+\frac{a^{\mathrm{U}}_\alpha+b^{\mathrm{L}}_\alpha c^{\mathrm{L}}_{r\alpha}+b^{\mathrm{L}}_\alpha c^{\mathrm{L}}_{m\alpha}}{2}w+\frac{c^{\mathrm{L}}_{r\alpha}b^{\mathrm{L}}_\alpha-a^{\mathrm{U}}_\alpha c^{\mathrm{L}}_{m\alpha}}{2}\end{aligned}\tag{13}$$

Calculate the first and second order derivatives of above equations regarding w

$$\frac{d\max_w(\Pi_{\mathrm{m}}(w,m^*(w)))^{\mathrm{U}}_\alpha}{dw}=-b^{\mathrm{L}}_\alpha w+\frac{a^{\mathrm{U}}_\alpha+b^{\mathrm{L}}_\alpha c^{\mathrm{L}}_{m\alpha}+b^{\mathrm{L}}_\alpha c^{\mathrm{L}}_{r\alpha}}{2},$$

$$\frac{d^2\max_w(\Pi_{\mathrm{m}}(w))^{\mathrm{U}}_\alpha}{dw^2}=-b^{\mathrm{L}}_\alpha<0$$

So, $\max_w(\Pi_{\mathrm{m}}(w,m^*(w)))^{\mathrm{U}}_\alpha$ is a concave function, and realize it max value in

$$w^*=\frac{a^{\mathrm{U}}_\alpha+b^{\mathrm{L}}_\alpha c^{\mathrm{L}}_{m\alpha}+b^{\mathrm{L}}_\alpha c^{\mathrm{L}}_{r\alpha}}{2b^{\mathrm{L}}_\alpha}\tag{14}$$

Apparently, m^* is a strict decreasing function regarding w. Put w^* into m^*, it can obtain

$$m^*=\frac{a^{U}_\alpha+b^{L}_\alpha c^{L}_{r\alpha}-b^{L}_\alpha c^{L}_{m\alpha}}{4b^{L}_\alpha}\tag{15}$$

Therefore, (w^*,m^*) is the only equilibrium solution of α optimistic value for manufacturer and retailer.

On the other hand, we can build the minimax chance-constrained programming model for the two-echelon supply chain.

$$\begin{cases}\max\limits_w\min\limits_{\Pi_{\mathrm{m}}}\Pi_{\mathrm{m}}\\ s.t.\\ \quad Pos\{(w-c_m)(a-b(m^*+w))\geq\Pi_{\mathrm{m}}\}\geq\alpha\\ \quad w-c_m>0\\ \quad m^*\text{ is the optimal solution of lower level plan}\\ \quad\begin{cases}\max\limits_m\min\limits_{\Pi_{\mathrm{M}}}\Pi_{\mathrm{M}}\\ s.t.\\ \quad Pos\{(m-c_r)(a-b(w+m))\geq\Pi_{\mathrm{r}}\}\geq\alpha\\ \quad Pos\{a-b(w+m)\leq 0\}=0\\ \quad Pos\{m-c_r\leq 0\}=0\end{cases}\end{cases}\tag{16}$$

Wherein, α is the predefined confidence level for the manufacturer and retailer, for all provided available (w,m) strategy, $\min\limits_w\Pi_{\mathrm{M}}$ and $\min\limits_m\Pi_{\mathrm{R}}$ are the profit α pessimistic values for respectively the manufacturer and retailer, therefore the model (16) is equivalent to following model:

$$\begin{cases}\max\limits_w((w-c_m)(a-b(m^*(w)+w)))^{\mathrm{L}}_\alpha\\ s.t.\\ \quad w-c_m>0\\ \quad m^*\text{ is the optimal solution for lower level plan}\\ \quad\begin{cases}\max\limits_m((m-c_r)(a-b(w+m)))^{\mathrm{L}}_\alpha\\ s.t.\\ \quad Pos\{a-b(w+m)\leq 0\}=0\\ \quad Pos\{m-c_r\leq 0\}=0\end{cases}\end{cases}\tag{17}$$

Wherein, $(\Pi_{\mathrm{m}}(w,m^{*}(w)))_{\alpha}^{\mathrm{L}}$, $(\Pi_{\mathrm{r}}(m))_{\alpha}^{\mathrm{L}}$ are the profit α pessimistic values for respectively the manufacturer and retailer.

Regarding model (16) and (17), there are tenable conclusion as below:

Proposition 8.

If $Pos\{c_r \geq \frac{a_{\alpha}^{\mathrm{L}} - b_{\alpha}^{U} c_{m\alpha}^{\ U} + b_{\alpha}^{U} c_{r\alpha}^{\ U}}{4b_{\alpha}^{U}}\} = 0$, and $Pos\{a - b\frac{3a_{\alpha}^{L} + c_{m\alpha}^{\ U} b_{\alpha}^{U} + 3b_{\alpha}^{U} c_{R\alpha}^{\ U}}{4b_{\alpha}^{U}} \leq 0\} = 0$,

the model (18) has the one and only α optimistic value, Stackelberg-Nash equilibrium solution $(\frac{a_{\alpha}^{\mathrm{L}} + b_{\alpha}^{U} c_{m\alpha}^{\ U} + b_{\alpha}^{U} c_{r\alpha}^{\ U}}{2b_{\alpha}^{\mathrm{U}}}, \frac{a_{\alpha}^{\mathrm{L}} - b_{\alpha}^{U} c_{m\alpha}^{\ U} + b_{\alpha}^{U} c_{r\alpha}^{\ U}}{4b_{\alpha}^{\mathrm{U}}})$.

Proof. Similar to the proof of Proposition 7.

According to the analysis above, a conclusion for game equilibrium of two-echelon supply chain is shown in the table below:

Table 1. Summary of Fuzzy Two-Echelon Supply Chain Model Dominated by the Manufacturer

Ranking criterion	Optimal wholesale price w^*	Optimal unit product profit m^*
Expectation criterion	$\frac{E[a]+E[bc_m]-E[bc_r]}{2E[b]}$	$\frac{E[a]-E[bc_m]+3E[bc_r]}{4E[b]}$
α optimistic value criterion	$\frac{a_{\alpha}^{\mathrm{U}} + b_{\alpha}^{L} c_{m\alpha}^{\ L} + b_{\alpha}^{L} c_{r\alpha}^{\ L}}{2b_{\alpha}^{\mathrm{L}}}$	$\frac{a_{\alpha}^{\mathrm{U}} - b_{\alpha}^{L} c_{m\alpha}^{\ L} + b_{\alpha}^{L} c_{r\alpha}^{\ L}}{4b_{\alpha}^{\mathrm{L}}}$
α pessimistic value criterion	$\frac{a_{\alpha}^{\mathrm{L}} + b_{\alpha}^{U} c_{m\alpha}^{\ U} + b_{\alpha}^{U} c_{r\alpha}^{\ U}}{2b_{\alpha}^{\mathrm{U}}}$	$\frac{a_{\alpha}^{\mathrm{L}} - b_{\alpha}^{U} c_{m\alpha}^{\ U} + b_{\alpha}^{U} c_{r\alpha}^{\ U}}{4b_{\alpha}^{\mathrm{U}}}$
	Max profit of manufacturer	Max profit of retailer
Expectation criterion	$\frac{(E[a]+E[bc_m]-E[bc_r])^2}{8E[b]} + \frac{E[a]+E[bc_r]}{2E[b]}E[bc_m] - \frac{1}{2}\int_0^1 (a_{\alpha}^{\mathrm{L}} c_{m\alpha}^{\ U} + a_{\alpha}^{\mathrm{U}} c_{m\alpha}^{\ L}) d\alpha$	$\frac{(E[a]-E[bc_m]+3E[bc_r])^2}{16E[b]} + \frac{E[bc_r](E[a]+E[bc_m]-E[bc_r])}{2E[b]} - \frac{1}{2}\int_0^1 (a_{\alpha}^{\mathrm{L}} c_{r\alpha}^{\ U} + a_{\alpha}^{\mathrm{U}} c_{r\alpha}^{\ L}) d\alpha$
α optimistic value	$\frac{(a_{\alpha}^{U} + b_{\alpha}^{L} c_{R\alpha}^{\ L} + b_{\alpha}^{L} c_{m\alpha}^{\ L})^2}{8b_{\alpha}^{L}} + \frac{b_{\alpha}^{L} c_{R\alpha}^{\ L} - b_{\alpha}^{L} c_{m\alpha}^{\ L}}{2}$	$\frac{(a_{\alpha}^{U} + b_{\alpha}^{L} c_{r\alpha}^{\ L} - b_{\alpha}^{L} c_{m\alpha}^{\ L})^2}{16b_{\alpha}^{L}} + \frac{b_{\alpha}^{L} c_{m\alpha}^{\ L} + b_{\alpha}^{L} c_{r\alpha}^{\ L} - a_{\alpha}^{U}}{2} c_{r\alpha}^{\ L}$
α pessimistic value	$\frac{(a_{\alpha}^{L} + b_{\alpha}^{U} c_{R\alpha}^{\ U} + b_{\alpha}^{U} c_{m\alpha}^{\ U})^2}{8b_{\alpha}^{U}} + \frac{b_{\alpha}^{U} c_{R\alpha}^{\ U} - b_{\alpha}^{U} c_{m\alpha}^{\ U}}{2}$	$\frac{(a_{\alpha}^{L} + b_{\alpha}^{U} c_{r\alpha}^{\ U} - b_{\alpha}^{U} c_{m\alpha}^{\ U})^2}{16b_{\alpha}^{U}} + \frac{b_{\alpha}^{U} c_{m\alpha}^{\ U} + b_{\alpha}^{U} c_{r\alpha}^{\ U} - a_{\alpha}^{L}}{2} c_{r\alpha}^{\ U}$

5. Numerical Experiment

The above content solve the pricing strategies in the two-echelon supply chain when the manufacturer is the dominant role, and then a numerical example will be given to illustrate the effectiveness of this game model.

For example, the manufacturing costs, operational costs, market capacity and demand change rate are normally evaluated by management decision makers and experts. During evaluation, it will often emerge oral words such as "low costs", "big market capacity", "sensitive demand changing rate" as an description for approximate evaluation values. The estimators depend on experience to determine the relation between fuzzy language variable and triangle fuzzy value, which is shown in Table 2.

Table 2.Relation between Linguistic Expression and Triangular Fuzzy Variable

	Language variable	Triangle fuzzy value
Manufacturing costs	low（approx.5）	（4,5,6）
	medium（approx.7）	（6,7,8）
	high（approx.9）	（8,9,10）
Operational costs	low（approx.3）	（2,3,4）
	medium（approx.5）	（4,5,6）
	high（approx.7）	（6,7,8）
Market capacity	Very big（approx.9000）	（8900,9000,9100）
	Rather small（approx.3000）	（2900,3000,3100）
Demand changing rate	Very sensitive（approx.500）	（450,500,550）
	Sensitive（approx.300）	（280,300,320）

Suppose the current condition is: the evaluated product market capacity is very big (Approx.9000), the demand changing rate is very sensitive (Approx.500), the manufacturing costs are medium (Approx.7), the operational cost of retailer is rather low (Approx.3), according to the expected value model and fuzzy variable equation, it can obtain the conclusions in Table.3 and Table.4

Table 3. The Pptimal Strategy of Stackelberg Game of Supply Chain When Manufacturer is the Dominant Role

Ranking criterion	Optimal wholesale price w^*	Optimal unit product profit m^*
Expected value criterion	33	9.2
	Max profit of manufacturers	Max profit of retailers
	1280582	573495

Table 4. Analysis on Optimal Pricing Strategy and the Sensitivity of α Variable

α value	Optimistic value criterion		Pessimistic value criterion	
	w^*	m^*	w^*	m^*
$\alpha = 1$	14.0000	3.5000	14.0000	3.5000
$\alpha = 0.95$	14.0003	3.5251	14.0002	3.4751
$\alpha = 0.9$	14.0010	3.5505	14.0010	3.4505
$\alpha = 0.85$	14.0023	3.5761	14.0022	3.4261
$\alpha = 0.8$	14.0041	3.6020	14.0039	3.4020

According to Table.3, we can see that in the Stackelberg game of two-echelon supply chain, the dominant manufacturer can obtain bigger profit. The reason is the manufacturer's dominant role, by which manufacturers can forces retailers to decrease production price, so that make an increase profit for manufacturers.

According to the Table.4, we can see that the Stackelberg game optimal strategies and the max profits change with the predefined confidence levels by manufacturers and retailers. Under the optimistic value criterion, as the decrease of confidence level, the optimal wholesale price and optimal unit margin profit gradually increase, the max profits of manufacturers gradually increase while the max profits of retailers gradually increase at the same time; Under the pessimistic value criterion, as the decrease of confidence level, the optimal wholesale prices gradually increase, and the max profits of manufacturers gradually increase, while optimal unit margin profits and retailers' max profits gradually decrease instead.

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