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Construction Method and Application for Threshold Function Family in Wavelet Threshold Denoising

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Abstract

Wavelet threshold filter is widely used because it is simple to implement and small computation. Traditional hard and soft threshold functions have their limitations. At the same time, design method of the threshold function is used only for a particular problem and general construction method of threshold function is rarely involved. In this paper, we study the basic requirements of the threshold function and the threshold objective function, and propose a general method for constructing the threshold function. The y=x method applies rotation, shift and other operation on the basic function, and takes as the objective function, constructs threshold function families with different thresholds and approximation rate. Then, choose different threshold functions according to the characteristics of the signal. According to this approach, we take y(x)=a/x as basic function and construct a new threshold function family. The new threshold function is continuous and differentiable at high order, which overcomes the shortcomings of hard threshold function and soft threshold function. Simulation results show that in different noise levels and different wavelet decomposition layers, it has different noise characteristics; signal-to-noise ratio (SNR) gain and minimum squared error (MSE) of this method is better than that of the traditional soft and hard threshold function and improved threshold function and made a better balance between the sensitive degree and smoothness.

Keywords*: Wavelet threshold denoising; threshold function; construction; application*

1. Introduction

With the development of signal and noise reduction technology, methods of the denoising varied such as particle denoising, wavelet denoising, mathematics morphology denoising, PDE denoising, Hilbert - Huang (Hilbert Huang transform HHT) denoising [1-2]. These filtering methods have their own characteristics, which can show different advantages when dealing with different types of noise signals. For example, mathematics morphology denoising method has the characteristics of nonlinear noise reduction, so it shows a good effect in the impulse noise suppression; PDE denoising has advantages of edge continuity and internal continuity, *etc*. Wavelet denoising method has been used in the field of signal denoising in recent years [3-6]. Donoho and Johnstone proposed wavelet threshold denoising algorithm on basis of wavelet transform, and proved that it was the optimal estimation approximating the original signal [7-8].

The research of wavelet threshold noise reduction generally lies in three aspects: determination of threshold, choice of wavelet basis and establishment of threshold noise reduction function. There is fixed threshold (sqtwolog), minimum maximum variance threshold (minimaxi), Stein unbiased likelihood estimation based threshold (rigrsure), selective heuristic threshold (heursure) and so on [9]. For the establishment of threshold de-noising function, Donoho and Johnstone *et al* proposed hard threshold and soft threshold method [7-8] and H. Gao, proposed not garrote thresholding function [10],

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Wang Ya proposed semi soft threshold method [11], H. Gao also improved two kinds of noise reduction methods based on hard threshold and soft threshold [12].

The traditional soft and hard threshold denoising have two different assumptions: (1) soft threshold denoising, assuming that the noise signal exists in all wavelet coefficients after wavelet transform, so as denoising, a fixed threshold are removed from all wavelet coefficients before reconstruction; (2) hard threshold denoising assumes that the signal after wavelet transform, noise generates wavelet coefficients which are smaller than the threshold, and the greater wavelet coefficients are generated by the actual signal. Therefore reconstruction will be established by directly removing the wavelet coefficients smaller than the threshold. But for the actual signal, assumptions of both soft and hard threshold function and the noise reduction process have problems. In the structure of the threshold function, most of the designs of the threshold noise reduction function are based on soft and hard threshold function, through the threshold weighted correction to achieve the purpose of improving the noise reduction effect [11]. The function constructed through this kind of construction method is too slow to approximate the hard threshold, which leads to the deviation of the wavelet coefficients in a large scale. Other scholars solve the optimization of polynomial coefficients to build a threshold function through the optimization of the noise reduction effect index [13]. However, this kind of method still has the disadvantage of the discontinuous threshold function.

This paper proposes a general method for constructing threshold function according to the problem of the traditional threshold function and the method of improving the threshold function. First, determine the threshold function target curve; and then through translation, rotation and scaling operations to meet the construction conditions of typical function curve, forming a function family with different thresholds and different approximation rates to the objective function. Then according to the characteristics of the signal, choose different threshold function used to reduce the noise. According to this construction method, the threshold function family is obtained by the rotation transformation. Simulation results show that the function constructed in this paper overcomes the shortcoming that the general threshold function is not continuous or the deviation in a large scale and it can achieve a better balance in the sensitivity and smoothness.

2. Wavelet Threshold Denoising

2.1. Principle and Basic Method of Wavelet Threshold Denoising

Suppose 1-D signal:

$$f(t) = s(t) + \varepsilon(t) \tag{1}$$

in which $s(t)$ is the original signal, $\varepsilon(t)$ is the noise. Perform discrete sample to $f(t)$ and get discrete signal $f(n)$, $n=0,1,2,\ldots,N-1$, whose wavelet transformation is

$$Wf(j,k) = 2^{-j/2}\sum_{n=0}^{N-1} f(n)\psi(2^{-j} - k) \ , \ j,k \in Z \tag{2}$$

in which $Wf(j,k)$ is wavelet coefficients. For convenience, set $w(j,k)=Wf(j,k)$. Because the wavelet transform is a linear transformation, the wavelet coefficient $w(j,k)$ is still composed of two parts: one part is the wavelet coefficient of the signal $s(t)$, and the other part is the generation of the wavelet coefficient from $\varepsilon(t)$.

Wavelet transform has a strong decorrelation for data, and the energy of the signal is mainly concentrated in a number of large limited wavelet coefficients [7]. It means the large wavelet coefficients is signal based, while the small coefficient is mainly noise based. Donoho and Johnstone followed this principle, and proposed the wavelet threshold denoising method with the process introduced below:

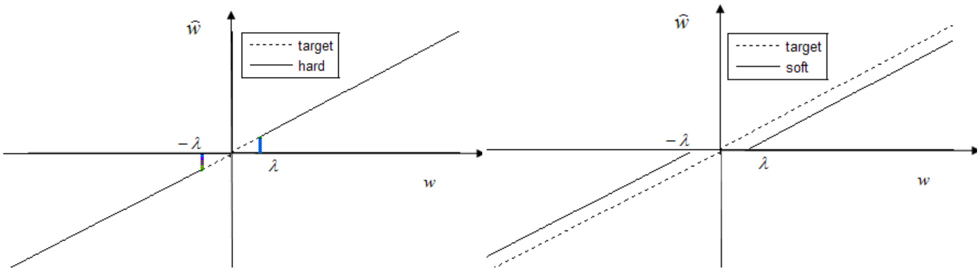
The first step is to set up a threshold λ; when $w(j,k) \geq \lambda$ it means that wavelet coefficients are generated by the signal, suppose the new wavelet coefficients $\hat{w}(j,k) = w(j,k)$ (hard threshold method), or soft threshold method $\hat{w}(j,k) = w(j,k) - K$ in which $K = \Psi(\lambda) = \lambda$; when $w(j,k) < \lambda$ consider $w(j,k)$ is mainly caused by noise and $\hat{w}(j,k) = 0$. Then the new wavelet coefficients $\hat{w}(j,k)$ are used to reconstruct the signal after noise reduction.

Donoho defined the hard threshold and soft threshold functions as shown in Figure 1, in which the hard threshold function is

$$\widehat{w} = \begin{cases} w_{j,k} \ , & |w_{j,k}| \geq \lambda \\ 0 \ , & |w_{j,k}| < \lambda \end{cases} \tag{3}$$

And soft threshold function is

$$\widehat{w} = \begin{cases} sign(w_{j,k}) * (|w_{j,k}| - \lambda) \ , & |w_{j,k}| \geq \lambda \\ 0 \ , & |w_{j,k}| < \lambda \end{cases} \tag{4}$$



(a) hard threshold function (b) soft threshold function

Figure 1. Hard Threshold Function and Soft Threshold Function

In which Sign ($w(j,k)$) gets sign of $w(j,k)$, and λ is the threshold value with for $\sigma(\log N)^{1/2}$ the general value, σ is the signal of the noise variance, N is the length of the signal [7]. In the literature [7], it was proved that the estimation error signal obtained by this method is valid in the sense of minimum mean square error.

2.2. Improvement of Threshold Function

In view of the shortcomings of traditional soft and hard threshold functions, many scholars have put forward some new threshold function. A soft and hard threshold function is proposed in the paper, which is a compromise of soft and hard threshold function:

$$w_{j,k} = \begin{cases} sign(w_{j,k}) * (|w_{j,k}| - \alpha\lambda(j)) & |w_{j,k}| \geq \lambda \\ 0 & |w_{j,k}| < \lambda \end{cases} \tag{5}$$

in which $0 \leq \alpha \leq 1$, $K = \Psi(\lambda) = \alpha\lambda(j)$. This improved method has weighted threshold λ arithmetic, makes a compromise line between the soft and hard threshold function curve, and improves the soft threshold function coefficient constant deviation problem, but there are still hard threshold function which is discontinuous and has the disadvantage of constant deviation in the soft threshold.

A modified method called "garrote nonnegative" is proposed in the literature [10], and the threshold value function is

$$\widehat{w}_{j,k} = \begin{cases} sign(w_{j,k}) * (\left|w_{j,k}\right| - \lambda^2 / \left|w_{j,k}\right|) & \left|w_{j,k}\right| \geq \lambda \\ 0 & \left|w_{j,k}\right| < \lambda \end{cases} \tag{6}$$

in which $K=\Psi(\lambda)=\lambda/|w_{j,k}|$. This method is an improvement of the threshold weighted method, which can produce a continuous curve between the soft and hard threshold function curves, which overcomes the breakpoints in hard threshold function.

A threshold value function is proposed based on the optimal polynomial coefficient threshold method, which is based on the [13].

$$\widehat{w}_{j,k} = \begin{cases} a_{N-1} w_{j,k} - a_N \operatorname{sgn}(w_{j,k})\lambda & \left|w_{j,k}\right| > \lambda \\ \sum_{k=0}^{N-2} a_k x^{2k+1} & \left|wj,k\right| \leq \lambda \end{cases} \tag{7}$$

in which N, a_0 ,$a_1 \ldots a_N$ is obtained through the optimization algorithm with objective function $MSE(\widehat{g}) = L2(g,\widehat{g}) = \frac{1}{K}\sum_{i=1}^{K} | gi - \widehat{g}i |^2$. This method first set the target of noise reduction, and then solve the optimization of polynomial coefficients from noise function, and use the optimization coefficient to determine the threshold function. This method need to estimate the optimal value to polynomial functions with different orders and different types of noise in order to achieve better noise reduction effect; and it has the shortcomings of non-differentiable point at the inflection point of threshold function, computational complexity and noise reduction effect and estimation of polynomial coefficient.

3. General Construction Method of Threshold Function

Soft and hard threshold method is widely used in signal denoising, but they still have some shortcomings. From Figure 1 (a), we can see that the hard threshold function is not continuous at λ and -λ, so Pseudo-Gibbs phenomenon will occur after the noise reduction at the singular point, which results in large mean square deviation and oscillation. From Figure 1(b) can be seen that although the soft threshold function is a continuous function, but the new wavelet coefficients and the original coefficients exist a constant deviation, the reconstructed signal may have a greater variance, resulting in the loss of high frequency information of the signal and fuzzy edge. At the same time, in the design method, the improved threshold function is mostly just for a particular problem or a specific function was improved. The threshold function is fixed, so we cannot apply different threshold function according to different characteristics from signal and the noise.

3.1. A General Design Method for Threshold Function

According to the characteristics of the wavelet coefficients, the wavelet coefficients around the threshold value should be processed according to the size of the threshold. For the different decomposition level, the threshold function of the different rate approaches to the objective function $y(x)=x$ should be used. Define a general threshold function $y(x)$, in which x represents the wavelet coefficients $w(j,k)$. According to definition in (3) ~ (7), these functions are in fact an approximation to $y(x)=x$ in different form of the threshold of different threshold. So, the constructed threshold function with this function principle should meet the following conditions.

Condition 1：

(1) To avoid the pseudo Gibbs phenomenon, it should be continuous at the threshold.

(2) Within the definition domain, threshold function should be monotonic and continuous which assures we can perform inverse transform to transformed coefficients and restore the original signal, as shown in Figure 1 (b) where $|w|>\lambda$.

(3) The value of the function is zero within the definition domain smaller than the threshold value, as shown in Figure 1 (b) where $|w|\leq\lambda$.

(4) Within the definition domain greatly higher than threshold, such as the region where $|w|>\lambda+\Delta$ in Figure 1 (b), the curve approximates to the line $y(x)=x$ quickly; and within the definition domain slightly higher than threshold, such as the region where $|w|\leq\lambda+\Delta$ in Figure 1 (b), the curve approximates to the line $y(x)=x$ at a certain speed. Size of Δ should be estimated according to the intensity of the noise, when the noise is large, Δ is relatively large, and curve approximates to the objective line slowly; when the noise is small, Δ is relatively small, and the curve approximates to the objective line fast.

The velocity of threshold function approximating to $y(x)=x$ is related to λ. When λ is small, the wavelet decomposition coefficients are less affected due to smaller amplitude of noise so the threshold curve should be fast approximating and it reduces the effects of soft threshold shrinkage to small coefficients; when λ is larger, the wavelet decomposition coefficients are greatly affected due to larger amplitude of noise, so the threshold curve should be approximating slowly.

According to the principles above, this paper proposes a general method for constructing the threshold function. First set $y(x)=x$ as the objective function, and choose basic function to satisfy condition 2; determine the threshold according to the characteristics of signal; and then use threshold to obtain different threshold function families with different curvatures and thresholds (zeroes) through translation, rotation and scaling transformation of the basic function of.

Condition 2：

(1) The function has asymptote line coincident with $y(x)=x$ after transformation.

(2) The function has intersection with the x-axis and the intersection is adjustable. It meets the different decomposition levels corresponding to different thresholds.

(3) The transformed function is continuous and monotone.

(4) The curves in the families are not cross, and have a faster approximation speed. It meets the Condition 1 (4).

According to the basic function listed in condition 2, the asymptote and the objective curve will coincide with $y(x)=x$ after rotation, translation and scaling operation, and meet the requirement that the threshold functions will approximate objective curve; at the same time curvature and threshold of transformed curves will controlled by parameters (intersection with x-axis). So the curve family formed through this method can meet the requirement of the threshold function, which can obtain good effect of wavelet threshold denoising.

The Main Steps of the Proposed Method is:

(1) Choose basic function $k=\varphi(x)$ meeting Condition 2.

(2) Take $y(x)=x$ as the objective curve, and perform rotation, translation, scaling and other operations to curves $k=\varphi(x)$ to obtain function $K=\Psi(x)$ which meets Condition 1.

(3) Reserve the curve of $K=\Psi(x)$ in the first and third quadrant; if it only exists in the first quadrant, rotate 180 degrees with center of the origin to reconstruct a three quadrant curve. The threshold function is constructed by this curve.

3.2. Example of Threshold Function Construction

In this paper, we take the function $y=a/x$ and construct a threshold curve family that satisfies the condition through the rotation operation. Suppose the point (x, y) in the basic function $y=a/x$, through θ degree counterclockwise rotation, its coordinates are:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \tag{8}$$

$$x' = x\cos\theta + y\sin\theta, \quad y' = -x\sin\theta + y\cos\theta \tag{9}$$

$$x = x'\cos\theta - y'\sin\theta, \quad y = x'\sin\theta + y'\cos\theta \tag{10}$$

Substitute (10) in function $y=a/x$ ($x>0$), we obtain the curve after θ rotation:

$$x'\sin\theta + y'\cos\theta = a/(x'\cos\theta - y'\sin\theta) \tag{11}$$

Solve the equation:

$$y' = -\frac{x' \pm \sqrt{x'^2 - 4a\sin\theta\cos\theta}}{2\cos\theta} * \sin\theta + \frac{2a\cos^2\theta}{x' \pm \sqrt{x'^2 - 4a\sin\theta\cos\theta}} \tag{12}$$

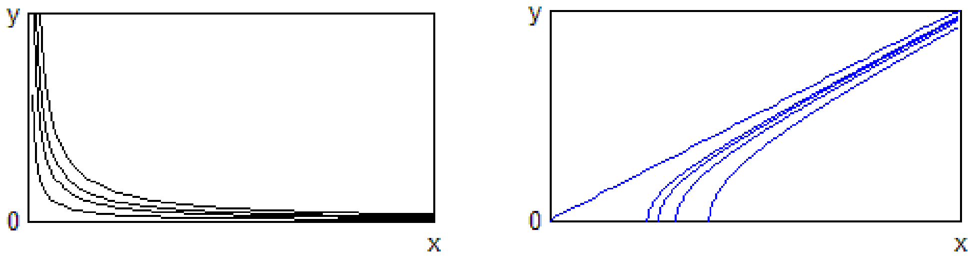
When the clockwise rotation π/4, that is $\theta=\pi/4$, we have:

$$y' = -\left(\frac{x \pm \sqrt{x'^2 - 2a}}{2}\right) + \frac{a}{x'^2 \pm \sqrt{x'^2 - 2a}} \tag{13}$$

We take the curve in the first quadrant after and get the curve function

$$y' = -\left(\frac{x' - \sqrt{x'^2 - 2a}}{2}\right) + \frac{a}{x' - \sqrt{x'^2 - 2a}} \tag{14}$$

Figure 2 (a) shows function family with different variable 'a' in function and there are two asymptotes $y=0$ and $x=0$. After π/4 clockwise rotation we get curve family shown in Figure 2 (b), which is the representation in the formula (14). The family has different approximating rate to $y(x)=x$ and different intersections with x-axis.



(a) function family (b) curve family in equation(14)

Figure 2. Threshold Function Family

It can be seen from the Figure2 (b) that the curve family meets Condition 1 and the curves with different starting points approximate the objective curve at different rates, representing different noise levels. The intersections of curves and x-axis are the threshold λ.

Consider the specific range of wavelet coefficients, and intersection between rotated $y(x)=a/x$ curve and x-axis $x=\sqrt{2a}$, so in formula (9), we set $\lambda=\sqrt{2a}$, $x'=w$, $y'=\widehat{w}_{j,k}$ curves in the first and third quadrant are symmetrical with reference of the origin, and finally get the threshold curve function:

$$\widehat{w}_{j,k} = \begin{cases} sign(w_{j,k}) * \left(-\dfrac{w_{j,k} - \sqrt{w_{j,k}^2 - \lambda_j^2}}{2} + \dfrac{\lambda_j^2/2}{w_{j,k} - \sqrt{w_{j,k}^2 - \lambda_j^2}}\right) & |w_{j,k}| \geq \lambda_j \\ 0 & |w_{j,k}| < \lambda_j \end{cases} \tag{15}$$

Function curve is shown in Figure 3:

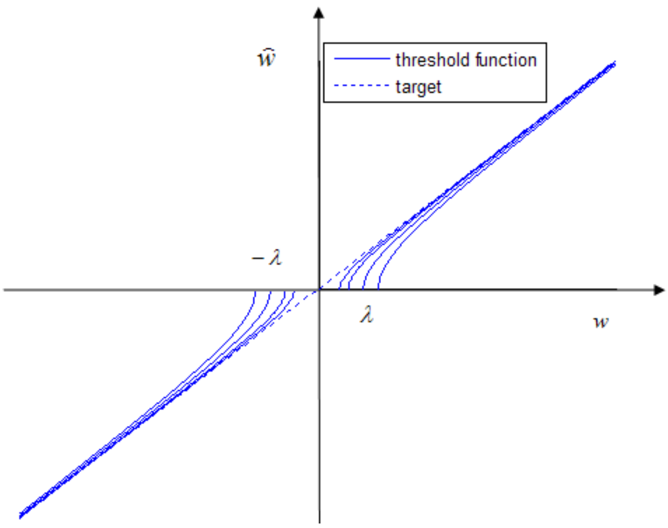
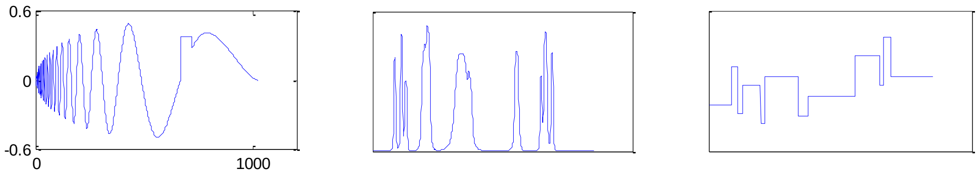


Figure 3. Curve Family in Equation（15）

From the structure process mentioned before, the threshold function family constructed from rotation of basic function y=a/x meets Condition 1. The threshold function family is continuous at the threshold, continuous and high order differentiable within the definition domain. It can not only overcome the constant deviation between decomposed and the estimated wavelet coefficients in soft threshold function, but also overcome Gibbs oscillation due to discontinuity in hard threshold function. Moreover it balances the effects threshold λ on threshold function.

4. Performance Comparisons

In this paper, comparative experiments are carried out for Bumps, Block and Doppler one-dimensional signals as shown in Figure 4, and then the two-dimensional signal Lena image is used for comparison of noise reduction effect. In the tests, the signals with different types and noise intensities are decomposed by using DB4 wavelet (decomposition of 5 layers), and is operated with different threshold functions.



(a) Doppler (b)Bumps (c)Blocks

Figure 4. Doppler, Bumps and Blocks Signals

4.1. Noise Reduction Effect Comparison among Different Noise Types and Different Signal to Noise Ratios

Figure 5 (a) shows the original Doppler with a Block signal added, Figure 5 (b) is the signal with noise (Gauss noise, SNR=15dB). Figure 5 (c), (d), (e), (f) are the experimental results from different methods. From the Figure (d), (f), the comparison in Block part our proposed method shows a smaller oscillation than that of the hard threshold method, which indicates Pseudo-Gibbs phenomena are suppressed; the new method is more effective in filtering out the noise for coefficients greater than the threshold compared with hard threshold method; from the map (c), (f) the contrast can be seen. In the overall noise reduction effect, the new method is better than the soft threshold method of small amplitude deviation; from Figure (c) and (f), the new method has smaller deviation compared with soft threshold method; Also it keeps more effective signals where the signal frequency varies severely compared with non negative garrote method and represents good performance for small high-frequency signal.

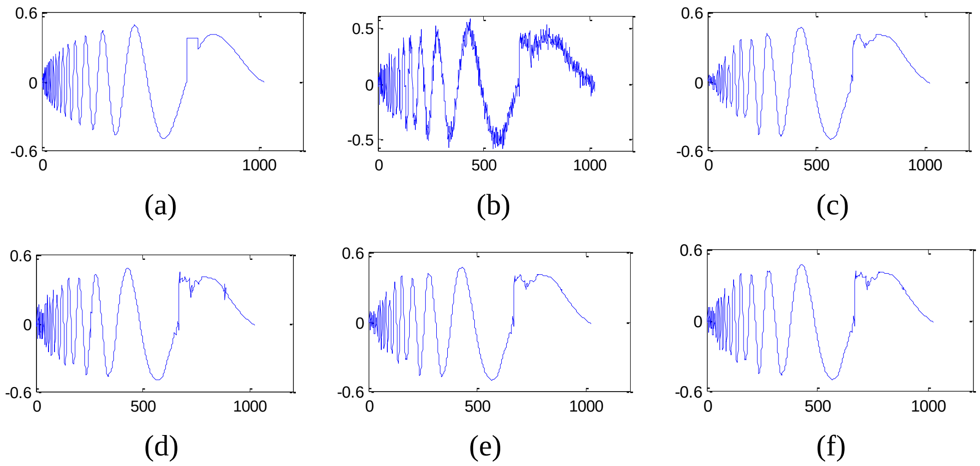


Figure 5. Denoising Effect Comparisons of Superimposed Doppler and Block Signal

(a) original signal, (b) signal with noise, (c) soft threshold method in equation(3), (d) hard threshold method in equation(4), (e) non negative garrote threshold method in equation(6), (f) new threshold method in equation(8).

Table 1 gives the Doppler signal with different types of noise and SNRs, and compares MSE of the 4 threshold functions after the noise reduction. When SNR is 0, the noise pollution is the most serious, pseudo Gibson phenomenon is more obvious for small high-frequency signal with hard threshold which results in a worse performance of hard threshold function, and MSE value is also the largest, reaching 0.0135 and 40% ~ 50% higher than other threshold methods; for SNR 5 and 10, noise pollution is relatively light, soft and hard threshold functions limit the noise reduction effect due to their inherent disadvantages. The performance of soft and hard threshold functions is closed. Generally speaking soft threshold method is still better from MSE comparison with different SNR.

Table 1. Doppler SNR and MSE Comparisons at Different Noise Type

SNR/ MSE	Gauss white noise			uniform white noise			periodic noise		
	0/ 0.082	5/ 0.026	10/0.00 8	0/ 0.081	5/ 0.026	10/ 0.0083	0/ 0.083	5/ 0.026	10/ 0.008
Soft	0.0095	0.0039	0.0021	0.0071	0.0044	0.0036	0.0085	0.0035	0.0020
Hard	0.0135	0.0054	0.0022	0.0082	0.0052	0.0040	0.0095	0.0034	0.0017
Non	0.0086	0.0032	0.0017	0.0064	0.0037	0.0028	0.0078	0.0023	0.0015
New	0.0085	0.0032	0.0016	0.0063	0.0035	0.0026	0.0073	0.0022	0.0014

Figure 4 (b) and (c) present Bumps signal and Blocks signal. Bumps signal has more pulse signal, while Blocks signal is a smooth transition signal. With different types and intensity noise, denoising methods mentioned above are compared, whose results are shown in Table 2 and 3.

Table 2. Bumps SNR and MSE Comparisons At Different Noise Type

SNR/ MSE	Gauss white noise			uniform white noise			periodic noise		
	0/ 2.2899	5/ 0.7450	10/ 0.2334	0/ 2.1839	5/ 0.7372	10/ 0.2272	0/ 2.3547	5/ 0.7177	10/ 0.2300
Soft	0.3777	0.1687	0.0859	0.36	0.1680	0.0880	0.3474	0.1854	0.0966
Hard	0.3886	0.1716	0.0515	0.3035	0.1102	0.0546	0.54	0.1616	0.0715
Non	0.2985	0.1264	0.0536	0.2888	0.1148	0.0563	0.3843	0.1342	0.0621
New	0.2677	0.1189	0.0464	0.2638	0.0987	0.0494	0.3210	0.1249	0.0579

Bumps signal has poor denoising effect because it contains more substantial amplitude changes, especially the presence of spikes. These spikes are weakened after wavelet transform in soft threshold function, resulting in poor denoising effect. The non negative ground threshold method, which has a slow approximating rate also has a deviation from the soft threshold value. Table 2 shows that the new method can perform better in the signal with more spikes.

Table 3. Blocks SNR and MSE Comparisons at Different Noise Type

SNR/ MSE	Gauss white noise			uniform white noise			periodic noise		
	0/ 3.8994	5/ 1.2213	10/ 0.3836	0/ 3.8216	5/ 1.2130	10/ 0.3908	0/ 3.827	5/ 1.2165	10/ 0.3886
Soft	0.5682	0.3598	0.1776	0.5627	0.3698	0.2156	0.5323	0.3667	0.2055
Hard	0.8012	0.2977	0.1155	0.5846	0.3053	0.1329	1.0200	0.3688	0.1138
Non	0.5635	0.3127	0.1219	0.5333	0.3269	0.1572	0.5522	0.3101	0.1443
New	0.5625	0.2721	0.1041	0.5104	0.2976	0.1344	0.5389	0.3019	0.1165

Blocks signal also contains more singular points, but compared with the Bumps signal, there are more stable regions. It can be observed that soft and hard threshold functions have a significant gap with the new method, while non negative twist method and the new threshold function is relatively closed.

4.2. Comparison of Noise Reduction Effect

Table 4 is the noise reduction effect of Gauss white noise, uniform white noise and periodic noise. When SNR equals 0, MSE based on denoising method are 0.0082, 0.2673, 0.4335 respectively, which is much smaller compared with the other methods. Performance is still good in case of SNR 5 and 10.

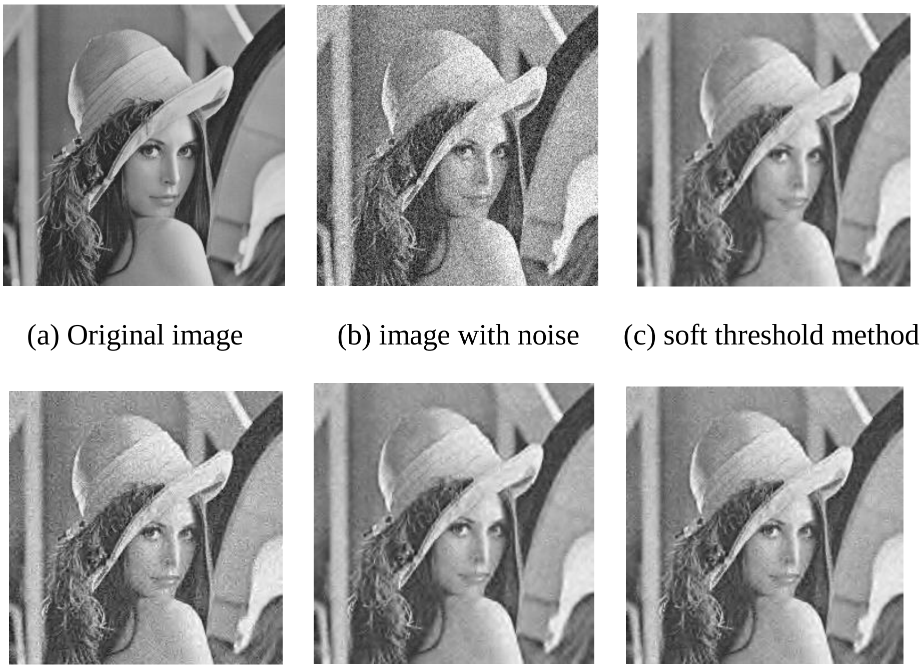
Table 4. MSE Comparison under Different Intensity of Denoising

SNR/ MSE	doppler			bumps			blocks		
	0/ 0.0830	5/ 0.0263	10/ 0.0080	0/ 2.3533	5/ 0.7355	10/ 0.2171	0/ 3.6563	5/ 1.2315	10/ 0.3782
Soft	0.0091	0.0048	0.0027	0.3618	0.1606	0.0858	0.5149	0.3565	0.1986
Hard	0.0084	0.0037	0.0018	0.3137	0.1616	0.0583	0.7869	0.3147	0.1264
Non	0.0086	0.0033	0.0015	0.2961	0.1127	0.0555	0.4618	0.3096	0.1259
New	0.0082	0.0032	0.0014	0.2673	0.1078	0.0498	0.4335	0.2867	0.1232

From Table 2, 3 and 4, the comparison of MSE performance can be observed, the new threshold method has significant performance improvement both in different signal types and different intensity of noise reduction over soft and hard, and the non negative garrote threshold functions.

4.3. Effect Comparison of 2-D signal Noise Reduction

Wavelet denoising has been widely used in image denoising [14-17]. Lena image with white Gaussian noise (PSNR is 33 and MSE is 29) and apply soft and hard threshold function, non negative garrote threshold function and the structure of the new threshold function for noisy image noise reduction. The noise reduction effect is shown in Figure 5 and 6. It can be seen that after noise reduction from new threshold function, pictures, the picture receives good performance in either the details of the performance or in the image smoothness so performance is better than other methods. From the MSE value, the new threshold noise reduction of MSE was 2.6506, while the soft and hard MSE are 3.3111 and 4.6019 respectively, and MSE from non negative twist method and the new threshold method are more closed, reaching 2.7234.



(a) Original image (b) image with noise (c) soft threshold method

(d) hard threshold method (e) non negative garrote threshold method (f) new method

Figure 6. Comparison of Noise Reduction

5. Conclusion

Since D.L. Donoho proposed wavelet threshold denoising method, the threshold function has been widely studied. In this paper, we analyze the traditional threshold denoising methods and related improvements, the general requirements of the threshold function are declared, and a general method for constructing the threshold function is proposed. With rotation operation upon basic functions, and as the objective function, the structure of the threshold function is achieved. Different functions have different approximations rate to objective function, which indicates the different estimation of the noise in the wavelet decomposition coefficients. In this paper, a new threshold function is constructed by using $y=a/x$ as a basic function to form a new threshold function family. Through the simulation tests, these methods were compared and it is proved that new threshold function avoids pseudo Gibbs oscillation caused by hard threshold function due to discontinuity point; also it solves the problem of large deviation coefficient in a considerable range caused by soft threshold methods. As to say it has good noise reduction effect.

The construction method provided in this paper is a general criterion, which can be used to obtain the function families with different thresholds. Approximation rate of threshold function to the objective function depends on the order and threshold of the basis functions, and has a direct impact on the effect of noise reduction. Relationship between speed and noise reduction effect, and selections of basic function and threshold according to SNR need further research.

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