

Dynamic Output Feedback Control for Networked Predictive Control Systems with Uncertainties

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Abstract

The paper studies the application of a prediction-based scheme for dynamic output feedback control in a networked control system with uncertainties, communication delay and data dropout. In order to make a compensation for communication delay and data dropout, a prediction-based output control scheme is developed. With the given theoretical derivation, the closed-loop networked predictive control system with uncertainties can be formulated into a robust system with a standard form, which makes it convenient to design the controller and analysis closed-loop stability. Simulation examples demonstrate the performance and stability of a networked control system using the proposed predictive output control scheme.

Keywords: *Networked Control Systems; Prediction-based Output Feedback; Uncertainties*

1. Introduction

For the last few years, control systems wherein the closed control loop is realized via a communication network have attracted increasing concern in the field of control science and engineering, because the rapid progress of microelectronics technology and network capabilities bring considerable convenience for monitoring and controlling large-scale distributed plants remotely [1,2]. These systems are generally referred to as networked control systems (NCSs). They have a lot of advantages such as reduced complexity, easy maintenance and low-power consumption, and also promote large-scale engineering applications, including traffic engineering, industrial chemical process systems, spacecraft formation control, power management and energy harvesting, and intelligent health-care service [3-6]. Nevertheless, the integration of various communication networks into control loops inevitably results in a few challenging issues, such as transmission delay, data loss and data packet disorder, which has a severe performance degradation or might even make the closed-loop system unstable in some cases. Such important issues have to be considered for the design and analysis of a NCS.

In order to improve the control quality of a NCS, there exist many control methods proposed in the published literature, such as Smith Predictor [7], switched system theory [8], stochastic optimal control [9] and impulsive control [10]. Nevertheless, their applicability is restricted by making some impractical assumptions on network characteristic and reducing the influence of communication delay and data dropout on controlled systems in a passive way. In point of fact, it is a really hard task how to design a less conservative networked controller, in the aim of making a compensation for communication delay and data dropouts from an active perspective.

In this paper, to make a compensation for communication constraints and uncertainties in NCSs actively, the networked predictive control (NPC) scheme has been introduced in

[11]. As described in Figure 1, the NPC system is mainly composed of two components: one is control prediction generator (CPG), the other is network delay compensator (NDC). The CPG is responsible for generating a set of control predictions which can achieve desired control performance. The NDC makes a compensation for the random communication delay. A state feedback controller together with perturbation in the controlled plant is designed and analyzed for an NPC system in [12]. However, of all the modern control strategies, the state feedback controller is a less flexible and more conservative structure. In this work, a novel dynamic output feedback controller design for an NPC system with uncertainties is discussed in detail hereinafter.

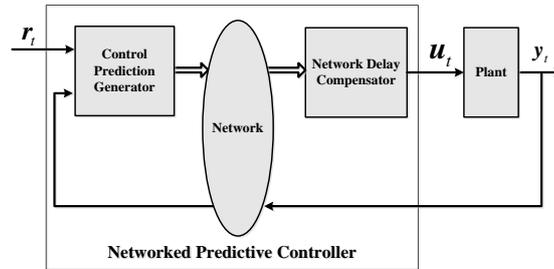


Figure 1. The NPC system

2. Problem Formulation and NPC Scheme

For the description of network characteristic, this section makes four reasonable assumptions as follows: i) The sensor, actuator and controller are time-driven and synchronous; ii) The communication delay in the feedback channel τ_{sc} and the forward channel τ_{ca} is bounded by n_b and n_f , respectively; iii) The number of consecutive data dropout in both forward and feedback channels has an upper bound n_d ; iv) All data over a communication network are time-stamped before transmission.

2.1. Problem Formulation

In many practical applications, there are many different kinds of uncertainties in a physical system, such as sampling noise, modelling errors and external disturbances. The modelling uncertainties are discussed below. Consider an uncertain discrete-time linear plant with the following state-space model:

$$\begin{aligned} x_{t+1} &= (A + \Delta A)x_t + (B + \Delta B)u_t \\ y_t &= (C + \Delta C)x_t, \end{aligned} \quad (1)$$

where $x_t \in \mathbf{R}^n$ is the state, $u_t \in \mathbf{R}^m$ is the control input, $y_t \in \mathbf{R}^l$ is the measured output of the controlled plant. $A \in \mathbf{R}^{n \times n}$, $B \in \mathbf{R}^{n \times m}$ and $C \in \mathbf{R}^{l \times n}$ are constant real matrices that describe the nominal system, and ΔA , ΔB , ΔC denote the perturbations of system matrices A , B and C , respectively. Here, a dynamic output feedback controller (*i.e.* a dynamic compensator) for the controlled system (1) is designed as:

$$\begin{aligned} z_{t+1} &= Fz_t + Hy_t \\ u_t &= Nz_t + My_t, \end{aligned} \quad (2)$$

where $z_t \in \mathbf{R}^q$ is the state of the dynamic compensator, $F \in \mathbf{R}^{q \times q}$, $H \in \mathbf{R}^{q \times l}$, $N \in \mathbf{R}^{m \times q}$, $M \in \mathbf{R}^{m \times l}$ are the parameter matrices of the dynamic output feedback controller. On the basis of the nominal model of (1), substituting y_t in (2) by $y_t = Cx_t$ leads to

$$\begin{aligned} z_{t+1} &= Fz_t + HCx_t \\ u_t &= Nz_t + MCx_t. \end{aligned} \quad (3)$$

2.2. NPC Scheme

To make a compensation for communication delay, a NPC scheme is employed. From assumptions ii) and iii), let $\mathbf{f} = \mathbf{n}_f + \mathbf{n}_d$, $\mathbf{b} = \mathbf{n}_b + \mathbf{n}_d$. Without loss of generality, it is assumed that the state of the plant \mathbf{x}_t cannot be measured but the system output \mathbf{y}_t is available. To observe the state variables of the plant at the controller side, the following observer is employed

$$\begin{aligned}\hat{\mathbf{x}}_{t-b+1|t-b} &= A\hat{\mathbf{x}}_{t-b|t-b-1} + B\mathbf{u}_{t-b} + L(\mathbf{y}_{t-b} - \hat{\mathbf{y}}_{t-b}) \\ \hat{\mathbf{y}}_{t-b} &= C\hat{\mathbf{x}}_{t-b|t-b-1},\end{aligned}\quad (4)$$

where $\mathbf{x}_{t-i|t-b} \in \mathbf{R}^n$ ($i < \mathbf{b}$) is the state prediction for time $t-i$ based on the data up to time $t-b$, $\hat{\mathbf{y}}_{t-b} \in \mathbf{R}^l$ denotes the output of the observer (4), and $\mathbf{L} \in \mathbf{R}^{n \times l}$ denotes the observer gain matrix.

Because there is a feedback delay, it will delay for \mathbf{b} steps when the measured output \mathbf{y} arrives at the controller side. Notwithstanding the observer (4) computes a one-step ahead states prediction of the controlled plant by the measured output signal \mathbf{y}_{t-b} , the states of the controlled plant from time $t-b+2$ to time $t+f$ are still not predicted. On the basis of the data at the controller side at time $t-b$, the other state predictions up to time $t+f$ can be obtained by

$$\hat{\mathbf{x}}_{t-b+i|t-b} = A\hat{\mathbf{x}}_{t-b+i-1|t-b} + B\mathbf{u}_{t-b+i-1}, \quad (5)$$

for $i=2, 3, \dots, b+f$. At time t , despite some of control inputs at the controller side are not used to drive the controlled plant, all control inputs from time $t-b$ to time $t+f-1$ are available. Thus, the state predictions of the controlled plant given by (5) can be calculated using the measured output signal \mathbf{y}_{t-b} . Similarly, the state predictions of the dynamic feedback compensator from $t-b$ to $t+f$ can be constructed

$$\begin{aligned}\hat{\mathbf{z}}_{t-b+1|t-b} &= F\hat{\mathbf{z}}_{t-b|t-b-1} + HC\hat{\mathbf{x}}_{t-b|t-b-1} \\ &\vdots \\ \hat{\mathbf{z}}_{t+f|t-b} &= F\hat{\mathbf{z}}_{t+f-1|t-b} + HC\hat{\mathbf{x}}_{t+b-1|t-b}.\end{aligned}\quad (6)$$

as

When the state predictions of the plant and the designed dynamic compensator are obtained, the observer-based dynamic output feedback control law is used. Therefore, a CPG at the controller side is constructed as

$$\mathbf{u}_{t+f|t-b} = N\hat{\mathbf{z}}_{t+f|t-b} + MC\hat{\mathbf{x}}_{t+f|t-b}, \quad (7)$$

that is, the CPG at the controller side is to produce control predictions calculated by (7).

For coping with data packet dropout and disorder, a information mechanism is employed below. One output data set $[\mathbf{y}_t \ \mathbf{y}_{t-1} \ \dots \ \mathbf{y}_{t-nd}]$ at time t is transmitted from the sensor to the controller if it happens that the output data in the feedback channel drops. In a similar way, a set of control predictions $[\mathbf{u}_{t+f|t-b} \ \mathbf{u}_{t+f-1|t-b-1} \ \dots \ \mathbf{u}_{t+nf|t-b-nd}]$ at time t , which are calculated by (7), will be sent from the controller to the actuator in case of the control data loss in the forward channel. To avoid data packet disorder, two data buffers are prepared to reorder the received data and keep the latest data. One is set at the actuator side for the control data and the other is set at the controller side for the measured output data. Thus, under Assumption iv), the latest output data at the controller and the latest control data at the actuator are always available.

The control input at the actuator is taken via

$$\mathbf{u}_t = \mathbf{u}_{t|t-b-f} = N\hat{\mathbf{z}}_{t|t-b-f} + MC\hat{\mathbf{x}}_{t|t-b-f} \quad (8)$$

which implies that the network delay compensator selects $\mathbf{u}_{t|t-b-f}$ from the latest control predictions in the data buffer on the actuator side. Without considering the modeling uncertainties, it is illustrated in [13] that the control performance of a

closed-loop NPC system is very similar to a conventional control system without the network.

3. Robustness Analysis of NPC Systems

Robustness analysis is a very important part of the control system design. Normally, the analysis of NCS is much harder than that of a conventional control system without network. The robustness issue of a closed-loop NPC system is discussed hereinafter.

Let $\varepsilon_t = [\mathbf{x}_t^T \quad \mathbf{z}_t^T]^T$. Combining (1) and (3) leads to

$$\begin{aligned}\varepsilon_{t+1} &= (A' + \Delta A')\varepsilon_t + (B' + \Delta B')u_t \\ y_t &= (C' + \Delta C')\varepsilon_t\end{aligned}\quad (9)$$

where

$$\begin{aligned}A' &= \begin{bmatrix} A & 0 \\ HC & F \end{bmatrix}, B' = \begin{bmatrix} B \\ 0 \end{bmatrix}, C' = [C \quad 0], \\ \Delta A' &= \begin{bmatrix} \Delta A & 0 \\ H\Delta C & 0 \end{bmatrix}, \Delta B' = \begin{bmatrix} \Delta B \\ 0 \end{bmatrix}, \Delta C' = [\Delta C \quad 0]\end{aligned}$$

Combining with (4) and (6), the one-step prediction for the controlled plant and dynamic feedback compensator can be rewritten as

$$\begin{aligned}\hat{\varepsilon}_{t-b+1|t-b} &= A'\hat{\varepsilon}_{t-b|t-b-1} + B'u_{t-b} + L'(y_{t-b} - \hat{y}_{t-b}) \\ \hat{y}_{t-b} &= C'\hat{\varepsilon}_{t-b|t-b-1}\end{aligned}\quad (10)$$

where

$$L' = \begin{bmatrix} L \\ 0 \end{bmatrix}, \hat{\varepsilon}_{t-b|t-b-1} = \begin{bmatrix} \hat{x}_{t-b|t-b-1} \\ \hat{z}_{t-b|t-b-1} \end{bmatrix}$$

When the time is shifted for b steps, it is obtained from (10) that

$$\begin{aligned}\hat{\varepsilon}_{t+1|t} &= A'\hat{\varepsilon}_{t|t-1} + B'u_t + L'(y_t - \hat{y}_t) \\ \hat{y}_t &= C'\hat{\varepsilon}_{t|t-1}\end{aligned}\quad (11)$$

Define the error vector be

$$e_t = \varepsilon_t - \hat{\varepsilon}_{t|t-1}\quad (12)$$

So, (11) is changed into

$$\begin{aligned}\hat{\varepsilon}_{t+1|t} &= A'\hat{\varepsilon}_{t|t-1} + B'u_t + L'C'e_t + L'\Delta C'\varepsilon_t \\ \hat{y}_t &= C'\hat{\varepsilon}_{t|t-1}\end{aligned}\quad (13)$$

From the prediction equation (10), it can be obtained that

$$\hat{\varepsilon}_{t+f|t-b} = (A')^{b+f-1}\hat{\varepsilon}_{t-b+1|t-b} + \sum_{i=2}^{b+f} (A')^{b+f-i} B'u_{t+i-b-1}\quad (14)$$

Similarly

$$\begin{aligned}\hat{\varepsilon}_{t+f|t-b+1} &= (A')^{b+f-1}\hat{\varepsilon}_{t-b+1|t-b} + \sum_{i=2}^{b+f} (A')^{b+f-i} B'u_{t+i-b-1} + (A')^{b+f-2} L'C'e_{t-b+1} \\ &\quad + (A')^{b+f-2} L'\Delta C'\varepsilon_{t-b+1}\end{aligned}\quad (15)$$

which uses (11). Subtracting (15) from (14) leads to the following:

$$\hat{\varepsilon}_{t+f|t-b} = \hat{\varepsilon}_{t+f|t-b+1} - (A')^{b+f-2} L'C'e_{t-b+1} - (A')^{b+f-2} L'\Delta C'\varepsilon_{t-b+1}\quad (16)$$

If the equation (16) is recursively used, it is derived that

$$\hat{\varepsilon}_{t+f|t-b} = \hat{\varepsilon}_{t+f|t+f-1} - \sum_{i=0}^{b+f-2} (A')^i L'(C'e_{t+f-i-1} + \Delta C'\varepsilon_{t+f-i-1})\quad (17)$$

Replacing $t+f$ by t yields

$$\hat{\varepsilon}_{t|t-b-f} = \hat{\varepsilon}_{t|t-1} - \sum_{i=0}^{b+f-2} (A')^i L'(C'e_{t-i-1} + \Delta C'\varepsilon_{t-i-1}) \quad (18)$$

Then, the expression of the control input (8) is

$$\begin{aligned} u_t &= u_{t|t-b-f} \\ &= G\hat{\varepsilon}_{t|t-b-f} \\ &= G\left(\hat{\varepsilon}_{t|t-1} - \sum_{i=0}^{b+f-2} (A')^i L'(C'e_{t-i-1} + \Delta C'\varepsilon_{t-i-1})\right) \\ &= G\left(\varepsilon_t - e_t - \sum_{i=0}^{b+f-2} (A')^i L'(C'e_{t-i-1} + \Delta C'\varepsilon_{t-i-1})\right) \end{aligned} \quad (19)$$

where $G = [MCN] \in \mathbf{R}^{m \times (n+q)}$. Combing (9), (13) and (19), the error equation can be rewritten as

$$\begin{aligned} e_{t+1} &= \varepsilon_{t+1} - \hat{\varepsilon}_{t+1|t} \\ &= (A' - L'C')e_t + (\Delta A' - L'\Delta C')\varepsilon_t + \Delta B'u_t \\ &= (A' - L'C')e_t + (\Delta A' + \Delta B'G - L'\Delta C')\varepsilon_t \\ &\quad - \Delta B'G\left(e_t + \sum_{i=0}^{b+f-2} (A')^i L'(C'e_{t-i-1} + \Delta C'\varepsilon_{t-i-1})\right) \end{aligned} \quad (20)$$

Using (19), the equation (9) can be rewritten as

$$\begin{aligned} \varepsilon_{t+1} &= (A' + \Delta A')\varepsilon_t + (B' + \Delta B')G\left(\varepsilon_t - e_t - \sum_{i=0}^{b+f-2} (A')^i L'(C'e_{t-i-1} + \Delta C'\varepsilon_{t-i-1})\right) \\ &= (A' + B'G)\varepsilon_t + (\Delta A' + \Delta B'G)\varepsilon_t - B'G\left(e_t + \sum_{i=0}^{b+f-2} (A')^i L'C'e_{t-i-1}\right) \\ &\quad - \Delta B'G\left(e_t + \sum_{i=0}^{b+f-2} (A')^i L'C'e_{t-i-1}\right) - (B' + \Delta B')G\left(\sum_{i=0}^{b+f-2} (A')^i L'\Delta C'e_{t-i-1}\right) \end{aligned} \quad (21)$$

Let $v_t = [\varepsilon_t^T \varepsilon_{t-1}^T \cdots \varepsilon_{t-b-f+1}^T]^T$ and $E_t = [e_t^T e_{t-1}^T \cdots e_{t-b-f+1}^T]^T$. Combing (20) and (21) gives the following equation:

$$\begin{bmatrix} v_{t+1} \\ E_{t+1} \end{bmatrix} = \left(\begin{bmatrix} T_{11} & T_{12} \\ 0 & T_{22} \end{bmatrix} + \begin{bmatrix} \Delta T_{11} & \Delta T_{12} \\ \Delta T_{21} & \Delta T_{22} \end{bmatrix} \right) \begin{bmatrix} v_t \\ E_t \end{bmatrix} \quad (22)$$

where

and ΔT_{11} , ΔT_{12} , ΔT_{21} , ΔT_{22} are expressed in (23)-(26).

$$\begin{aligned} \Delta T_{11} &= \begin{bmatrix} \Delta A' + \\ \\ \\ \end{bmatrix} & T_{11} &= \begin{bmatrix} A' + B'G & & & \\ & I & 0 & \\ & & \ddots & \ddots \\ & & & I & 0 \end{bmatrix}, \\ \Delta T_{22} &= \begin{bmatrix} -\Delta E \\ \\ \\ \end{bmatrix} & T_{22} &= \begin{bmatrix} A' - L'C' & & & \\ & I & 0 & \\ & & \ddots & \ddots \\ & & & I & 0 \end{bmatrix}, \\ \Delta T_{21} &= \begin{bmatrix} \Delta A' + \\ \\ \\ \end{bmatrix} & & & \\ \Delta T_{12} = \Delta T_{22} & T_{12} &= \begin{bmatrix} -B'G & -B'GL'C' & -B'GA'L'C' & \cdots & -B'G(A')^{b+f-2}L'C' \\ & & & & \mathbf{0} \end{bmatrix} \end{aligned}$$

Consequently, the stability analysis of the closed-loop NPC system with uncertainties is changed into a robust system with the normal form as:

$$S_{t+1} = (T + \Delta T)S_t, \quad (27)$$

where

$$S_t = \begin{bmatrix} v_t \\ E_t \end{bmatrix}, \quad T = \begin{bmatrix} T_{11} & T_{12} \\ 0 & T_{22} \end{bmatrix}, \quad \Delta T = \begin{bmatrix} \Delta T_{11} & \Delta T_{12} \\ \Delta T_{21} & \Delta T_{22} \end{bmatrix}$$

The robustness analysis on the standard system (27) is studied by many scholars and mature results can be followed, e.g., [15]. Because the uncertainty ΔT is related to the dynamic feedback gain N, M, F, H and the observer gain L , the design of the gains N, M, F, H and L must also match the conditions of the uncertainty ΔT . Since it is very hard to give an analytical solution to N, M, F, H and L in most NPC systems, it means that the above robust controller design problem can be solved numerically. Furthermore, if $\Delta A=0, \Delta B=0$ and $\Delta C=0$, that is, no uncertainty exists, the closed-loop stability of the NPC system is only dependent on the eigenstructure of two matrices $(A'+B'G)$ and $(A'-L'C')$, that is, if and only if their eigenvalues lie in the unit circle, the closed-loop system is stable.

4. Simulation Results

In order to analysis the robustness of an NPC system, a DC motor control system in [14] is considered. The system input is the voltage (V) and the system output are the speed (rpm). The plant model for the DC motor at a sampling period of 0.05 s was

$$G_p(z^{-1}) = \frac{A(z^{-1})}{B(z^{-1})} = \frac{18.3296z^{-1} + 65.2067z^{-2}}{1 - 0.3383z^{-1} - 0.3737z^{-2}}$$

The above transfer function model is converted into a state-space form as follows:

$$A = \begin{bmatrix} 0.8034 & 0 \\ 0 & -0.4651 \end{bmatrix}, \quad B = \begin{bmatrix} 8.09 \\ -6.955 \end{bmatrix}, \\ C = [7.789 \quad 6.424].$$

When the desired poles for the state observer are chosen as [0.16, 0.48], the observer gain is designed as $L = [0.0211 \quad -0.0725]^T$, which can use standard observer design approaches. The dynamic output feedback control gains are designed to be $F = 1; H = 0.0017; M = 0.0024$ and $N = 0.5$ by pole assignment technique, when the desired poles of the closed-loop system are set to $[-0.3771; 0.8357 + 0.1567i; 0.8357 - 0.1567i]$.

The stability and control performance of NPC systems are illustrated in the following three cases.

Case I) Conventional control experiment without considering communication delay compensation and uncertainty. It is assumed two three-step communication delays exist in the forward and feedback channel respectively, and there exists also no perturbation in the system modelling. In this example, without compensating the delay, but a normal dynamic output feedback controller is employed. Then, the conventional controller without NPC scheme is described as

$$z_{t-2} = Fz_{t-3} + Hy_{t-6} \\ u_t = Nz_{t-3} + My_{t-6}$$

The simulation results, as given in Figure 2, indicate that the speed output is divergent.

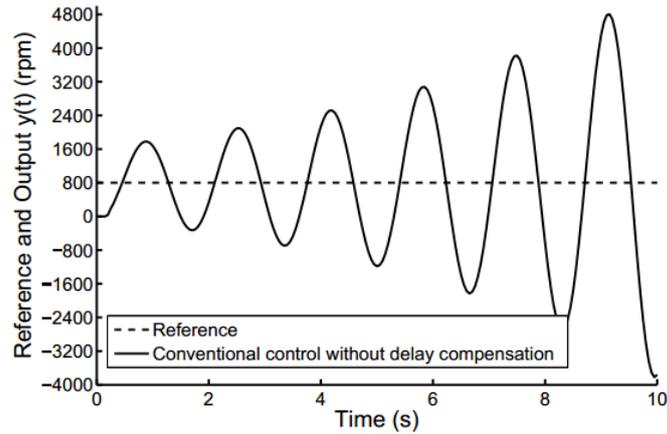


Figure 2. Conventional Control without Delay Compensation

Case II) NPC without uncertainty. The NPC strategy is applied to make a compensation for communication delay and data dropout. The characteristic parameters for the communication network shared in the forward and feedback channels are $n_f = 3$, $n_b = 3$ and $n_d = 1$. As given in Figure 3, it has been demonstrated that the close-loop NPC system is stable.

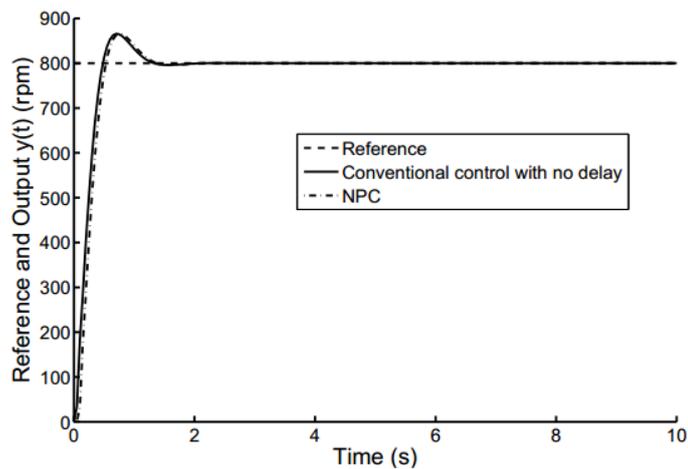


Figure 3. NPC without Uncertainties

Case III) NPC with uncertainties. In this simulation, the network characteristics are the same as the one in **Case II**. What differs from **Case II** is that the uncertainties are inserted into the plant model. The perturbed system matrices are $1.1A$, $1.1B$ and $0.9C$, respectively, that is, the system dynamics matrices $\pm 10\%$ uncertainty. The simulation results, as shown in Figure 4, exhibit good robustness of the NPC system.

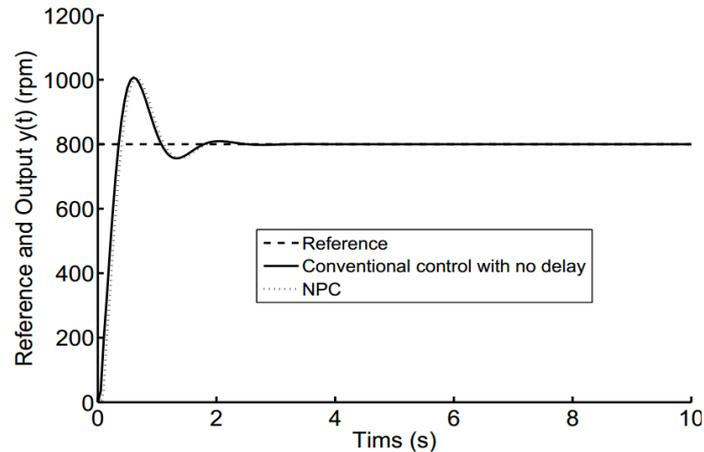


Figure 4. NPC Systems with Uncertainties

5. Conclusions

In summary, the problem of dynamic output feedback controller design for NPC systems with uncertainties has been addressed. The dynamic output feedback controller together with the NPC scheme has been designed for making compensation for communication delay and data dropout. On the basis of robustness analysis, the communication delay can be compensated in an active way and the control performance of NPC system cannot be influenced by the communication delay if there are no uncertainties in plants. Considering uncertainties, the stable controller design of the NPC system has been achieved using standard robust control techniques. The efficiency is illustrated with three simulations. This work will be applied to network-based motion control of stepping motor for laser beam alignment system.

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