

Research of Train Operation Adjustment on Double Track-line Based on Differential Evolution

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Abstract

With the features of high density, speed and too many restrictions, train operation adjustment on double track-line is a typical large-scale combinatorial optimization problem and the NP-hard problem. In this paper, based on the characteristics of China's railway operation, a mathematical model is built to shorten the train group's total delayed time and reduce the delayed time to the minimum, and which is solved and optimized with differential evolution algorithm. As the standard differential evolution algorithm can easily lead to the problem of 'premature convergence', an improved differential evolution algorithm is put forward. In the improved algorithm, the adaptive nonlinear regressive mutation operator and crossover operator of t-distribution are adapted to improve the mutation and the design of crossover operator in the standard differential evolution algorithm. The experimental results show the improved differential evolution algorithm has a high rationality and feasibility in solving the problem of train operation adjustment on double track-line, and it has better results than applying the genetic algorithm and standard differential evolution algorithm.

Keywords: Train operation adjustment on double track-line; Differential evolution; Adaptive nonlinear regressive; t-distribution

1. Introduction

In normal conditions, the train operates according to the train working graph. Some unpredictable factors such as natural disasters, railway accidents and some emergencies may turn away a train from its predetermined value, causing some disorders. Therefore, the train operation schedule need to be reorganized as soon as possible to make the train run orderly [1]. Owing to a higher speed and the density of the railways, the complexity of the railway network structure and the dynamic change of the railway transportation, the adjustment of the train operation should obey the running organization rules, and these rules need to be converted into constraints during modeling. Therefore, the train operation adjustment is a large-scale combinatorial optimization problem, and it is difficult to find out the optimal solution only by traditional mathematical methods. In recent years, several emerging intelligent optimization algorithms, such as genetic algorithm and particle swarm optimization algorithm, have been applied to solve the problem of train operation adjustment [2-6], but these algorithms take a long search time and are easy to result in such problems as premature convergence.

Differential Evolution algorithm (DE) is a new evolutionary computing technology, which is a new one proposed by Storn and Price, the professors at the University of Berkeley in 1995 [7]. Having less controlled parameters and simpler running process, the differential evolution algorithm is been regarded as a significant progress in the aspect of algorithm structure [8]. However, the algorithm also has its shorts, such as the prematurity, slow convergence speed and huge computation. Therefore, this paper

proposed an Improved Differential Evolution algorithm (the IDE). The simulation results show that improved differential evolution algorithm can efficiently solve the problem of train operation adjustment on double track-line, and has better results than the standard differential evolution algorithm. This paper provides a more reasonable and efficient way to solve the problem of train operation adjustment on double track-line.

2. The Mathematical Model of Train Operation Adjustment on Double Track-Line

2.1. The Mathematical Model of Train Operation Adjustment on Double track-line

Based on the train operation adjustment on double track-line, this paper suppose that in the double track-line section there are M stations, N trains, and the N_1 trains are the up trains, N_2 trains are the down trains. Then $N=N_1+N_2$. A_{ij}^0 and S_{ij}^0 represent respectively for the time that the $i(i=1,2,\dots,N)$ train arrives in and departs from the $j(j=1,2,\dots,M)$ station in the train working diagram. A_{ij} and S_{ij} represent respectively for the real time that the i train arrives in and departs from the j station. β_j^s and β_j^a represent the least interval time between the departure and arrival in j station. λ_{ij} is the minimum dwell time that the i train in the j station. $T_{j(j+1)}^i$ is the minimum travel time of the train j that runs between the station j and the station $j+1$. ω_i is the precedence level of train i , and the greater weight of ω will increase the train's precedence level. The value will be calculated according to the operation rules and the experts' experiences. The variable τ_{ij} is defined in formula (1), which illustrates that whether the train i will stop in station j .

$$\tau_{ij} = \begin{cases} 1 & S_{ij} \neq A_{ij} \\ 0 & S_{ij} = A_{ij} \end{cases} \quad (1)$$

The symbolic operation is defined in formula (2).

$$\text{sgn}(a,b) = \begin{cases} 1 & a > b \\ 0 & a \leq b \end{cases} \quad (2)$$

Thus the mathematical model of train operation adjustment on double track-line is proposed in formula (3).

$$f = \sum_{j=1}^M \sum_{i=1}^N (k_1 \omega_i (|A_{ij} - A_{ij}^0|) + k_2 \tau_{ij} \times \text{sgn}(A_{ij}, A_{ij}^0)) \quad (3)$$

In formula (3), $\sum_{j=1}^M \sum_{i=1}^N \omega_i (|A_{ij} - A_{ij}^0|)$ shows the train's delayed time, and

$\sum_{j=1}^M \sum_{i=1}^N \tau_{ij} \times \text{sgn}(A_{ij}, A_{ij}^0)$ is the number of the delayed trains. As the two dimensions have different targets, the coefficients K_1 and K_2 are weighted to convert the two targets into an optimal object, and its value is depend on the specific situations.

2.2. The Constraint Conditions of the Train Operation Adjustment on Double Track-Line

Based on the real condition of the high speed train, the constraint conditions of the train operation adjustment on double track-line are as follows,

The constraint of the earliest departure time is showed in formula (4)

$$S_{ij} \geq S_{ij}^0 \quad (4)$$

The constraint of the travel time is showed in formula (5)

$$A_{i(j+1)} - S_{ij} \geq T_{j(j+1)}^i \quad (5)$$

The constraint of the time interval for departure is showed in formula (6) and (7)

$$A_{(i+1)j} - A_{ij} \geq \beta_j^a \quad (6)$$

$$S_{(i+1)j} - S_{ij} \geq \beta_j^s \quad (7)$$

The constraint of train overtaking is showed from formula (8) to formula (10)

$$\omega_k > \omega_i \quad (8)$$

$$A_{kj} - A_{ij} \geq \tau_{ij} \beta_j^a \quad (9)$$

$$S_{ij} - A_{kj} \geq \tau_{ij} \beta_j^s \quad (10)$$

The constraint of dwell time is showed in formula (11)

$$S_{ij} - A_{ij} \geq \tau_{ij} \lambda_{ij} \quad (11)$$

3. Differential Evolution Algorithms

Differential evolution is a kind of swarm intelligence evolutionary algorithms [6] based on swarm iteration, and it can be converted to calculate the minimization problem as follow,

$$\min f(X)$$

The $X = [x_1, x_2, \dots, x_{NP}]$ is the decision space, $x_i(g) = [x_{1,g}, x_{2,g}, \dots, x_{d,g}]$, $i = 1, 2, \dots, NP$, the $x_i^l \leq x_i \leq x_i^u$, x_i^u , x_i^l represent respectively for the upper bound and the lower bound of the decision space, NP is the size of the decision space, d is the space dimensionality, and g is the current evolution algebra. The fundamental principles of DE are as follows

Firstly, initial population is randomly generated.

Secondly, progeny population is generated by mutating, crossing and choosing the individual species.

Finally, after iteration over and over, the results are acquired.

4. The Improvement and Solution of DE

4.1. The Code Design

By using real number coding method, for simplicity, differential evolution algorithms will convert the moment system into an integer minute system through setting up the time of a day for 1, 440 min. A moment is represented from midnight to some point by the number of minutes, for example, the moment 3:30 will be converted for 210 min.

The result of iterative algorithm is converted into an integer value with integer, for example, converting the result of the iterative 187.1 into 188. According to each schedule, L trains use the $2^M \times N$ N-matrix and code M stations in the adjustment segment based

on a real number coding. $X_{(k,i)}$ represents the moment that the i train arrives in k station; while $X_{(M+k,i)}$ is the moment that the i train departs from the k station.

4.2. The Design and Improvement of Mutation

In the calculation, the mutation operator of the differential evolution is generally a constant. However, during the search process of algorithm, if the mutation rate is too big, the algorithm will be close to the random search, resulting low search efficiency and a less accurate global optimal solution; If the mutation rate is too small, the population diversity will decrease, leading to the "premature" phenomenon easily. To solve the above problems, the mutation strategy using the regressive strategy and its formula is in (13).

$$v_i(g+1) = x_{best}(g) + F(x_{r2}(g) - x_{r3}(g)) + F(t)(x_{r4}(g) - x_{r5}(g)) \quad (12)$$

$$F = F_{min} + (F_{max} - F_{min}) * \frac{\sqrt{g_{max}^2 - g^2}}{g_{max}} \quad (13)$$

In the formula, $v_i(g+1)$ is the variation vector of the $g+1$ generation, and F is the zoom factor; $r_1, r_2, r_3, r_4, r_5 \in [1, 2, \dots, NP]$ are the different integers which exclude the i , and $x_{best}(g)$ means the best one in the population of the g generation; F_{min} and F_{max} are respectively the minimum and maximal mutations. In this formula, $F_{min} = 0.6$, $F_{max} = 0.9$.

4.3. The Design and Improvement of Crossover Operator

Differential evolutionary can amend the crossover operator, but if the population is equal approximately on one dimension, then, the algorithm can't continue to optimize. Therefore, only by keeping the diversity of population, the early maturity of the algorithm can be avoided. Cauchy's distribution has a global search ability which can effectively maintain the population diversity, while Gaussian's distribution is good at the local development which can guarantee the speed of its late evolution convergence. In this paper, the study is based on the t -distribution of the disturbance operator.

The t -distribution, also called the student distribution, is the probability density function with the t -distribution which including the DOF n , as shown in formula(14), and it was published by the British statistician Gosset in 1908 with the pseudonym of "student".

$$p_t(y) = \frac{\Gamma(\frac{n+1}{2})}{\sqrt{n\pi}\Gamma(\frac{n}{2})} (1 + \frac{y^2}{n})^{-\frac{n+1}{2}}, \quad -\infty < y < \infty \quad (14)$$

If $n=1$, the t -distribution will become a standard Cauchy distribution, namely $t(n=1) = C(0,1)$; if n is approaching to ∞ , the t -distribution is a standard Gaussian distribution, namely $t(n \rightarrow \infty) = N(0,1)$, and the general deviation will be ignored if $n \geq 30$. It is not easy to find that the standard Cauchy distribution and Gaussian distribution are the two boundary exceptions of the t -distribution [11]. Therefore, t -distribution operator can join Cauchy distribution operator and Gaussian operator together, and if an appropriate DOF is selected, the advantages of Cauchy mutation and Gaussian mutation will be given full play so as to improve the algorithm's flexibility and effect. In this paper, the DOF n is chose according to the formula (15), then the

disturbance operator which based on t-distribution is crossed, the formula as shown in (16).

$$n = 30 * \left\lfloor \frac{g}{g_{max}} \right\rfloor \quad (15)$$

$$u_i(g+1) = \begin{cases} v_{ij}(g+1) & \text{if } (rand(j) \leq CR) \text{ or } j = k \\ x_{ij}(g) \times (1 + t(n)) & \text{otherwise} \end{cases} \quad (16)$$

4.4. The Design Strategy of Selection Operator

After the mutation operation, the DE will have the crossover operator, and which may destroy the good genes. To avoid it, the algorithm is improved: if the variation individuals are more adaptive than the original ones, the crossover operation is skipped to choose the next generation of the variation individuals; otherwise, the crossover operation based on the mutation will be carried out, shown in formula (18).

$$x_i(g+1) = \begin{cases} u_i(g+1) & \text{if } f(u_i(g+1)) < f(x_i(g)) \\ x_i(g) & \text{otherwise} \end{cases} \quad (18)$$

F is the fitness function, and $f(u_i(g+1))$ is the corresponding value of the experimental individual $u_i(g+1)$

4.5. The Total Process of Improving the ED

Step1: Initialize the parameters. The preset iterations $g=0$, the maximum iterations g_{max} , population size NP , exaggeration factor F , the crossover operator CR , input the train's adjustment zone, the departure time and the adjustment time to confirm the total number of the trains M , the priority weight of each train ω_i and k_1, k_2 .

Step2; initialize the population according to the steps in 3.1 and calculate the fitness function of each individual, and the optimal individual $x_{best}(g)$ will be selected.

step3: $g = g + 1$.

Step4: $i=1$

step5: Besides the targeted individual x_i , five different individuals are randomly selected.

step6: The variation individual v_i will be produced based on the mutation design in 3.2

step7: Calculate the fitness of v_i , carry out the selecting operation. If v_i is better than x_i , then there will be step 10. Otherwise, there will be the step8.

step8: Based on the crossover operator in 3.3, having a crossover operation of the x_i and v_i to get the crossover individual u_i .

step9: Work out the fitness function of the u_i , then carry out the selecting operation based on the design of selecting operator.

step10: Get back to step5 until $i = NP$; Otherwise there will be step11.

step11: If the iterations g is greater than g_{max} , then ends the cycle and the optimal solution will be produced. Otherwise, the next iterations will begin from step3.

5. The Experiment

This paper takes the operation plan of Beijing-Shanghai high speed railways from Beijing South Station to Jinan West Station between 7:00 and 11:00 as the research object, as shown in table 1. Assuming that G101 and G185 train delayed 10 min as the underbody of these trains was for servicing operation in Beijing south railway station, while D333, G113 and G115 delayed 10 min to arrive in Tianjin South Station from Langfang Station as they are disturbed by the unstable railway control signal.

According to the design of ordering model and algorithm in this paper, the author adjusted the train schedule, and the results of the train operation plan as shown in Figure 1, the curve of the objective function which changes with the number of iterations is shown in Figure 2 (in this paper, $k_1 = 1, k_2 = 10$), the train operation timetable data is shown in Table 2. Now the explanations and analysis of the results are as follows:

G101 departed from Beijing South at 7:10 and has delayed for 10 minutes, and then it has an accelerated run from Beijing south to Cangzhou West to make it arrive in Cangzhou West station according to the timetable. The delayed G101 would cause the phenomenon of delay propagation which affected D331 and G261. Then D331 and G261 need to be adjusted to arrive in Tianjin South station according to the timetable. D333 delayed 10 minutes when arrived in Tianjin South station, after the operation adjustment it arrived in Cangzhou West at 9:50. The original provision of its departure time from Cangzhou West is 10:04, and it can recover its operational schedule. But in the assumptions, G113 also delayed to arrive in Tianjin South Station for 10 minutes, and the time of arrival in Cangzhou West is adjusted for 10:02. Due to the constraint of train tracking interval, the departure time of D333 in Cangzhou West station was delayed to 10:07. G115 was delayed for 10 minutes in arriving Tianjin south station, and after adjusting it arrived in Cangzhou West station at 10:19, which had the delay propagation effects on the G41. The G41 delayed for 1 minute to arrive in the Tianjin south station and 1 minute delayed to depart from Tianjin south, and it resumed running according to the schedule in Cangzhou west station. G115 had an accelerate run between Tianjin South station and Dezhou East station, and it assumed its schedule in Dezhou East station.

From the above analysis, the model and the algorithm put forward by this article can effectively adjust the train operation plan, and the acquired optimal value of time and efficiency were better than by genetic algorithm and standard differential evolution algorithm. Therefore, it is concluded that the reasonable and feasible results ensures the train operate according to the plan, and provide a new efficient and feasible method for the train operation adjustment on double track-line.

Table 1. The Original Train Operation Plan

Train Number \Station	Beijing South	Langfang		Tianjin South		Cangzhou West		Dezhou East		Jian West	
	From	To	From	To	From	To	From	To	From	To	
G101	7:00	7:18	7:18	7:31	7:31	7:51	7:52	8:15	8:15	8:38	
D331	7:10	7:29	7:29	7:44	7:46	8:07	8:07	8:30	8:32	9:21	
G261	7:15	7:34	7:34	7:49	7:51	8:12	8:12	8:35	8:37	9:03	
G57	7:25	7:44	7:44	7:59	8:01	8:22	8:22	8:45	8:47	9:11	
G185	7:30	7:49	7:49	8:04	8:06	8:28	8:30	8:53	8:53	9:16	
G263	7:55	8:12	8:12	8:25	8:25	8:44	8:44	9:06	9:06	9:27	
G11	8:00	8:17	8:17	8:30	8:30	8:49	8:49	9:11	9:11	9:32	
G107	8:08	8:26	8:26	8:39	8:39	8:59	9:01	9:28	9:30	9:54	
G55	8:13	8:32	8:32	8:47	8:49	9:10	9:10	9:33	9:43	10:07	
D315	8:18	8:38	8:38	8:54	9:14	9:38	9:38	10:05	10:12	10:39	
D333	8:23	8:44	9:01	9:19	9:21	9:44	10:04	10:30	10:43	11:07	
G31	8:30	8:49	8:49	9:03	9:03	9:25	9:25	9:48	9:48	10:02	
G113	9:05	9:23	9:23	9:36	9:36	9:56	9:56	10:18	10:20	10:44	
G115	9:16	9:37	9:39	9:53	9:53	10:14	10:14	10:36	10:38	11:02	
G41	9:33	9:53	9:53	10:07	10:09	10:30	10:30	10:53	10:55	11:19	
D317	9:38	9:59	10:01	10:18	10:18	10:42	10:56	11:26	—	—	
G13	10:00	10:17	10:17	10:30	10:30	10:49	10:49	11:12	—	—	
D335	10:30	10:51	10:53	11:10	—	—	—	—	—	—	

Table 2. The Train Operation Plan After Adjustment

Train Number \Station	Beijing South	Langfang		Tianjin South		Cangzhou West		Dezhou East		Jinan West	
	From	To	From	To	From	To	From	To	From	To	
G101	7:10	7:25	7:25	7:37	7:37	7:51	7:52	8:15	8:15	8:38	
D331	7:15	7:31	7:31	7:44	7:46	8:07	8:07	8:30	8:32	9:21	
G261	7:20	7:37	7:37	7:49	7:51	8:12	8:12	8:35	8:37	9:03	
G57	7:25	7:44	7:44	7:59	8:01	8:22	8:22	8:45	8:47	9:11	
G185	7:40	7:56	7:56	8:04	8:06	8:28	8:30	8:53	8:53	9:16	
G263	7:55	8:12	8:12	8:25	8:25	8:44	8:44	9:05	9:06	9:27	
G11	8:00	8:17	8:17	8:30	8:30	8:49	8:49	9:11	9:11	9:32	
G107	8:08	8:26	8:26	8:39	8:39	8:59	9:01	9:28	9:30	9:54	
G55	8:13	8:32	8:32	8:47	8:49	9:10	9:10	9:33	9:43	10:07	
D315	8:18	8:38	8:38	8:54	9:14	9:38	9:38	10:05	10:12	10:39	
D333	8:23	8:44	9:01	9:29	9:31	9:44	10:04	10:30	10:43	11:07	
G31	8:30	8:49	8:49	9:03	9:03	9:25	9:25	9:48	9:48	10:02	
G113	9:05	9:23	9:23	9:46	9:46	10:02	10:02	10:18	10:20	10:44	
G115	9:16	9:37	9:39	10:03	10:03	10:19	10:19	10:36	10:38	11:02	
G41	9:33	9:53	9:53	10:08	10:10	10:30	10:30	10:53	10:55	11:19	
D317	9:38	9:59	10:01	10:18	10:18	10:42	10:56	11:26	—	—	
G13	10:00	10:17	10:17	10:30	10:30	10:49	10:49	11:12	—	—	
D335	10:30	10:51	10:53	11:10	—	—	—	—	—	—	
G119	10:45	11:06	—	—	—	—	—	—	—	—	

Note: the italic moments in the table are the arrivals and departures of trains

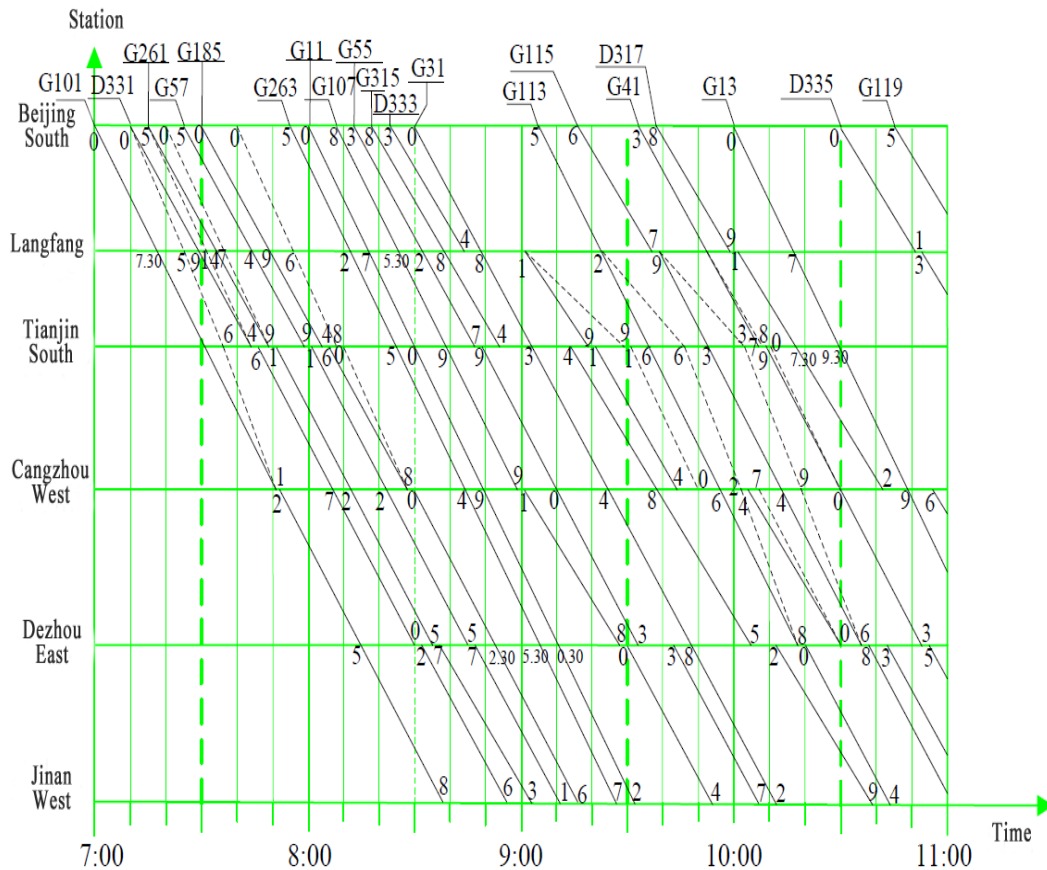


Figure 1. The Train Running Figure after Adjusting

6. Conclusions

(1) Considering of the constraint conditions, such as the earliest departure time, the operation time, interval time between departures, and the overtaking time, this model is a reasonable train operation adjustment on double track-line, which laid a foundation for train operation adjustment on double track-line.

(2) The designed difference algorithm is suitable for solving the problem of train operation adjustment on double track-line model established in this paper.

(3) The improved differential evolution algorithm has a good effect on the problem of double track-line operation, and the experiment proves the feasibility and effectiveness of this algorithm in solving the problem the train operation.

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